“YANGI O‘ZBEKISTON” UNIVERSITETI HUZURIDAGI ILG‘OR TADQIQOTLAR INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSC.03/07.07.2025.FM/T.192.01 RAQAMLI ILMIY KENGASH

ASTRONOMIYA INSTITUTI

SARIQULOV FURQAT SHERALI O‘G‘LI

ASTROFIZIK KUZATUVLAR ORQALI MUQOBIL GRAVITATSIYA DOIRASIDA QORA O‘RALAR XOSSALARINI TEKSHIRISH

01.03.01 – Astronomiya

Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi AVTOREFERATI

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Sariqulov Furqat Sherali o‘g‘li
Astrofizik kuzatuvlar orqali muqobil gravitatsiya doirasida qora o‘ralar xossalarini tekshirish.....3

Sarikulov Furkat Sherali ugli
Testing black hole nature in alternative gravity through astrophysical observations.....23

Сарикулов Фуркат Шерали ўғли
Тестирование природы чёрных дыр в альтернативных теориях через астрофизические наблюдения.....41

E’lon qilingan ishlar ro‘yxati
List of published works
Список опубликованных работ.....47

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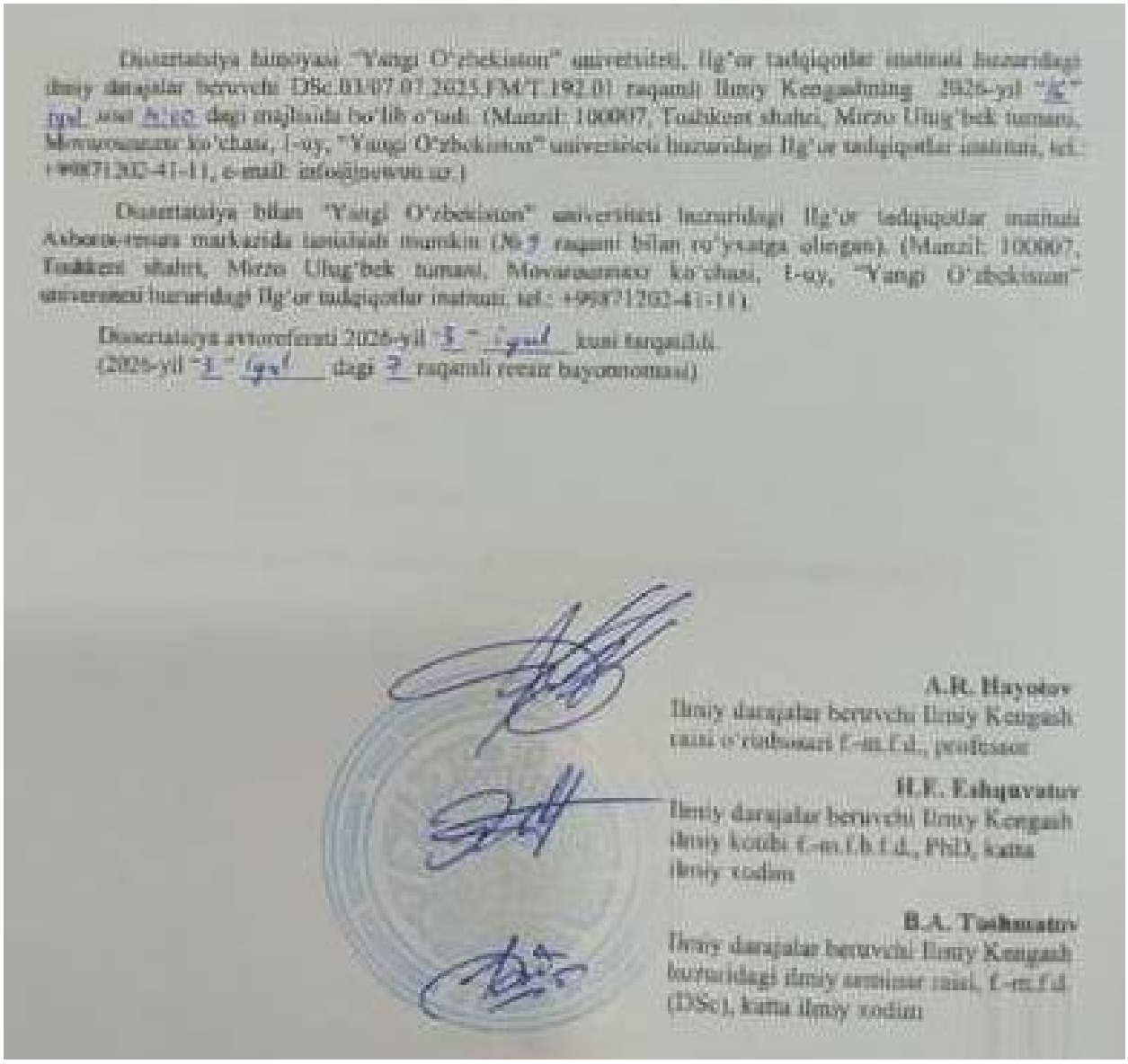
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KIRISH (PhD dissertatsiya annotatsiyasi)

Dissertatsiyaning dolzarbligi va talab yuqoriligi. Gravitatsiyaning muqobil nazariyalarida qora o‘ralarni o‘rganish zamonaviy astrofizika va nazariy fizikaning juda dolzarb va tez rivojlanayotgan sohasidir. Hodisalar gorizonti teleskopidan (EHT) olingan kashfiyot kuzatuvlari, shu jumladan M87* va Sgr A* supermassiv qora o‘ralarining birinchi tasvirlari bilan umumiy nisbiylik nazariyasini ekstremal gravitatsion rejimlarda tekshirish zarurati tug‘ildi. Klassik qora o‘ra yechimlaridan og‘ishlarning kashf etilishi kvant gravitatsiyasi, qorong‘u materiya va nochiziqli elektrodinamika (NED) haqida muhim tushunchalar berishi mumkin.

Ushbu dissertatsiya zamonaviy fizikaning asosiy masalalarini hal qiluvchi 4 o’lchovli Eynshteyn-Gauss-Bonnet (EGB) gravitatsiya, nochiziqli elektrodinamika nazariyasi kabi kengaytirilgan gravitatsiya modellarida yangi qora o‘ra yechimlarini o‘rganadi. Umumiy nisbiylik nazariyasi muqobillarini tekshirish: O‘zgartirilgan gravitatsiya nazariyalari (masalan, EGB, NED) qora o‘ralarning umumiy nisibiylik nazariyasidan farq qiladigan xususiyatlarini bashorat qiladi, bu esa ularni EHT kuzatuvlari bilan tekshirish imkonini beradi. Qorong‘u materiya ta’siri: qora o‘ralar va mukammal suyuqlik qorong‘u materiyasi o‘rtasidagi o‘zaro ta’sir qora o‘ra soyalariga ta’sir ko‘rsatib, galaktika markazlari yaqinidagi qorong‘u materiya taqsimotlarini tekshirishning yangi usulini taklif etadi. Metodologiyani rivojlantirish: Qora o‘ra soyasi tahliliga asoslangan parametrlarni baholash yondashuvi takomillashtirildi va yanada rivojlantirildi, bu esa qora o‘ra hamda uni o‘rab turgan modda xossalarini aniqlashning aniqligi va ishonchliligini oshirdi.

Bunday tadqiqotlarga bo‘lgan ehtiyoj yaqin kelajakda ishga tushiriladigan yuqori aniqlikdagi kuzatuv qurilmalari (masalan, keyingi avlod EHT, LISA) bilan yanada kuchaymoqda, bu esa qora o‘ra parametrlari va gravitatsiya modellarini yanada aniqroq cheklash imkonini beradi. Qora o‘ra soyasini tahlil qilishni kuzatuv ma’lumotlari bilan uyg‘unlashtirish orqali ushbu ish umumiy nisbiylik (GR) va muqobil nazariyalarni farqlash uchun yagona yondashuvni taklif etadi va kelajakdagi astrofizik sinovlar hamda nazariy rivojlanishlar uchun muhim ahamiyat kasb etadi. Shunday qilib, ushbu dissertatsiya fundamental fizikaning dolzarb masalalarini hal qilish bilan birga, eng so‘nggi kuzatuv ma’lumotlarini talqin qilishga hissa qo‘shadi.

Bundan tashqari, mazkur dissertatsiya tadqiqoti O‘zbekiston Respublikasi va xalqaro ilmiy hamjamiyatning ustuvor ilmiy yo‘nalishlariga mos keladi. Tadqiqot natijalari O‘zbekiston Respublikasining 2019-yil 29-oktabrdagi O‘RQ–576-son “Ilm-fan va ilmiy faoliyat to‘g‘risida”gi Qonuni, O‘zbekiston Respublikasi Prezidentining 2020-yil 29-oktabrdagi PF–6097-son “Ilm-fanni 2030-yilgacha rivojlantirish konsepsiyasini tasdiqlash to‘g‘risida”gi Farmoni, 2021-yil 19-martdagi PQ–5032-son “Fizika sohasidagi ta’lim sifatini oshirish va ilmiy tadqiqotlarni rivojlantirish chora-tadbirlari to‘g‘risida”gi qarori hamda mazkur faoliyatga tegishli boshqa me’yoriy-huquqiy hujjatlarda belgilangan vazifalarni amalga oshirishga muayyan darajada xizmat qiladi.

Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi. Dissertatsiya tadqiqoti respublika fan va texnologiyalari rivojlanishining II. «Energetika, energiya va resurs tejamkorligi» ustuvor yo’nalishi doirasida bajarilgan.

Muammoning o‘rganilganlik darajasi. Gravitatsiyaning muqobil nazariyalarida qora o‘ralarni o‘rganish umumiy nisbiylik nazariyasining dastlabki rivojlanishidan beri faol tadqiqot yo‘nalishi bo‘lib kelgan. Schwarzschild va Kerr metrikalari kabi klassik qora o‘ra yechimlari keng tahlil qilingan, ammo yuqori egrilik tuzatmalari (Boulware, Glavan, Lin, Fernandes, Kumar, Ghosh, Lu, Pang, Hennigar), nochiziqli elektrodinamika (Born, Infeld, Ayon-Beato, Garcia, Bronnikov, Dymnikova, Balart) va qorong‘u materiya (Haroon, Jusufi, Hou, Narzilloev, Atamurotov) o‘zaro ta’sirlari tufayli o‘zgartirishlar kamroq o‘rganilgan. Glavan va Lin (2020) ning 4 o’lchovli Eynshteyn-Gauss-Bonnet gravitatsiya va Born-Infeld tipidagi nochiziqli elektrodinamika modellari bo‘yicha so‘nggi ishlari nazariy asoslarni kengaytirdi, ammo Hodisalar gorizonti teleskopi kuzatuv cheklovlari faqat yaqinda ushbu modellarni empirik sinovdan o‘tkazishga imkon berdi (Synge, Bardin, Hioki, Maeda, Iogansen, Psaltis, Event Horizon Telescope Collaboration, Vagnozzi). Modifikatsiyalangan gravitatsiyada qora o‘ra yechimlarini olishda sezilarli yutuqlarga erishilgan bo‘lsada, nazariy bashoratlar bilan astrofizik kuzatuvlar o‘rtasidagi bog‘liqlik, ayniqsa qora o‘ra soyasi va parametrlarni baholash usullari bilan bog‘liq jihatlar, yanada takomillashtirilishini talab etadi.

Adabiyotlardagi yutuqlarga qaramay, asosiy bo‘shliqlar saqlanib qolmoqda. Aksariyat tadqiqotlar statik yoki sferik simmetrik qora o‘ralarga qaratilgan, astrofizik qora o‘ralar uchun ko‘proq ahamiyatga ega bo‘lgan aylanuvchi yechimlar esa muqobil gravitatsiyada kamroq o‘rganilgan. Bundan tashqari, qorong‘u materiyaning qora o‘ra soyalariga ta’siri soddalashtirilgan ssenariylarda o‘rganilgan, ammo mukammal suyuqlik qorong‘u materiyasi yoki Power-Yang-Mills maydonlarida qora o‘ra dinamikasining har tomonlama tahlili to‘liq emas. Ushbu dissertatsiya muqobil gravitatsiya nazariyalarida aylanuvchi qora o‘ralarni tizimli tahlil qilish, nazariy bashoratlarni EHT kuzatuvlari bilan bog‘lash va umumiy nisbiylik nazariyasidan og‘ishlarga yangi cheklovlar berish orqali ushbu bo‘shliqlarni bartaraf etadi. Bularning barchasini inobatga olgan holda, biz nochiziqli elektrodinamika, mukammal suyuqlik qorong‘u modda bilan o’ralgan to‘rt o‘lchamli Einstein–Gauss–Bonnet gravitatsiyasi hamda darajali Yang–Mills maydoni doirasidagi muqobil gravitatsiya nazariyalarida qora o‘ralarni o‘rganishni maqsad qildik. Ish mavjud tadqiqotlarga asoslanib, ularni yanada real, kuzatuv orqali tekshiriladigan ssenariylarga kengaytiradi.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya Innovatsion rivojlanish vazirligi tomonidan moliyalashtirilgan F-FA-2021-510 "Modifikatsiyalangan gravitatsiyada neytron yulduzlar yadroviy materiyasini tadqiq qilish" ilmiy loyihasi doirasida bajarilgan.

Tadqiqotning maqsadi: Qora o‘ra soyalari va hodisalar gorizonti teleskopi (EHT) kuzatuv cheklovlari yordamida muqobil gravitatsiya nazariyalarini sinab ko‘rishdir.

Tadqiqotning vazifalari:

nochiziqli elektrodinamika (NED) va to’rt o’lchovli Eynshteyn-Gauss-Bonnet (4D EGB) gravitatsiyalarida yangi qora o‘ra yechimlarini olish;

Nyuman-Yanis algoritmi yordamida aylanuvchi holatlarga statik yechimlarni kengaytirish va gorizont strukturalarini tahlil qilish;

noturg‘un doiraviy orbitalar va soya chegaralarini aniqlash uchun fotonlar harakati va effektiv potensiallarni o‘rganish.

turli gravitatsiya modellari uchun qora o’ra soyalarini hisoblash va ularni EHT kuzatuvlari bilan taqqoslash.

qora o‘ra parametrlarini (spin, zaryad, bog‘lanish konstantalari) soya kuzatuvlaridan (yuza, tekislik) baholash.

umumiy nisbiylik nazariyasidan chetga chiqishlarni cheklash uchun nazariy bashoratlarni EHT ma’lumotlari bilan taqqoslash.

Tadqiqotning obyekti: nochiziqli elektrodinamikada aylanuvchi va statik qora o‘ralar; Mukammal suyuqlik qorong‘u moddasi bilan o‘ralgan 4D Eynshteyn-Gauss-Bonnet qora o‘ralari; o‘zgartirilgan gravitatsiya maydonidagi darajali Yang-Mills qora o‘ralari; M87* va Sgr A* supermassiv qora o‘ralar kuzatuv test holatlari sifatida.

Tadqiqot predmeti: muqobil gravitatsion qora o‘ralarning metrik yechimlari va gorizont strukturalari; o‘zgartirilgan fazo-vaqtlarda foton sferalari va beqaror aylana orbitalari; qora o‘ra soyasining morfologiyasi va kuzatiladigan parametrlari; parametrlarni baholash; Hodisalar gorizonti teleskopi ma’lumotlaridan foydalangan holda usullar; astrofizik kuzatuvlardan olingan o‘zgartirilgan gravitatsiya parametrlariga cheklovlar; turli muqobil gravitatsiya ssenariylarining qiyosiy tahlili.

Tadqiqot usullari: nochiziqli elektrodinamika va o‘zgartirilgan gravitatsion maydon tenglamalarini qo‘llash uchun analitik modellashtirish usullari; Eynshteyn-Gauss-Bonnet gravitatsion modellari yordamida metrik yechimlarni keltirib chiqarish; aylanuvchi qora o‘ra yechimlarini yaratish uchun Nyuman-Yanis algoritmini amalga oshirishning sonli usullari; Gamilton-Yakobi formalizmi sinov zarralari harakati uchun samarali potensiallarni hisoblash; qora o‘ra soyasini qayta tiklashning standart usullari; turli gravitatsiya modellarini tizimli baholash uchun qiyosiy tahlil.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

Birinchi marta, 4D Eynshteyn-Gauss-Bonnet gravitatsiyasi mukammal suyuq qorong‘u materiya va darajali Yang-Mills maydoni bilan bog‘langan holatda ilgari noma’lum bo’lgan aylanayotgan qora o‘ra yechimlari keltirib chiqarildi. Bu natijalar klassik Kerr–Newman metrikalaridan kengaygan holda olinadi.

Mukammal suyuqlik qorong‘u modda bilan o‘ralgan 4D Einstein–Gauss–Bonnet qora o’ralari uchun parametr baholashlari amalga oshirildi.

Qora o‘ra soyalarida qorong‘u materiyaning noyob kuzatuv belgilari aniqlandi, bu esa yuqori aniqlikdagi tasvirlar orqali qorong‘u materiya zichligi profillarini cheklash mumkinligini ko‘rsatadi.

Tadqiqotning amaliy natijalari quyidagilarni o‘z ichiga oladi:

Hodisalar gorizonti teleskopi ma’lumotlaridan foydalangan holda birinchi marta qora o‘ra spini (a), NED zaryadi (q) va Gauss-Bonnet bog‘lanishi (α) uchun miqdoriy cheklovlar taqdim etildi, bu esa kelajakdagi kuzatuvlar uchun sinovdan o‘tkaziladigan bashoratlarni taklif qiladi.

U muqobil gravitatsiya ssenariylarida qora o‘ra soyalarini modellashtirish va parametrlarni baholash uchun ochiq kirishli raqamli kodlarga moslashtirildi, bu esa keng ilmiy hamjamiyat tomonidan tekshirish imkonini beradi.

Yangi avlod EHT uchun o‘ziga xos soya buzilishlari bashorat qilingan bo‘lib, ular umumiy nisbiylik nazariyasi va muqobil gravitatsiya nazariyalari o‘rtasidagi farqni aniqlash uchun kuzatuv maqsadlariga ustuvorlik beradi.

Tadqiqot natijalarining ishonchliligi. Matematik fizika, hisoblash matematikasi va relyativistik astrofizikaning zamonaviy sinalgan usullarini qo‘llash bilan ta’minlangan. Barcha olingan qora o‘ra yechimlari o‘zlarining o‘zgartirilgan tortishish nazariyalarining asosiy maydon tenglamalarini qondiradi, bu esa matematik qat’iylik va o‘rnatilgan tizimlarga muvofiqlikni ta’minlaydi. Muhim natijalar (masalan, soya radiuslari) mustaqil analitik va sonli usullar (Gamilton-Yakobi formalizmi) yordamida hisoblab chiqilgan, moslik va barqarorlikni tasdiqlaydi. Parametr cheklovlari Hodisalar gorizonti teleskopining (M87*, Sgr A*) empirik ma’lumotlariga mos keladi va umumiy nisbiylik nazariyasi bashoratlaridan og‘ishlar joriy kuzatuv noaniqliklari doirasida qoladi. Taxminlar (masalan, asimptotik tekislik, o‘ziga xos qorong‘u materiya profillari) va ularning potentsial ta’siri aniq ko‘rib chiqiladi va qo‘llanilish doirasini belgilaydi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati.

Olingan natijalar qora o‘ra yechimlarini kuzatuv ma’lumotlari bilan taqqoslash orqali muqobil gravitatsiya modellarining (masalan, 4D EGB, NED) qat’iy sinovlarini ta’minlaydi, bu esa umumiy nisbiylik nazariyasiga kengaytmalarni tasdiqlash yoki cheklashga yordam beradi.

Natijalar qora o‘ra soyalari va dinamikasi qorong‘u materiyaning mavjudligi va tarqalishini qanday aniqlay olishini ko‘rsatadi, bu esa koinotning ushbu tutqich bermas tarkibiy qismini o‘rganish uchun yangi astrofizik vositani taqdim etadi.

Tadqiqot hozirgi va kelajakdagi loyihalar uchun aniq, kuzatuv orqali tekshiriladigan bashoratlarni (soya shakllari) taqdim etadi, bu esa umumiy nisbiylik nazariyasidan og‘ishlarni aniqlash bo‘yicha sa’y-harakatlarga yetaklaydi.

Tahlil modifikatsiyalangan gravitatsiya doirasida qora o‘ra soyasi va parametrlarini baholash uchun ochiq kirishli analitik yondashuvlar hamda raqamli kodlarni ishlab chiqadi, bu esa kengroq tadqiqot olib borish va hamkorlik asosida tekshirish imkonini beradi.

Tadqiqot natijalarining qo‘llanilishi: Modifikatsiyalangan gravitatsiya nazariyalarida qora o‘ralar atrofidagi zarrachalar dinamikasini o‘rganish natijalari quyidagicha qo‘llanilgan:

"Furkat Sarikulov, Farruh Atamurotov, Ahmadjon Abdujabbarov, Bobomurat Ahmedov, Shadow of the Kerr-like black hole / / The European Physical Journal C, Vol. 82 (2022), 9, 771 (Web of Science: IF=4.2)", nomli ilmiy ishda chop etilgan va F.Sarikulovning doktorlik (PhD) dissertatsiyasida keltirilgan nazariy tadqiqot natijalari va usullaridan Fudan universiteti tomonidan qo‘llab-quvvatlanadigan dasturlar doirasida foydalanilgan (prof. Kozimo Bambi tomonidan xat).

Tadqiqot natijalarining aprobatsiyasi. Mazkur tadqiqotning asosiy natijalari 1 ta xalqaro va 1 ta Respublika miqyosidagi ilmiy-amaliy konferensiyalarda ma’ruzalar qilinib, tegishli muhokamalardan o‘tkazilgan.

Tadqiqot natijalarining e’lon qilinganligi. Tadqiqot natijalariga asosan 14 ta ilmiy nashr chop etilgan bo‘lib, ularning barchasi xorijiy jurnallarda e’lon qilingan maqolalardir.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish, uch bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan iborat. Dissertatsiya hajmi 83 bet.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi, muammoning o‘rganilganlik darajasi, dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi, tadqiqotning maqsadi, vazifalari, obyekti, predmeti, usullari, ilmiy yangiligi, amaliy natijasi, natijalarning ishonchliligi, ilmiy va amaliy ahamiyati, natijalarning amaliyotga joriy etilishi, aprobatsiyasi, natijalarning e’lon qilinganligi hamda dissertatsiyaning tuzilishi va hajmi ko‘rsatilgan.

Dissertatsiyaning "**Kengaytirilgan gravitatsiya modellarida qora o‘ralarning yangi yechimlari va ular atrofidagi fotonlar harakati**" deb nomlangan birinchi bobida kengaytirilgan gravitatsiya nazariyalari doirasida qora o‘ralar yechimlarining yangi sinflari olingan va tahlil qilingan. Biz Eynshteynning umumiy nisbiylik nazariyasi va NED modeli o‘rtasidagi bog‘liqlikni quyidagi amal orqali aniqlashdan boshlaymiz:

$$S = \int dx^4 \sqrt{-g} \left[\frac{R}{16\pi G} + \mathcal{L}(\mathcal{F}) \right] \tag{1}$$

bu yerda metrik g tenzorning determinantini ifodalaydi, R Richchi skalyarini bildiradi, G Nyutonning gravitatsion doimiysidir va NED modelini aniqlaydigan Lagranjian quyidagicha ifodalanadi

$$\mathcal{L}(\mathcal{F}) = \frac{-4b\mathcal{F}}{\left(\sqrt{b} + \sqrt{b - 2\sqrt{-\frac{\mathcal{F}}{2}}} \right)^2}. \tag{2}$$

Bu kontekstda $4\mathcal{F} = F_{\mu\nu}F^{\mu\nu} = 2(B^2 - E^2)$ Makswell invariantini ifodalaydi, va *b* nochiziqlilik parametrini bildiradi. Elektr maydon *E(r)* magnit maydon *B=0* deb olinib, $\nabla_\mu \left(F^{\mu\nu}\frac{\partial \mathcal{L}}{\partial \mathcal{F}}\right) = 0$ tenglamani yechish orqali aniqlanadi. Elektr maydoni *E(r)* quyidagicha aniqlanadi:

$$E(r) = \frac{bq(2br^2 + q)}{2(br^2 + q)^2} \tag{3}$$

bu yerda *q* elektr zaryadini bildiradi. Statik va sferik simmetrik metrika uchun (1) ta'sirning variatsiyasi va Eynshteyn maydon tenglamalarining yechimi

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + sin^2\,\theta d\phi^2\right) \tag{4}$$

quyidagi funksiya orqali qora o‘ra yechimini beradi

$$f(r) = 1 - \frac{2M}{r} + \frac{\pi q\sqrt{bq}}{2r} - \frac{q\sqrt{bq}}{r}tan^{-1}\left(\frac{\sqrt{b}r}{\sqrt{q}}\right), \tag{5}$$

bu yerda *M* qora o‘raning massasini ifodalaydi.

Nochiziqli elektrodinamika doirasida electromagnit maydonning nochiziqliligi sababli u bilan o‘zaro ta'sirlashadi va oddiy null geodezikalarga amal qilmaydi. Shu sababli, fotonlarning harakatini umumiy nisbiylik va NED bilan bog‘langan qora tuynuk fazo-vaqtida ko‘rib chiqish zarur. Faqat elektr maydon konfiguratsiyasida elektromagnit tenzorning nolga teng bo‘lmagan yagona komponenti elektr maydon kuchlanganligidir va elektromagnit invariant quyidagi ko‘rinishga ega bo‘ladi:

$$F = -\frac{1}{2}E^2(r) \tag{6}$$

bu yerda *E(r)* (3)-tenglama bilan aniqlanadi. Ushbu holatda fotonlarning tarqalishini belgilovchi effektiv metrik (chiziqli element) quyidagicha yozilishi mumkin:

$$ds^2_{eff} = -\frac{f(r)}{\mathcal{L}_F}dt^2 + \frac{dr^2}{f(r)\mathcal{L}_F} + \frac{r^2}{\Phi(r)}\left(d\theta^2 + sin^2\,\theta d\phi^2\right) \tag{7}$$

bu yerda *Φ(r)* funksiyasi quyidagicha aniqlanadi:

$$\Phi(r) = \mathcal{L}_F - 2F\mathcal{L}_{FF} \tag{8}$$

Effektiv geometriyada null geodezikalar ikkita saqlanuvchi kattalikka ega, ya'ni fotonning energiyasi *E* va burchak impulsi *L*. Harakatni ekvatorial tekislik bilan (*θ=π/2*) cheklab, radial harakat tenglamasini quyidagicha yozish mumkin:

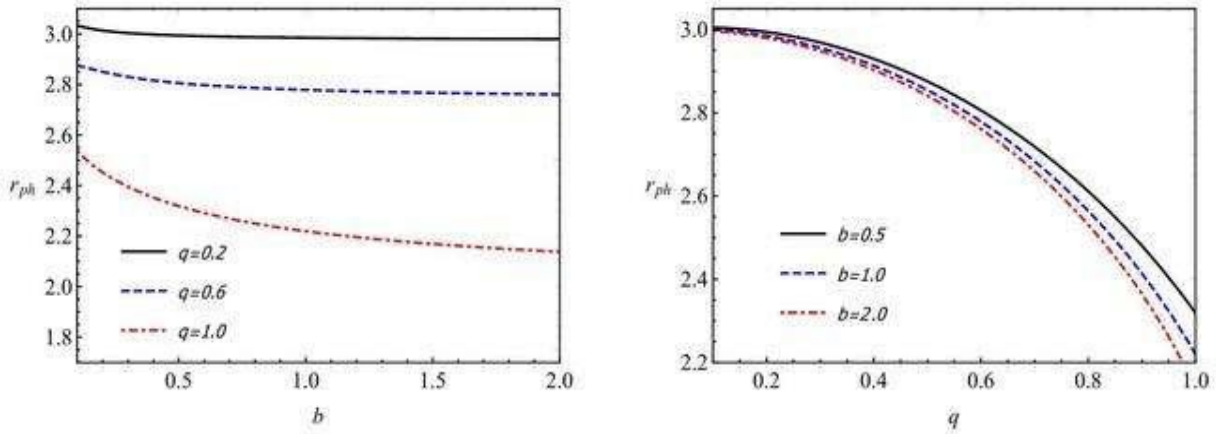
$$\dot{r}^2 + V_{eff}(r) = 0 \tag{9}$$

effektiv potensial

$$V_{eff}(r) = \frac{f(r)}{r^2}\frac{\Phi(r)}{\mathcal{L}_F}L^2 - E^2 \tag{10}$$

Qora o‘ra soyasi va foton sferasini belgilovchi aylana foton orbitlari quyidagi shartlarni qanoatlantiradi:

$$V_{eff}(r) = 0,\ \frac{dV_{eff}(r)}{dr} = 0. \tag{11}$$



1-rasm: NED doirasida qora o'ra atrofidagi foton orbitasi radiuslarining *b* va *q* parametrlariga bog'liqligi (*M=1* holatda).

Ushbu shartlar quyidagi asosiy tenglamaga olib keladi:

$$\frac{d}{dr}\left(\frac{r^2\mathcal{L}_F}{f(r)\Phi(r)}\right) = 0 \tag{12}$$

Ushbu tenglama yordamida foton sferasining radiuslarini aniqlash mumkin. Foton radiuslarining qora o'ra parametrlari q va b ga bog'liqligi 1-rasmda keltirilgan. Ushbu grafiklardan ko'rinadiki, elektr zaryad q ortishi foton sferasi radiusining kamayishiga olib keladi, b parametrining ta'siri esa faqat q yetarlicha katta qiymatlarga ega bo'lganda sezilarli bo'ladi.

Shuningdek, biz kengaytirilgan 4D Einstein–Gauss–Bonnet qora o'ra ustida ham ishladik. Mukammal suyuqlik qorong'u modda (PFDM) bilan o'ralgan 4D Einstein–Gauss–Bonnet nazariyasi doirasidagi qora o'ra yechimi quyidagicha ifodalanadi:

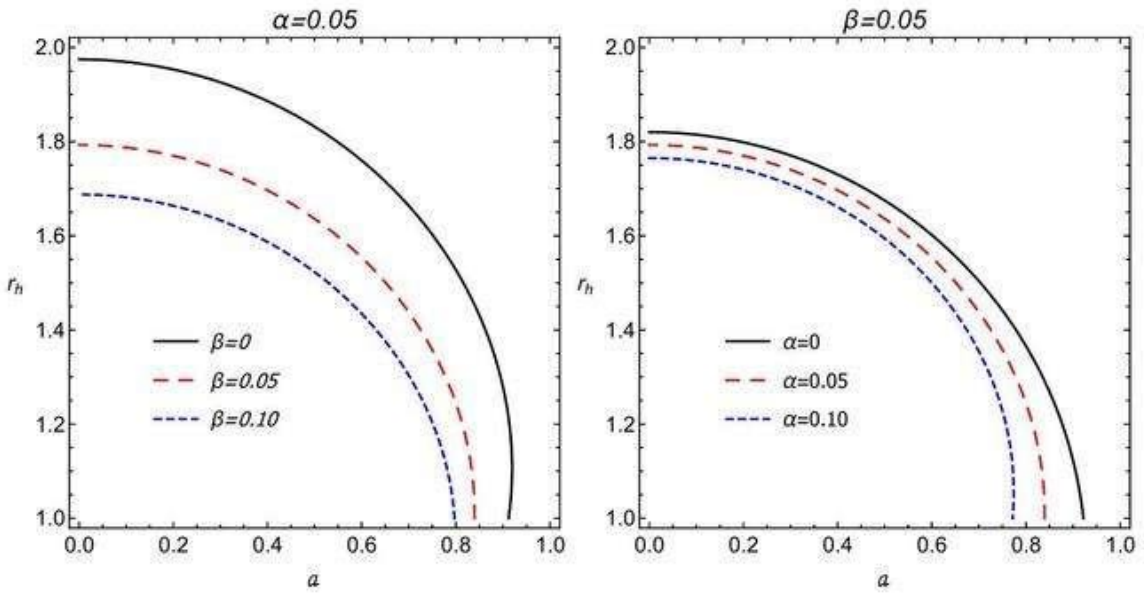
$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2\left(d\theta^2 + sin^2\,\theta d\phi^2\right) \tag{13}$$

bu yerda

$$f(r) = 1 + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 + \frac{4\alpha}{r^3}\left(2M - \beta ln\,\left(\frac{r}{\beta}\right)\right)}\right), \tag{14}$$

Newman–Janis algoritmi (NJA) umumiy nisbiylik (GR) doirasida statik qora o'ra metrikalarining aylanma analoglarini hosil qilish uchun ishlab chiqilgan. Kerrga o'xshash aylanma qora o'ra metrika quyidagicha beriladi:

$$\begin{aligned} ds^2 = & -\frac{\Delta(r) - a^2 sin^2\,\theta}{\rho^2}dt^2 + \frac{\rho^2}{\Delta(r)}dr^2 + \rho^2 d\theta^2 \\ & + \frac{sin^2\,\theta}{\rho^2}\left((r^2 + a^2)^2 - \Delta(r)a^2 sin^2\,\theta\right)d\phi^2 \\ & - \frac{2asin^2\,\theta}{\rho^2}(a^2 + r^2 - \Delta(r))dtd\phi \end{aligned} \tag{15}$$



2-rasm: Mukammal suyuqlik qorong‘u modda (PFDM) muhitidagi aylanma 4D Einstein–Gauss–Bonnet qora o‘ra parametrlari funksiyasi sifatida voqealar gorizonti radiusi, *M=1* holatda.

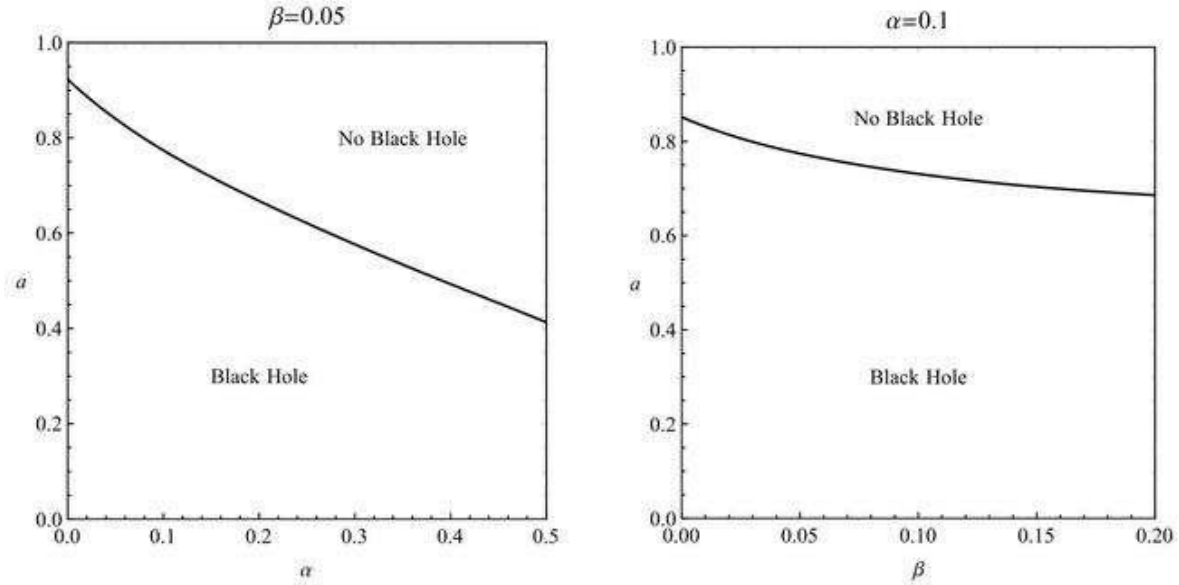
Hamda metrik funksiyalar

$$\rho^2 = r^2 + a^2 cos^2\,\theta \tag{16}$$

$$\Delta = r^2 + a^2 + \frac{r^4}{2\alpha}\left(1 - \sqrt{1 + \frac{4\alpha}{r^3}\left(2M - \beta ln\left(\frac{r}{\beta}\right)\right)}\right) \tag{17}$$

Biz 4D Einstein–Gauss–Bonnet gravitatsiyasi doirasidagi (15)-metrikaning murakkab tuzilishga ega aylanma qora o‘ralarni ifodalash qobiliyatiga ega ekanini va u Event Horizon Telescope (EHT) kuzatuvlaridan olingan qora o‘ra soya xususiyatlari bilan mos kelishini ko‘rsatamiz. 2-rasmda gorizont radiusining turli qora o‘ra parametrlari bilan bog‘liqligi tasvirlangan. Grafiklardan ko‘rinadiki, barcha holatlarda qora o‘ra parametrlari ortishi voqealar gorizonti radiusining kamayishiga olib keladi. Ruxsat etilgan parametrlar sohasi 3-rasmda keltirilgan.

Keyingi bosqichda biz aylanma qora oʻra atrofidagi zarracha harakatini, ayniqsa qora oʻra soyasini belgilovchi aylana foton orbitlarini oʻrganishni maqsad qilamiz. Hamilton–Jacobi usuli yordamida masala soddalashtirilib, energiya, burchak impulsi va Karter konstantasi kabi saqlanuvchi kattaliklar kiritiladi. Bu yondashuv harakat tenglamalarini radial va burchak qismlarga ajratishga imkon beradi, bu esa geodezikalarni tahlil qilishni osonlashtiradi. Natijada quyidagi geodezik tenglamalar hosil qilindi:



3-rasm: NED doirasidagi qora o‘ra uchun voqealar gorizonti mavjud bo‘ladigan parametrlar chegaralarini ko‘rsatuvchi grafiklar, *M=1* holatda.

$$\rho^2\frac{dt}{d\lambda} = a\left(L - aEsin^2\,\theta\right) + \frac{r^2+a^2}{\Delta}\left(E(r^2+a^2) - aL\right), \tag{18}$$

$$\rho^2\frac{dr}{d\lambda} = \pm\sqrt{\mathcal{R}}, \tag{19}$$

$$\rho^2\frac{d\theta}{d\lambda} = \pm\sqrt{\Theta}, \tag{20}$$

$$\rho^2\frac{d\phi}{d\lambda} = \left(Lcsc^2\,\theta - aE\right) + \frac{a}{\Delta}\left(E(r^2+a^2) - aL\right). \tag{21}$$

bu yerda

$$\mathcal{R} = \left[(r^2+a^2)E - aL\right]^2 - \Delta[m^2r^2 + \mathcal{K} + (L-aE)^2] \tag{22}$$

$$\Theta = \mathcal{K} + cos^2\,\theta\left(a^2(E^2-m^2) - L^2sin^{-2}\,\theta\right) \tag{23}$$

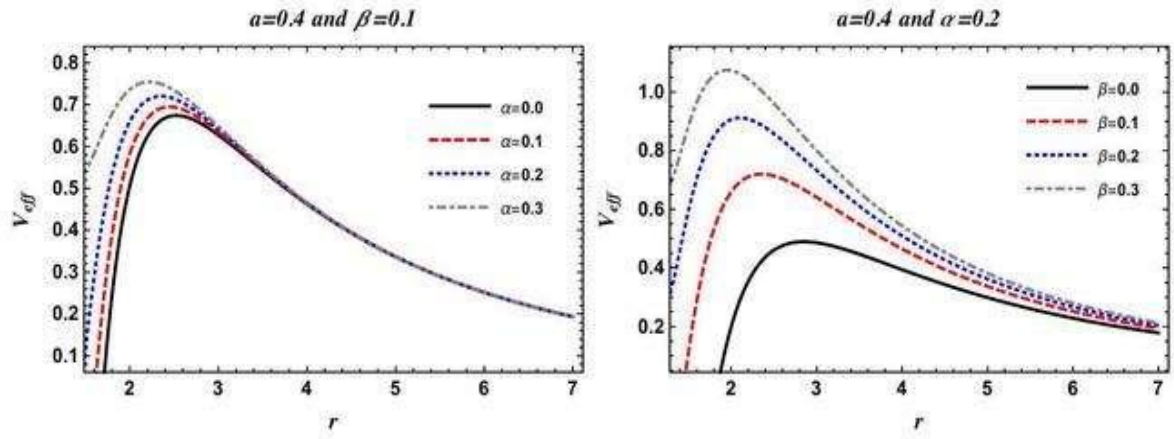
Radial harakat (19) tenglamasi quyidagicha qayta ifodalanishi mumkin:

$$\left(\rho^2\frac{dr}{d\lambda}\right)^2 + V_{eff} = 0. \tag{24}$$

Effektiv potensial quyidagi ko‘rinishda ifodalanadi:

$$V_{eff} = -\frac{\left((a^2+r^2)E - aL\right)^2 - \Delta(\mathcal{K} + (L-aE)^2)}{2r^4} \tag{25}$$

4-rasmda taqdim etilgan diagrammalar turli qora o‘ra parametrlarida samarali potensialning radial koordinataga nisbatan o‘zgarishini ko‘rsatadi. Ushbu chizilgan egri chiziqlarning cho‘qqilarini tahlil qilish orqali beqaror aylana null orbitlarni aniqlash mumkin (4-rasm).



4-rasm: Qora o‘ra parametrlarining tanlangan qiymatlarida samarali potensialning radial bog‘liqligini ko‘rsatuvchi grafiklar.

Beqaror aylana foton orbitalari quyidagi shartlarni qanoatlantiradi:

$$V_{eff} = 0, \ \frac{\partial V_{eff}}{\partial r} = 0, \ \frac{\partial^2 V_{eff}}{\partial r^2} < 0. \tag{26}$$

Ushbu shartlarni yechish orqali quyidagi natijaga ega bo‘lamiz:

$$\xi = \frac{(a^2 + r^2)\Delta' - 4\Delta r}{a\Delta'} \tag{27}$$

$$\eta = \frac{r^2\left(16\Delta(a^2 - \Delta) - r^2\Delta'^2 + 8\Delta r\Delta'\right)}{a^2\Delta'^2}, \tag{28}$$

bu yerda $'$ belgisi r ga nisbatan hosilani bildiradi.

"**Kengaytirilgan gravitatsiya nazariyalarida qora o‘ra soyalari**" deb nomlangan ikkinchi bobda turli kengaytirilgan gravitatsiya modellarida aylanuvchi va aylanmaydigan qora o‘ralarning soya tuzilmalari tahlil qilingan. Gauss-Bonnet bog‘lanish doimiysi, qorong‘u materiya parametri, nochiziqli elektrodinamika parametri va Yang-Mills zaryadi kabi qo‘shimcha parametrlarning ta’siri batafsil o‘rganildi. Ushbu parametrlar qora o‘ra soyasining shakli, o‘lchami va buzilishiga sezilarli ta’sir ko‘rsatishi va shu bilan kuzatiladigan xususiyatlarni o‘zgartirishi ko‘rsatilgan. Har bir holat uchun soya radiusi va buzilish parametri kabi kuzatiladigan parametrlar hisoblab chiqilgan. Bundan tashqari, soya profillaridan qora o‘ra parametrlarini baholash uchun yangi yondashuv qo‘llanildi. Natijalar shuni ko‘rsatadiki, soya xarakteristikalari umumiy nisbiylik nazariyasidan og‘ishlarni aniqlash va muqobil gravitatsion modellarning parametrlarini cheklash uchun samarali vosita bo‘lib xizmat qilishi mumkin.

Qora o‘ra soyasi hodisalar gorizontiga tushgan fotonlardan hosil bo‘lgan qorong‘u hudud bo‘lib, kuchli gravitatsion linzalash tufayli cheksizlikka tarqalib ketgan fotonlar tomonidan yaratilgan yorqin foton halqasi bilan taqqoslanadi. Ushbu soya qora tuynuklarning atrof-muhitga gravitatsion ta’siri haqida muhim kuzatuv ma’lumotlarini beradi.

Parametrlarning turli qiymatlari uchun 4D EGB qora o‘ralarning soya tasvirlari 5-rasmda tasvirlangan. Grafiklardan ko‘rinib turibdiki, Gauss-Bonnet va

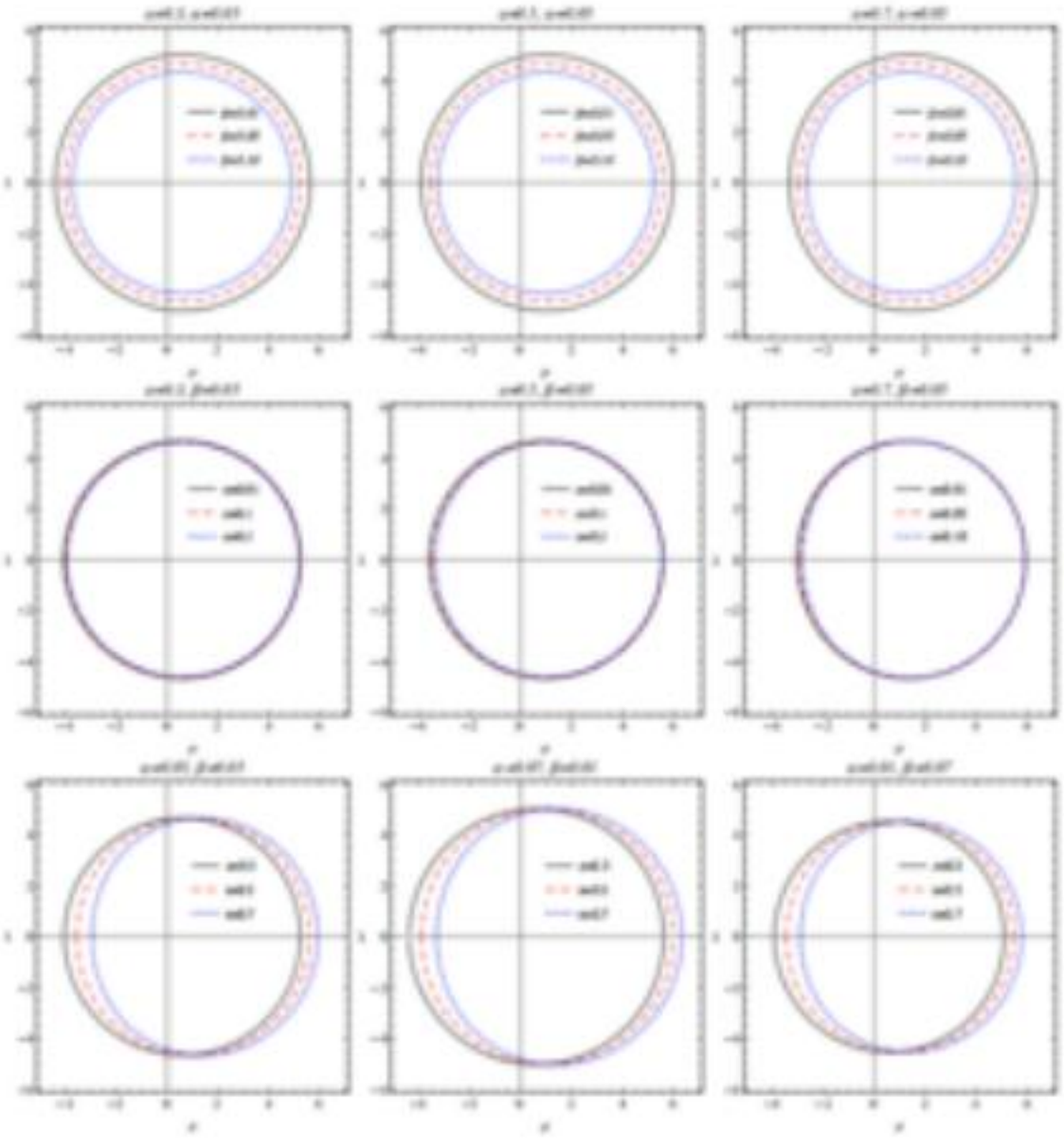
qorongu materiya parametrlari soya maydonining kamayishiga olib keladi, spin parametrlarining katta qiymatlarida esa, kutilganidek, ba’zi buzilishlar bo‘lishi mumkin.

Qora o‘ralarning xususiyatlarini baholash soya tahlilidan foydalanish orqali amalga oshirilishi mumkin. Ishni davom ettirish uchun qora o‘ra soyasining o‘lchamlari va konfiguratsiyasini tavsiflovchi aniq o‘lchovli o‘zgaruvchilarni o‘rnatish kerak. A bilan belgilangan qora o‘ra soyasining maydoni va D bilan belgilangan qora o‘ra soyasining deformatsiyasi bilan chambarchas bog‘liq yassilanish parametrlari:

$$A = 2\int_{r_-}^{r_+} \left(\lambda(r)\frac{d\mu(r)}{dr}\right) dr \tag{27}$$

$$D = \frac{\Delta\mu}{\Delta\lambda} \tag{28}$$

bu yerda r_+ va r_- beqaror doiraviy orbitalar radiuslari.



5-rasm: Turli holatlar uchun aylanuvchi 4D EGB qora o‘ra soya tasvirlari.

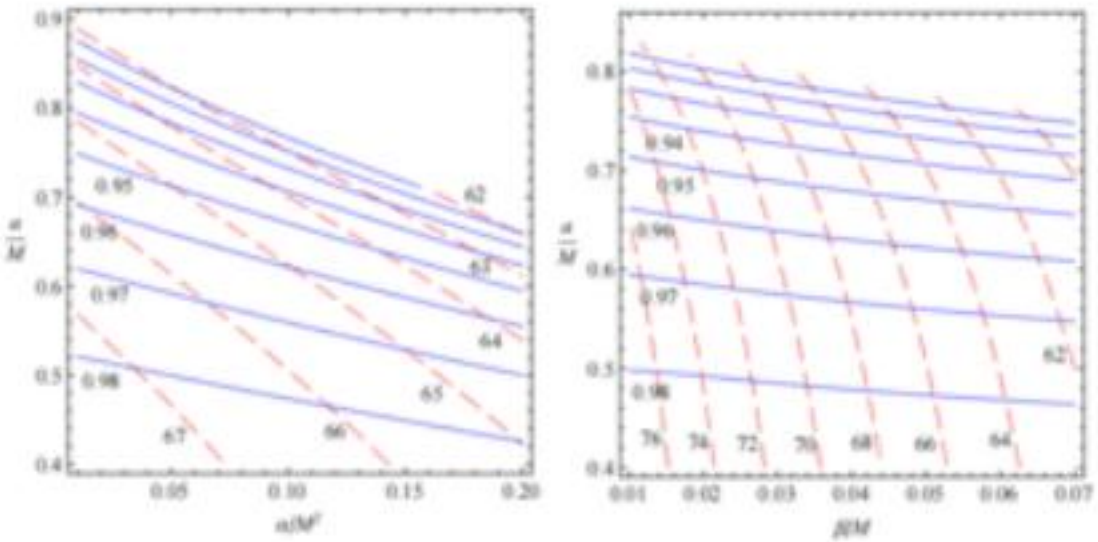
Turli holatlar uchun baholashlar 6-rasmda qora o‘ra parametrlarining konturli grafiklari bilan tavsiflangan. Ushbu tahlilda ko‘rsatilgan raqamlar shuni ko‘rsatadiki, kesishish koordinatalari har bir tegishli holatda ikkita qora o‘ra parametrlari uchun yagona aniqlanishlar bo‘lib xizmat qiladi. Bu baholash usuli qora o‘raning soyasi bo‘yicha qora o‘ra parametrlarning qiymatlarini aniqlash uchun foydalanish mumkin. Olingan natijalarni yaxshiroq ko‘rsatish uchun sonli natijalarni 1- va 2-jadvallarda ham keltiramiz. Olingan natijalar kuzatuv o‘lchovlari orqali aniqlanishi mumkin bo‘lgan soya xususiyatlarini tahlil qilish orqali qora o‘ra haqida ma’lumot olish uchun qimmatli vosita bo‘lib xizmat qilish potentsialiga ega. Bundan tashqari, natijalardan qora o‘ralarning xususiyatlari va xatti-harakatlarini o‘rganishda, ularning shakllanishi, evolyutsiyasi va atrofdagi materiya bilan o‘zaro ta’sirini yoritishda foydalanish mumkin. Ushbu baholash usulining samarali qo‘llanilishi astrofizikadan tashqariga chiqib, u umumiy nisbiylik nazariyasi va fizikaning asosiy qonunlarini tushunishimizga hissa qo‘shishi mumkin.

A/M^2	D	a/M	α/M^2	R_a
68	0.99	0.3868	0.0028	4.6524
67	0.98	0.5081	0.0342	4.6180
66	0.97	0.5819	0.0651	4.5835
65	0.96	0.6207	0.1016	4.5486
64	0.95	0.6404	0.1386	4.5135
63	0.94	0.6384	0.1810	4.4781

1-jadval: Fiksirlangan $\boldsymbol{\beta = 0.05M}$ qiymat uchun soya kuzatuv parametrlari $\boldsymbol{A/M^2}$ va $\boldsymbol{D}$ dan qora o‘ra parametrlari $\boldsymbol{a/M}$va $\boldsymbol{\alpha/M^2}$ning taxminiy qiymatlari.

A/M^2	D	a/M	β/M	R_a
74	0.98	0.4926	0.0197	4.8533
72	0.97	0.5807	0.0241	4.7873
70	0.96	0.6399	0.0290	4.7203
68	0.96	0.6325	0.0369	4.6524
66	0.95	0.6763	0.0429	4.5835
64	0.95	0.6684	0.0522	4.5135

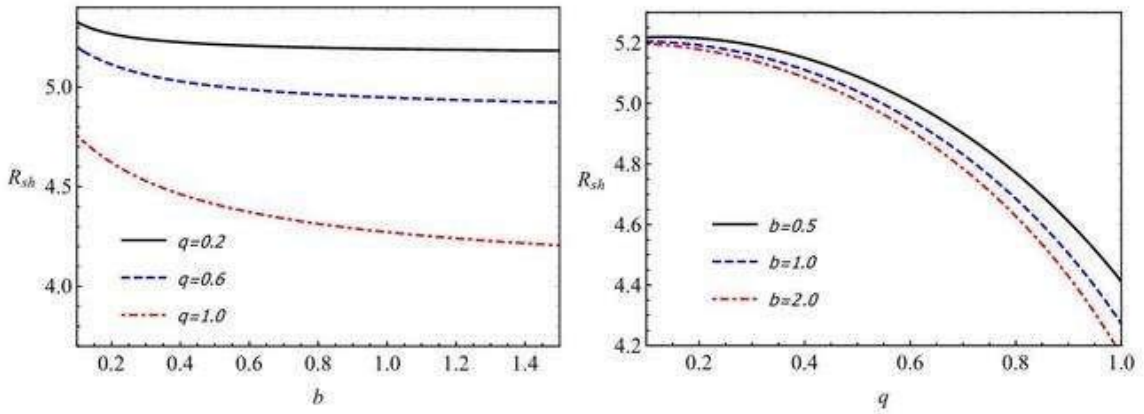
2-jadval: Fiksirlangan $\boldsymbol{\alpha = 0.1M^2}$ qiymat uchun soya kuzatuv parametrlari $\boldsymbol{A/M^2}$ va $\boldsymbol{D}$ dan qora o‘ra parametrlari $\boldsymbol{a/M}$ va $\boldsymbol{\beta/M}$ning taxminiy qiymatlari.



6-rasm: Konturli diagramma ma'lum bir parametrlar fazosida aylanuvchi 4D EGB qora o'ra uchun kuzatiladigan A/M^2 va D ni ko'rsatadi. parametrlarning o'zgarmas qiymatlari $\beta = 0.05M, \alpha = 0.1M^2$. Uzlukli qizil chiziqlar A/M^2 ning doimiy qiymatlarini, uzluksiz ko'k chiziqlar esa D ning doimiy qiymatlarini ko'rsatadi.

Qorong'u modda parametri β ning mavjudligi qora o'ra soyasining o'lchami va shakliga sezilarli ta'sir ko'rsatadi. β ko'paygan sari soyaning o'lchami kichrayadi, bu mukkamal suyuqlik qorong'u materiya qora o'ra soyasining ko'rinadigan o'lchamini kamaytirishini ko'rsatadi. Aylanmaydigan qora o'ralar mukammal aylana soyalarni ko'rsatadi, aylanadigan qora o'ralar esa parametrlar a va β ning oshishi bilan yanada aniqroq bo'ladigan buzilgan soyalarni ko'rsatadi. Qora moddaning aniq soya xususiyatlari qora tuynuklar atrofida qora moddaning tarqalishi va xususiyatlarini tekshirish imkonini beradi.

Biz a va β ning ortishi soya o'lchamini kamaytirishini, yassilanish D esa ortishini kuzatdik. Qora materiya bilan o'ralgan aylanuvchi Gauss Bonnet qora o'rasi uchun α va β parametrlarini baholash uchun biz a ning turli qiymatlari uchun α va β ning funksiyalari sifatida kuzatiladigan A (soya maydoni) va D (yassilanish)ni qurdik. Har bir grafik o'zgarmas A or D qiymatli egri chiziqlarni ifodalaydi. Ushbu egri chiziqlarning yagona kesishish nuqtalari α va β qora o'ra parametrlari uchun ma'lum qiymatlarni beradi.



7-rasm: NED doirasidagi qora o'raning soya radiuslari b va q parametrlarining funksiyasi sifatida, ($M = 1$ holatda).

Biz shuningdek nochiziqli elektrodinamika doirasidagi aylanmaydigan qora o‘ra soyasini ham o‘rgandik. Qora o‘ra soyasi radiusining q va b parametrlariga bog‘liqligi 7-rasmda keltirilgan. Ushbu grafiklardan ko‘rinadiki, elektr zaryad q ortishi qora o‘ra soya radiusining kamayishiga olib keladi. Aksincha, nochiziqli elektrodinamika parametri b faqat kichik qiymatlar sohasida ($0.1 \lesssim b \lesssim 1.5$) sezilarli ta’sir ko‘rsatadi, bu yerda nochiziqli effektlar muhim rol o‘ynaydi; b ning kattaroq qiymatlarida esa uning ta’siri asta-sekin kamayib, tizim chiziqli elektrodinamika limitiga yaqinlashadi.

Nochiziqli elektrodinamikaning foton tarqalishiga ta’sirini ko‘rsatish uchun biz asl fazo-vaqt metrikasi va foton harakatini boshqaruvchi samarali geometriya asosida hisoblangan qora o‘ra soya radiuslarini taqqosladik. Natijalar shuni ko‘rsatadiki, samarali geometriya doirasida olingan soya radiusi asl metrika asosida aniqlangan qiymatdan tizimli ravishda kattaroqdir. Bu esa nochiziqli effektlar fon elektromagnit maydoni bilan o‘zaro ta’sir orqali foton trayektoriyalarini o‘zgartirib, qora o‘ra soyasini kengaytirishini anglatadi.

"**EHT kuzatuvlari yordamida qora o‘ra parametrlariga cheklovlar**" deb nomlangan uchinchi bobda M87* va Sgr A* o‘ta massiv qora o‘ralar uchun hodisalar gorizonti teleskopining kuzatuv ma’lumotlaridan foydalangan holda qora o‘ra soyasiga asoslangan cheklovlar olingan. Tahlil muqobil gravitatsiya modellari doirasida, jumladan, nochiziqli elektrodinamikadagi zaryadlangan qora o‘ralar va mukammal suyuq qorong‘u materiya va darajali Yang-Mills maydonlari bilan o‘ralgan 4D Eynshteyn-Gauss-Bonnet qora o‘ralari doirasida amalga oshirildi. Nazariy soya radiuslarini EHT ma’lumotlari bilan taqqoslash orqali har bir model uchun parametr chegaralari olindi.

Biz EHT jamoasi tomonidan xabar qilingan so‘nggi kuzatuv ma’lumotlariga asoslanib, Sgr A* va M87* supermassiv qora o‘ralari cheklovlarini olishga e’tibor qaratamiz. Birinchidan, biz qora o‘ra soyasining o‘lchami va shaklini tavsiflash uchun foydalanadigan kuzatiladigan parametrlarni aniqlaymiz. qora o‘radan uzoqdagi kuzatuvchi uchun soya tasvirining burchak diametri quyidagicha ifodalanishi mumkin:

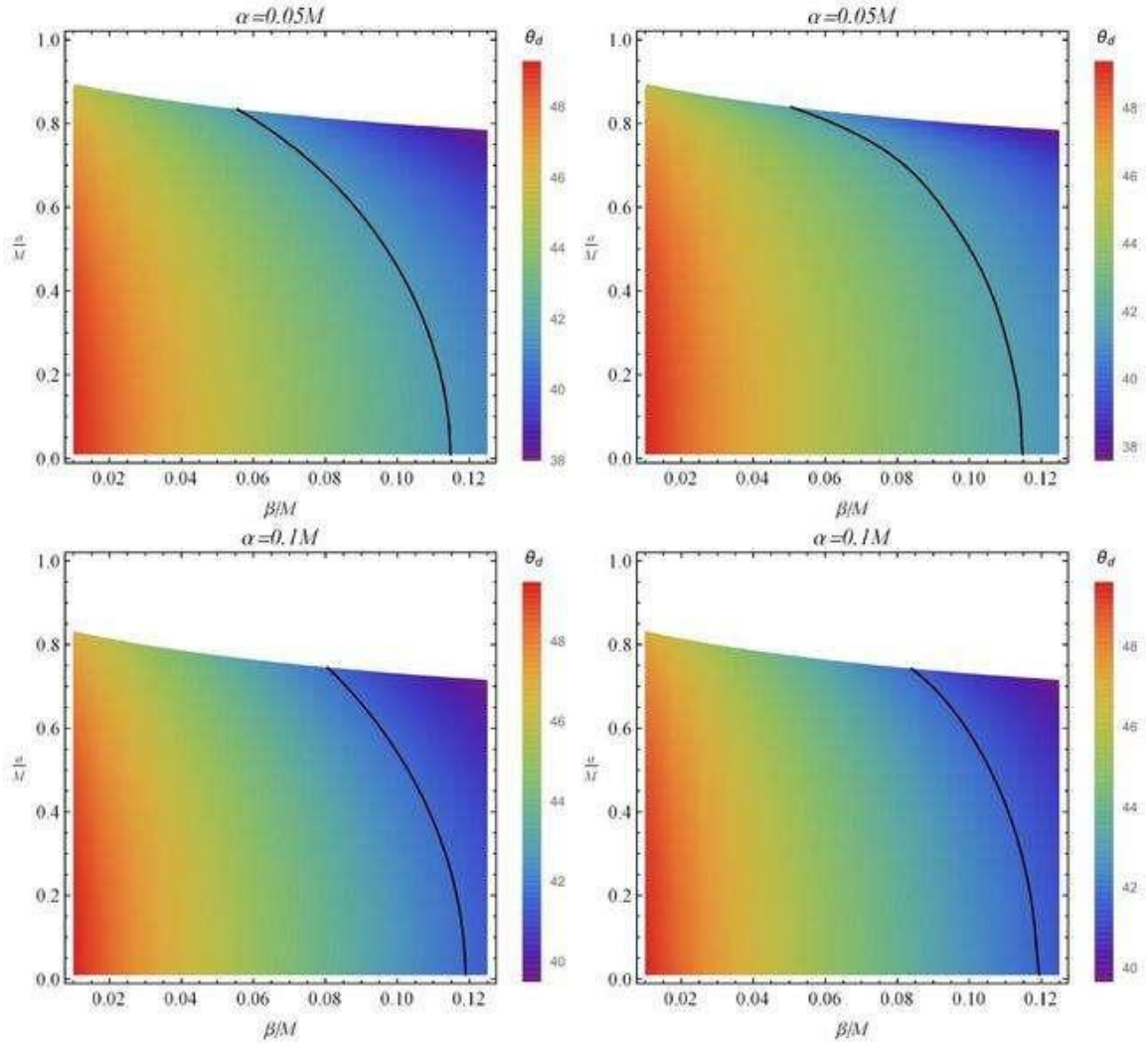
$$\theta_d = 2\frac{R_a}{d}, \quad R_a = \sqrt{\frac{A}{\pi}}, \tag{29}$$

bu yerda R_a maydon soyasi radiusi.

Cheklovlarni olish uchun quyidagi kuzatuv ma’lumotlaridan foydalandik. M87* ning massasi va Yergacha bo‘lgan masofani mos ravishda $M = 6.5 \times 10^9 M_\odot$ va $d = 16.8 Mpc$ deb hisoblash mumkin. Soddalik uchun biz o‘ta massiv qora o‘ralarning massa va masofa o‘lchovlarining noaniqliklarini hisobga olmaymiz. Supermassiv qora o‘ra M87* tasvirining burchak diametri $1-\sigma$ darajada $\theta_d = 42 \pm 3\mu as$. O‘ta massiv qora o‘ra Sgr A* soyasining burchak diametri $\theta_d = 48.7 \pm 7\mu as$ ga teng. Quyosh tizimidan masofasi va massasi mos ravishda $M \simeq 4 \times 10^6 M_\odot$ va $d \simeq 8 kpc$ deb hisoblangan.

Avvalo, biz o‘zimizning soya tahlili bo‘yicha nazariy natijalarimizni EHT ma’lumotlaridan olingan Sgr A* soya tasviri bilan taqqoslash maqsadida Sgr A* supermassiv qora o‘rasini 4D Einstein–Gauss–Bonnet (EGB) qora o‘rasi sifatida

PFDM muhitida ko‘rib chiqamiz. Tadqiqot doirasida biz supermassiv qora o‘raning massasi va masofasi o‘lchovlari bilan bog‘liq ehtimoliy noaniqliklarni soddalashtirilgan yondashuvda e’tibordan chetda qoldiramiz. Natijalarimiz Sgr A* ning EHT kuzatuvlaridan olingan burchak diametri bilan bog‘liq noaniqlik sababli ma’lum darajada xatoliklarni o‘z ichiga oladi. Shunga qaramay, ushbu tahlil qora o‘ra va qorong‘u modda parametrlarining kuzatiladigan kattaliklarga qanday ta’sir qilishini aniq aniqlash imkonini beradi. 8-rasmda Sgr A* uchun turli holatlardagi cheklovlar keltirilgan. Qora o‘ra burchak diametrining moslashtirilgan zichlik taqsimotlari Sgr A* uchun o‘lchangan burchak diametri ($\theta_d = 48.7 \pm 7\mu as$) ichida joylashgan. Shuning uchun, biz 4D EGB qora o‘rasi PFDM muhitida olingan nazariy natijalarimiz Somon Yo‘li galaktikasi markazidagi supermassiv qora o‘ra bilan mos kelishi mumkin degan xulosaga kelamiz.



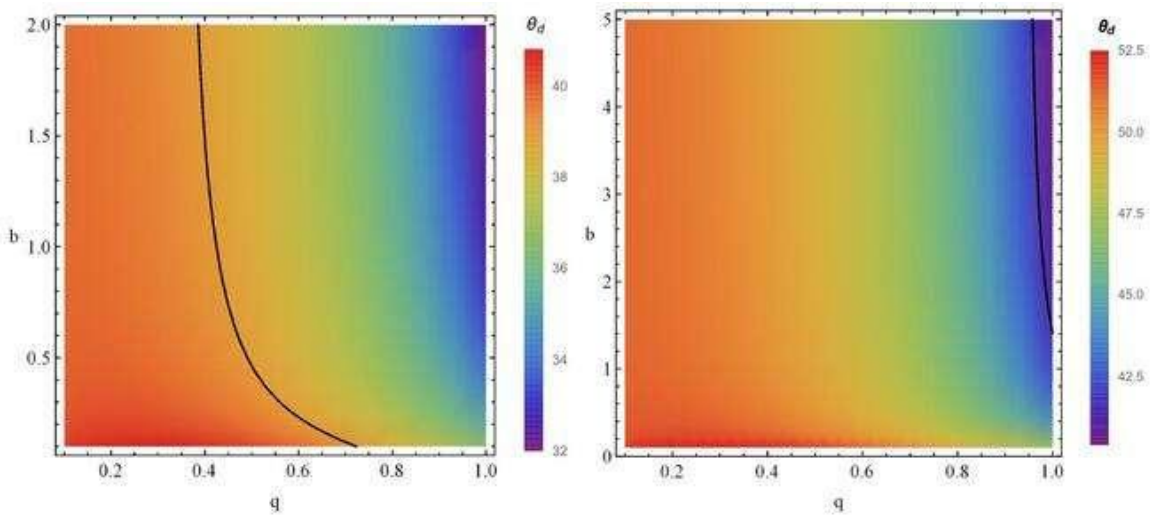
8-rasm: PFDM bilan o‘ralgan aylanma 4D Gauss–Bonnet qora o‘ra uchun burchak diametri θ_d (a/M , β/M^2) parametrlar fazosida, 20° (chap) va 50° (o‘ng) og‘ish burchaklarida.

Bugungi kunda qora o‘ra fizikasi sohasida turli modifikatsiyalangan va muqobil gravitatsiya nazariyalari uchun ko‘plab qora o‘ra yechimlari mavjud. Yangi gravitatsiya modellarini tadqiq qilish davom etar ekan, ularning fizik jihatdan maqbulligi kuzatuv ma’lumotlari asosida baholanishi kerak bo‘ladi, va bu boradagi muhim kuzatuvlardan biri, Event Horizon Telescope tomonidan olingan

qora o‘ra soyasi tasvirlaridir. Ushbu tadqiqotda biz EHT kuzatuvlari asosida 4D Einstein–Gauss–Bonnet qora o‘rasi parametrlariga cheklovlar qo‘ydik. Olingan natijalar shuni ko‘rsatadiki, bunday cheklash usuli yangi gravitatsiya modellaridagi qora o‘ra yechimlarini o‘rganishda foydali vosita bo‘lishi mumkin. Bundan tashqari, ushbu yondashuv orqali qorong‘u moddaning qora o‘ra fizikasi ustidagi ta’sirini ham chuqurroq tahlil qilish imkoniyati mavjud.

Nochiziqli elektrodinamika doirasidagi yangi zaryadlangan qora o‘ralar. Ushbu fazo-vaqt ikki o‘lchamsiz parametr bilan tavsiflanadi: NED maydoniga bog‘liq zaryad parametri q va standart Maksvell elektrodinamikasidan og‘ishlarni ifodalovchi nochiziqli bog‘lanish parametri b. (b, q) parametrlar fazosidagi har bir nuqta uchun qora o‘ra soyasining burchak diametri NED doirasida foton tarqalishini boshqaruvchi samarali geometriya hisobga olingan holda, beqaror aylana foton orbitasi radiusidan aniqlanadi.

9-rasmda M87* (chap panel) va Sgr A* (o‘ng panel) uchun (b, q) tekisligida burchak diametrining kontur grafiklari keltirilgan. Rang shkalasi θ_d ning kattaligini ifodalaydi, qora uzluksiz chiziqlar esa EHT o‘lchovlaridan olingan pastki kuzatuv chegaralarini ko‘rsatadi. Ushbu chiziqlardan pastda joylashgan parametr fazosi sohalari chiqarib tashlanadi, chunki ular kuzatuvlar tomonidan ruxsat etilgan qiymatlardan kichik qora o‘ra soya o‘lchamlarini bashorat qiladi.



9-rasm: NED doirasidagi yangi zaryadlangan qora o‘raning kuzatiladigan burchak diametri b–q parametrlar fazosida M87* (chap) va Sgr A* (o‘ng) uchun. Qora egri chiziqlar EHT tomonidan xabar qilingan M87* va Sgr A* supermassiv qora o‘ralarining o‘lchangan burchak diametrlarining pastki chegaralarini ifodalaydi.

M87* ga mos keluvchi chap paneldan ko‘rinadiki, b ning o‘zgarmas qiymatlarida zaryad parametri q ortishi bilan burchak diametri monoton ravishda kamayadi. Bu holat qora o‘ra yaqinida NED zaryadi mavjudligi tufayli gravitatsion tortishishning samarali zaiflashishi bilan izohlanadi; natijada foton sferasi radiusi kamayadi va mos ravishda soya ham kichrayadi. Nochiziqli parametr b esa ushbu ta’sirni yanada modulyatsiya qiladi: b ning kattaroq qiymatlari Schwarzschild limitidan og‘ishni kuchaytiradi va shu orqali q ga qo‘yilgan cheklovlarni yanada toraytiradi.

Sgr A* uchun ham xuddi shunga o‘xshash sifat jihatdan o‘xshash xatti-harakat 9-rasmning o‘ng panelida kuzatiladi. Biroq, bu holatdagi cheklovlar nisbatan kuchsizroq bo‘lib, Sgr A* soya o‘lchovlari bilan bog‘liq katta kuzatuv noaniqliklarini aks ettiradi. Shunga qaramay, chiqarib tashlash egri chizig‘i b ning keng diapazonida zaryad parametriga yuqori chegara qo‘yadi va juda katta q qiymatlariga ega bo‘lgan konfiguratsiyalarni inkor etadi, chunki bunday holatlar kuzatuvlar bilan mos kelmaydigan darajada kichik qora o‘ra soyasini hosil qiladi.

XULOSA

Dissertatsiya bo‘yicha olib borilgan tadqiqotlar asosida quyidagi xulosalar taqdim etildi.

1. Birinchi marta kengaytirilgan 4D Einstein–Gauss–Bonnet (EGB) gravitatsiyasi doirasida aylanma qora o‘ra metrikalari olinib, tahlil qilindi hamda ular klassik Kerr–Newman metrikasidan og‘ishlarni ko‘rsatishi aniqlandi.
2. Birinchi marta nochiziqli elektrodinamika (NED) va kengaytirilgan 4D EGB gravitatsiyasi doirasidagi qora o‘ralar uchun foton orbitalari, soya chegaralari va deformatsiya parametrlari o‘rganildi. Muqobil nazariyalardagi qora o‘ra soyalari umumiy nisbiylik bilan solishtirganda o‘lcham, deformatsiya va foton halqasi tuzilishi bo‘yicha sezilarli farqlar ko‘rsatishi aniqlandi.
3. Qora o‘ra xossalarini kuzatiladigan kattaliklar bilan bog‘lash imkonini beruvchi samarali usul sifatida soya kuzatuv parametrlari (yuzasi va yassiligi) orqali 4D EGB qora o‘ralari uchun parametr baholash grafiklari olindi.
4. NED mavjudligi zaryadlangan qora o‘ralarning soya radiusini standart Einstein–Maksvell holatiga nisbatan oshirishi aniqlandi, bu esa NED tuzatishlari uchun kuzatiladigan xususiyatni taqdim etadi.
5. Qora o‘ralar atrofidagi mukammal suyuqlik qorong‘u modda (PFDM) soya radiuslarini sezilarli darajada kamaytirishi aniqlanib, bu qorong‘u modda zichlik profillarini cheklash uchun kuzatuv signali bo‘lishi ko‘rsatildi.
6. Birinchi marta kengaytirilgan 4D EGB gravitatsiyasi va NED doirasidagi qora o‘ra parametrlariga cheklovlar EHT tomonidan olingan M87* va Sgr A* soya kuzatuvlari asosida topildi.
7. Umumiy nisbiylik va muqobil gravitatsiya nazariyalarini farqlash uchun keyingi avlod teleskoplari (ngEHT) orqali qo‘llanilishi mumkin bo‘lgan maxsus kuzatuv belgilari aniqlandi.

SCIENTIFIC COUNCIL DSc. 03/07.07.2025.FM/T.192.01 ON AWARD OF SCIENTIFIC DEGREE AT INSTITUTE FOR ADVANCED STUDIES AT "NEW UZBEKISTAN" UNIVERSITY

ASTRONOMICAL INSTITUTE

SARIKULOV FURKAT SHERALI UGLI

TESTING BLACK HOLE NATURE IN ALTERNATIVE GRAVITY THROUGH ASTROPHYSICAL OBSERVATIONS

01.03.01 – Astronomy

Doctor of Philosophy in Physical and Mathematical Sciences (PhD)
ABSTRACT OF THE DISSERTATION

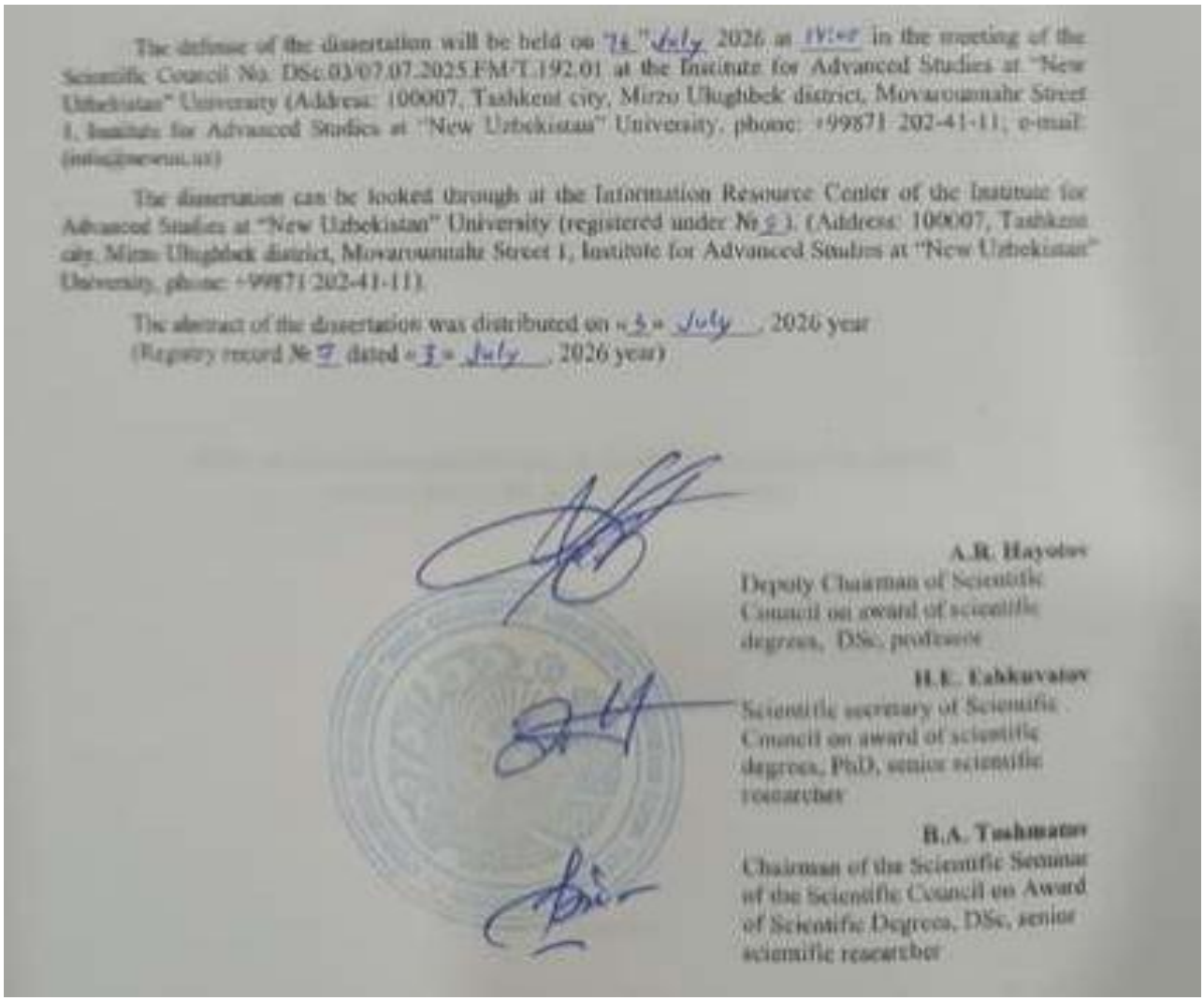
Tashkent – 2026

The theme of dissertation of the doctor of philosophy (PhD) on physical and mathematical sciences is registered at Higher Attestation Commission at the Academy of Sciences of the Republic of Uzbekistan under the number B2025.4.PhD/FM1412.

Dissertation has been prepared at Astronomical Institute of Uzbekistan Academy of Sciences.

The abstract of the dissertation is posted in three languages (Uzbek, English, Russian (resume)) on the website (ias.newuu.uz) and the "Ziyonet" information and educational portal (www.ziyonet.uz).

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Leading organization:	**Fesenkov Astrophysical Institute**



The defense of the dissertation will be held on "16" July 2026 at 11:00 in the meeting of the Scientific Council No. DSc.03/07.07.2025.FM/T.192.01 at the Institute for Advanced Studies at "New Uzbekistan" University (Address: 100007, Tashkent city, Mirzo Ulugbek district, Movarounnahr Street 1, Institute for Advanced Studies at "New Uzbekistan" University, phone: +99871 202-41-11, e-mail: (info@newuu.uz)

The dissertation can be looked through at the Information Resource Center of the Institute for Advanced Studies at "New Uzbekistan" University (registered under № 9). (Address: 100007, Tashkent city, Mirzo Ulugbek district, Movarounnahr Street 1, Institute for Advanced Studies at "New Uzbekistan" University, phone: +99871 202-41-11).

The abstract of the dissertation was distributed on «3» July 2026 year
(Registry record № 7 dated «3» July 2026 year)

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INTRODUCTION (Annotation of PhD dissertation)

Topicality and demand of the theme of the dissertation. The study of black holes (BHs) in alternative theories of gravity is a highly relevant and rapidly evolving field in modern astrophysics and theoretical physics. With groundbreaking observations from the Event Horizon Telescope (EHT), including the first images of supermassive BHs M87* and Sgr A*, there is an urgent need to test general relativity (GR) in extreme gravitational regimes. The discovery of deviations from classical BH solutions could provide crucial insights into quantum gravity, dark matter, and nonlinear electrodynamics (NED).

This dissertation explores novel BH solutions in extended gravity models, such as 4D Einstein-Gauss-Bonnet (EGB) gravity, NED addressing key questions in contemporary physics. Testing GR Alternatives: Modified gravity theories (e.g., EGB, NED) predict BH properties that differ from GR , making them testable with EHT observations. Dark Matter Influence: The interaction between BHs and perfect fluid dark matter (PFDM) affects BH shadows, offering a new way to probe dark matter distributions near galactic centers. Method Development: The parameter estimation approach based on black hole shadow analysis has been improved and further refined to enhance the accuracy and reliability of extracting black hole and surrounding matter properties.

The demand for such research is underscored by upcoming high-resolution observatories (e.g., next-generation EHT, LISA), which will further constrain BH parameters and gravity models. By combining shadow analysis with observational constraints, this work provides a unified framework for distinguishing between GR and alternative theories, making it essential for future astrophysical tests and theoretical advancements. Thus, this dissertation addresses pressing questions in fundamental physics while contributing to the interpretation of cutting-edge observational data.

In addition, this dissertation research is aligned with the priority scientific directions of the Republic of Uzbekistan and the broader international scientific community. The results of the research contribute, to a certain extent, to the implementation of the tasks set forth in the Law of the Republic of Uzbekistan No. O‘RQ-576 “On Science and Scientific Activity” dated October 29, 2019, the Decree of the President of the Republic of Uzbekistan No. PF-6097 “On Approval of the Concept for the Development of Science until 2030” dated October 29, 2020, the Resolution of the President of the Republic of Uzbekistan No. PQ-5032 “On Measures to Improve the Quality of Education and Develop Scientific Research in the Field of Physics” dated March 19, 2021, as well as other regulatory and legal documents related to this area of activity.

Relevance of the research to the priority areas of science and technology development of the Republic of Uzbekistan. Dissertation research has been carried out in accordance with the priority areas of science and technology in the Republic of Uzbekistan: II. "Power, energy and resource-saving".

Degree of study of the problem. The study of black holes in alternative theories of gravity has been an active area of research since the early developments of general relativity. Classical BH solutions, such as the Schwarzschild and Kerr

metrics, have been extensively analyzed, but modifications due to higher-curvature corrections (Boulware, Glavan, Lin, Fernandes, Kumar, Ghosh, Lu, Pang, Hennigar), nonlinear electrodynamics (Born, Infeld, Ayon-Beato, Garcia, Bronnikov, Dymnikova, Balart), and dark matter (Haroon, Jusufi, Hou, Narzilloev, Atamurotov) interactions remain less explored. Recent works, including those by Glavan and Lin (2020) on 4D Einstein-Gauss-Bonnet gravity and Born-Infeld-type NED models, have expanded the theoretical framework, yet observational constraints from the Event Horizon Telescope have only recently allowed for empirical testing of these models (Synge, Bardeen, Hioki, Maeda, Johannsen, Psaltis, Event Horizon Telescope Collaboration, Vagnozzi). Although significant progress has been made in deriving BH solutions in modified gravity, the connection between theoretical predictions and astrophysical observations-particularly regarding BH shadows and parameter estimation methods-requires further refinement.

Despite advances, key gaps persist in the literature. Most studies focus on static or spherically symmetric BHs, whereas rotating solutions, more relevant for astrophysical BHs, are less understood in alternative gravity. Additionally, the influence of dark matter on BH shadows has been explored in simplified scenarios, but a comprehensive analysis of BH dynamics in perfect fluid dark matter or Power-Yang-Mills (PYM) fields remains incomplete. This dissertation addresses these gaps by systematically analyzing rotating BHs in NED, EGB, linking theoretical predictions to EHT observations, and providing new constraints on deviations from GR. Taking all of this into account, we aimed to study black holes in alternative theories of gravity, including nonlinear electrodynamics, fourdimensional Einstein-Gauss-Bonnet gravity with perfect fluid dark matter, and in power Yang-Mills field. The work builds upon existing studies while extending them to more realistic, observationally testable scenarios.

Connection of the topic of the dissertation to the scientific works of higher education and research institutions, where the dissertation is carried out. The dissertation was carried out within the framework of scientific projects funded by the Ministry of Innovative Development. F-FA-2021-510 "Investigations of Nuclear Matter of Neutron Stars in Modified Gravity".

The aim of the research is to test alternative gravity theories using black hole shadows and EHT observational constraints.

The tasks of the research:

to derive novel black hole solutions in nonlinear electrodynamics (NED) and 4D Einstein-Gauss-Bonnet (EGB) gravity;

to extend static solutions to rotating cases using the Newman-Janis algorithm and analyze horizon structures;

to study photon motion and effective potentials to determine unstable circular orbits and shadow boundaries.

to calculate black hole shadows for different gravity models and compare them with EHT observations.

to estimate black hole parameters (spin, charge, coupling constants) from shadow observables (area, oblateness).

to confront theoretical predictions with EHT data to constrain deviations from general relativity.

The objects of the research are static black holes in nonlinear electrodynamics; 4D Einstein-Gauss-Bonnet black holes surrounded by perfect fluid dark matter; black holes in Power-Yang-Mills fields within modified gravity; Supermassive black holes M87* and Sgr A* as observational test cases.

The subjects of the research are metric solutions and horizon structures of alternative gravity black holes; photon spheres and unstable circular orbits in modified spacetimes; black hole shadow morphology and observable parameters; parameter estimation techniques using Event Horizon Telescope data; constraints on modified gravity parameters from astrophysical observations; and comparative analysis of different alternative gravity scenarios.

The methods of the research are methods of analytical modeling for application of nonlinear electrodynamics and modified gravity field equations; numerical techniques for implementation of the Newman-Janis algorithm for generating rotating black hole solutions; Hamilton-Jacobi formalism for calculation of effective potentials for test particle motion; standard methods for black hole shadow reconstruction; and comparative analysis for systematic evaluation of different gravity models.

The scientific novelty of the research is the following:

For the first time, it has been derived previously unknown rotating black hole solutions in 4D Einstein-Gauss-Bonnet gravity coupled with perfect fluid dark matter and power Yang-Mills field, extending beyond classical Kerr-Newman metrics.

Parameter estimations have been obtained for 4D Einstein–Gauss–Bonnet black holes surrounded by perfect fluid dark matter.

Unique observational signatures of PFDM in black hole shadows have been identified, demonstrating that dark-matter density profiles can be constrained through high-resolution imaging.

Practical results are the following:

For the first time quantitative constraints have been provided on black hole spin (a), NED charge (q), and Gauss-Bonnet coupling (α) using Event Horizon Telescope data, offering testable predictions for future observations.

It has been modified to open-access numerical codes for simulating shadows and parameter estimations in alternative gravity scenarios, enabling broader community verification.

Specific shadow distortions have been predicted for ngEHT, prioritizing observational targets to discriminate between GR and alternative theories.

Reliability of the research results provided by applying modern, proven methods of mathematical physics, computational mathematics, and relativistic astrophysics. All derived black hole solutions satisfy the fundamental field equations of their respective modified gravity theories, ensuring mathematical rigor and consistency with established frameworks. Critical results (e.g., shadow radii, distortions) were computed using independent analytical and numerical methods, with agreement confirming robustness. Parameter constraints align with empirical data from the Event Horizon Telescope (M87*, Sgr A*), and deviations

from GR predictions remain within the current observational uncertainties. Assumptions (e.g., asymptotic flatness, specific dark matter profiles) and their potential impacts are explicitly addressed, defining the scope of applicability.

The scientific and practical significance of the research results.

Obtained results provide rigorous tests of alternative gravity models (e.g., 4D EGB, NED) by comparing their black hole solutions with observational data, helping to validate or constrain extensions to general relativity.

The results demonstrate how black hole shadows and dynamics can reveal the presence and distribution of dark matter, offering a novel astrophysical tool to study this elusive component of the universe.

The study delivers concrete, observational testable predictions (shadow shapes) for current (EHT) and future (ngEHT) instruments, guiding efforts to detect deviations from GR.

Analysis develops open-access analytical frameworks and numerical codes for black hole shadow and parameter estimations in modified gravity, enabling broader research and collaborative verification.

Applications of the research results. The results of the study of the dynamics of particles around black holes in modified gravity theories have been applied as follows:

The theoretical research results and methods, published in the scientific paper "Furkat Sarikulov, Farruh Atamurotov, Ahmadjon Abdujabbarov, Bobomurat Ahmedov, Shadow of the Kerr-like black hole / / The European Physical Journal C, Vol. 82 (2022), 9, 771 (Web of Science: IF=4.2), https://doi.org/10.1140/epjc/s10052-022-10711-4" and presented in the Doctorate (PhD) thesis of F. Sarikulov have been used in the framework of programs supported by the Fudan University (Letter from Prof. Cosimo Bambi).

Approbation of the research results. The dissertation results have been discussed in 1 international and 1 local conference.

Publication of the research results. 14 scientific publications have been made on research results, and 14 of them are research papers in refereed journals.

Volume and structure of the dissertation. The dissertation consists of an introduction, three chapters, a conclusion, and references, all in 83 pages.

THE MAIN CONTENT OF DISSERTATION

The introduction of the dissertation indicates the relevance and necessity of the topic, the correspondence of the research to the priority directions of development of science and technology of the republic, the degree of knowledge of the problem, its connection with the research plans of the higher educational institution in which the dissertation was carried out, and the purpose, objectives, object of research, brief information about the subject, methods, scientific novelty, practical result, reliability, scientific and practical significance of the results, introduction of the results into practice, approval of the results, publication of the results, as well as the structure and scope of the dissertation.

In the first chapter, entitled "Novel black hole solutions in extended gravity models and photon motion around them" new classes of black hole

solutions were obtained and analyzed within the framework of extended gravitational theories. We begin by formulating the coupling between Einstein's general relativity and a NED model, defined through the following action:

$$S = \int dx^4\sqrt{-g}\left[\frac{R}{16\pi G} + \mathcal{L}(\mathcal{F})\right] \tag{1}$$

where g represents the determinant of the metric tensor, R denotes the Ricci scalar, G is Newton's gravitational constant, and the Lagrangian that defines the NED model is expressed as

$$\mathcal{L}(\mathcal{F}) = \frac{-4b\mathcal{F}}{\left(\sqrt{b} + \sqrt{b - 2\sqrt{-\frac{\mathcal{F}}{2}}}\right)^2}. \tag{2}$$

In this context, $4\mathcal{F} = F_{\mu\nu}F^{\mu\nu} = 2(B^2 - E^2)$ represents the Maxwell invariant and b denotes the nonlinearity parameter. The electric field $E(r)$ was determined by setting the magnetic field $B = 0$ and solving the equation $\nabla_\mu\left(F^{\mu\nu}\frac{\partial\mathcal{L}}{\partial\mathcal{F}}\right) = 0$, the electric field $E(r)$ is determined as

$$E(r) = \frac{bq(2br^2 + q)}{2(br^2 + q)^2} \tag{3}$$

where q denotes the electric charge. The variation of the action (1) and the resolution of the Einstein field equations for the static spherically symmetric metric

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2\left(d\theta^2 + sin^2\,\theta d\phi^2\right) \tag{4}$$

yields the BH solution given by the lapse function:

$$f(r) = 1 - \frac{2M}{r} + \frac{\pi q\sqrt{bq}}{2r} - \frac{q\sqrt{bq}}{r}tan^{-1}\left(\frac{\sqrt{b}r}{\sqrt{q}}\right), \tag{5}$$

where M represents the mass of the black hole.

In nonlinear electrodynamics, a photon interacts with the field because of its nonlinearity and does not adhere to null geodesics. Consequently, it is necessary to consider the motion of photons in the spacetime of the black holes, as described by GR coupled with NED. For a purely electric field configuration, the only nonvanishing component of the electromagnetic tensor is electric field strength, and the electromagnetic invariant becomes

$$F = -\frac{1}{2}E^2(r) \tag{6}$$

where $E(r)$ is given by Eq. (3). In this case, the effective line element governing photon propagation can be written as

$$ds^2_{eff} = -\frac{f(r)}{\mathcal{L}_F}dt^2 + \frac{dr^2}{f(r)\mathcal{L}_F} + \frac{r^2}{\Phi(r)}\left(d\theta^2 + sin^2\,\theta d\phi^2\right) \tag{7}$$

where the function $\Phi(r)$ is given by

$$\Phi(r) = \mathcal{L}_F - 2F\mathcal{L}_{FF} \tag{8}$$

The null geodesics in the effective geometry admit two conserved quantities, namely the energy E and angular momentum L of the photon. Restricting the motion to the equatorial plane $\theta = \pi/2$, the radial equation of motion can be written as

$$\dot{r}^2 + V_{eff}(r) = 0 \tag{9}$$

with the effective potential

$$V_{eff}(r) = \frac{f(r)}{r^2}\frac{\Phi(r)}{\mathcal{L}_F}L^2 - E^2 \tag{10}$$

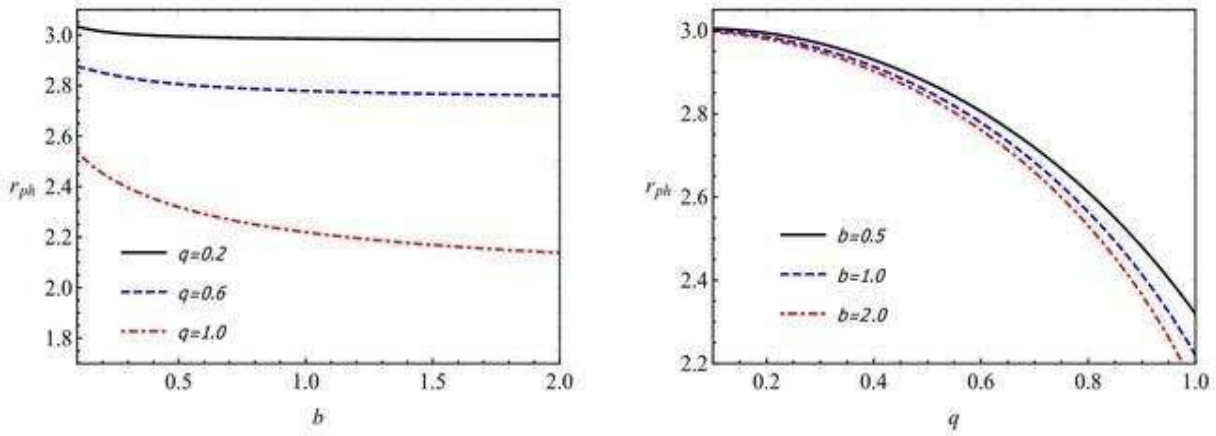


Figure 1: The dependencies of photon radii around BH in NED on parameters b and q, with $M = 1$.

Circular photon orbits, which determine the photon sphere and the black hole shadow, satisfy the conditions

$$V_{eff}(r) = 0, \ \frac{dV_{eff}(r)}{dr} = 0. \tag{11}$$

These conditions lead to the master equation

$$\frac{d}{dr}\left(\frac{r^2\mathcal{L}_F}{f(r)\Phi(r)}\right) = 0 \tag{12}$$

We can determine the radii of the photon sphere by using this equation. The dependencies of photon radii on the black hole parameters q and b are shown in Fig. 1. From these plots, we observe that increasing the electric charge q results in a reduction of the photon sphere radius, whereas the influence of the parameter b becomes more significant only for sufficiently large values of q .

We also worked on the extended 4D Einstein-Gauss-Bonnet black hole. The black hole solution, situated within the context of 4D Einstein-Gauss-Bonnet theory and surrounded by perfect fluid dark matter, is expressed as follows:

$$ds^2 = f(r)dt^2 - \frac{dr^2}{f(r)} - r^2\left(d\theta^2 + sin^2\,\theta d\phi^2\right) \tag{13}$$

with

$$f(r) = 1 + \frac{r^2}{2\alpha}\left(1 - \sqrt{1 + \frac{4\alpha}{r^3}\left(2M - \beta ln\left(\frac{r}{\beta}\right)\right)}\right), \tag{14}$$

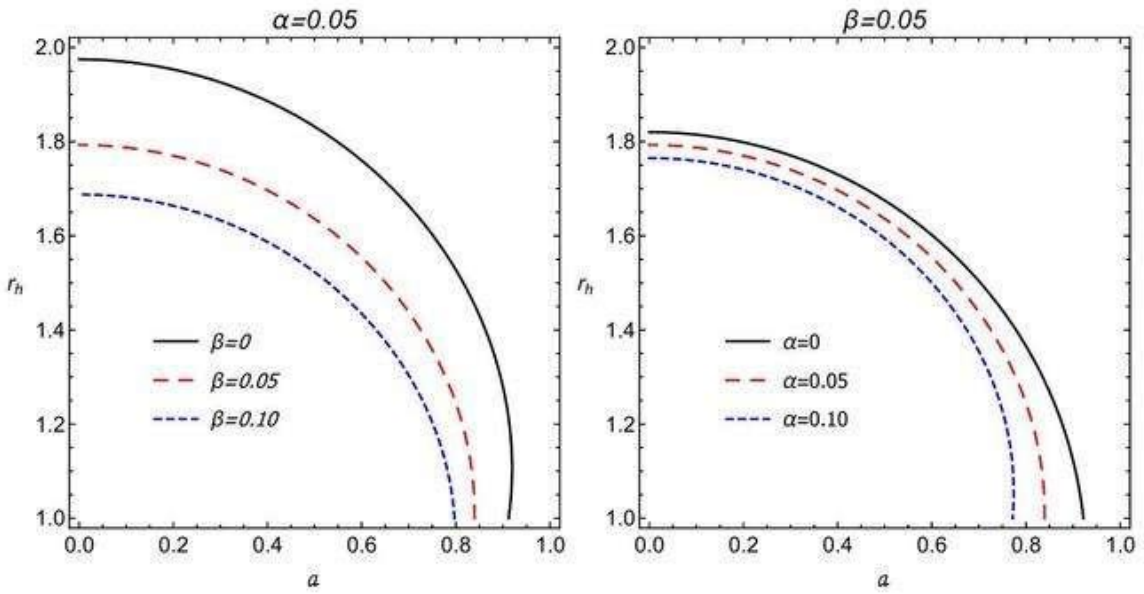


Figure 2: The event horizon radius as a function of parameters of rotating 4D EGB BH in PFDM, with *M =1*.

The Newman-Janis algorithm (NJA) was formulated to derive the rotating analogs of static black hole metrics within the framework of GR. the Kerr-like rotating BH metric is given by

$$ds^2 = -\frac{\Delta(r) - a^2 sin^2\,\theta}{\rho^2}dt^2 + \frac{\rho^2}{\Delta(r)}dr^2 + \rho^2 d\theta^2 + \frac{sin^2\,\theta}{\rho^2}\left((r^2+a^2)^2 - \Delta(r)a^2 sin^2\,\theta\right)d\phi^2 - \frac{2a sin^2\,\theta}{\rho^2}(a^2 + r^2 - \Delta(r))dtd\phi \quad (15)$$

with the metric functions

$$\rho^2 = r^2 + a^2 cos^2\,\theta \quad (16)$$

$$\Delta = r^2 + a^2 + \frac{r^4}{2\alpha}\left(1 - \sqrt{1 + \frac{4\alpha}{r^3}\left(2M - \beta ln\left(\frac{r}{\beta}\right)\right)}\right) \quad (17)$$

We will demonstrate that metric (15) for 4D EGB gravity is capable of representing rotating black holes with a complex structure and aligns with the black hole shadow characteristics derived from EHT observations. Figure 2 illustrates the relationship between the horizon radius and various black hole parameters. The graphs indicate that the event horizon radius decreases as the black hole parameters increase across all cases. The allowed space of parameters is described in Fig. 3.

Next, we aim to study particle motion around the rotating black hole, focusing on circular photon orbits that determine the black hole shadow. Using the Hamilton-Jacobi method, the problem is simplified by introducing conserved quantities such as energy, angular momentum, and the Carter constant.

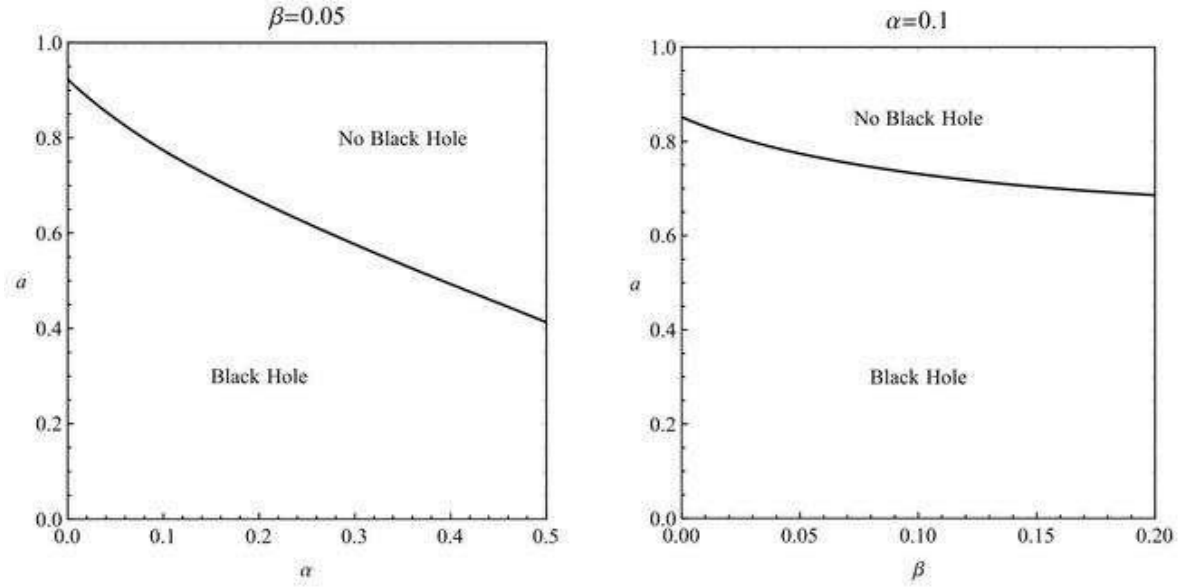


Figure 3: Plots showing the limits of parameters for which the event horizon exists for the black hole in NED, with $M = 1$.

This allows the equations of motion to be separated into radial and angular parts for easier analysis of geodesics. We derived the following geodesic equations:

$$\rho^2 \frac{dt}{d\lambda} = a\left(L - aEsin^2\,\theta\right) + \frac{r^2 + a^2}{\Delta}\left(E(r^2 + a^2) - aL\right), \tag{18}$$

$$\rho^2 \frac{dr}{d\lambda} = \pm\sqrt{\mathcal{R}}, \tag{19}$$

$$\rho^2 \frac{d\theta}{d\lambda} = \pm\sqrt{\Theta}, \tag{20}$$

$$\rho^2 \frac{d\phi}{d\lambda} = \left(Lcsc^2\,\theta - aE\right) + \frac{a}{\Delta}\left(E(r^2 + a^2) - aL\right). \tag{21}$$

where

$$\mathcal{R} = \left[(r^2 + a^2)E - aL\right]^2 - \Delta[m^2r^2 + \mathcal{K} + (L - aE)^2] \tag{22}$$

$$\Theta = \mathcal{K} + cos^2\,\theta\left(a^2(E^2 - m^2) - L^2sin^{-2}\,\theta\right) \tag{23}$$

The radial motion (19) can be reformulated as

$$\left(\rho^2 \frac{dr}{d\lambda}\right)^2 + V_{eff} = 0. \tag{24}$$

the effective potential reads

$$V_{eff} = -\frac{\left((a^2 + r^2)E - aL\right)^2 - \Delta(\mathcal{K} + (L - aE)^2)}{2r^4} \tag{25}$$

The diagrams presented in Fig. 4 illustrate the variation of the effective potential V_{eff} with respect to the radial coordinate r for different black hole parameters. By analyzing the peaks of these plotted curves, one can discern the unstable circular null orbits (Fig. 4).

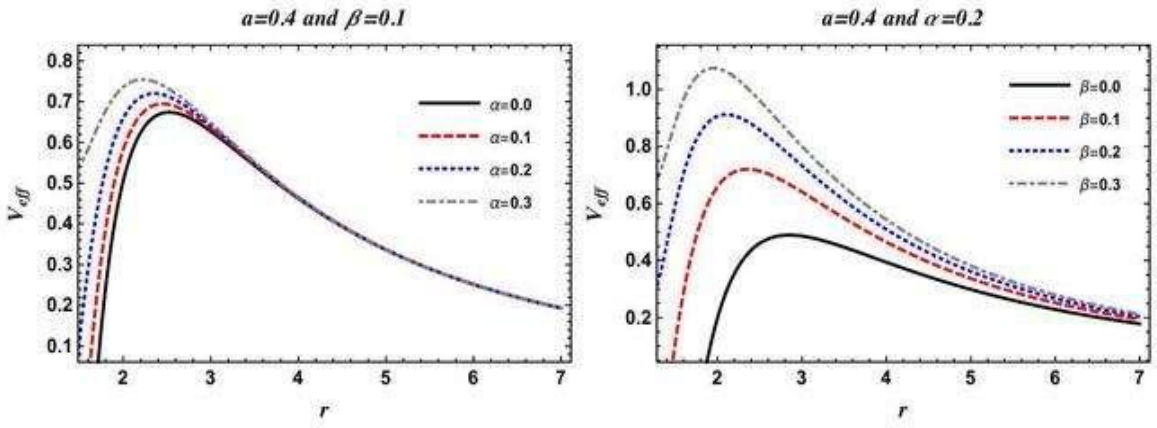


Figure 4: Plots showing the radial dependence of the effective potential for chosen values of BH parameters.

The unstable circular photon orbit satisfies the following conditions

$$V_{eff} = 0, \ \frac{\partial V_{eff}}{\partial r} = 0, \ \frac{\partial^2 V_{eff}}{\partial r^2} < 0. \tag{26}$$

The third condition ensures the orbit is unstable. Solving them, we obtain

$$\xi = \frac{(a^2 + r^2)\Delta' - 4\Delta r}{a\Delta'} \tag{27}$$

$$\eta = \frac{r^2\left(16\Delta(a^2 - \Delta) - r^2\Delta'^2 + 8\Delta r\Delta'\right)}{a^2\Delta'^2}, \tag{28}$$

where the prime symbol indicates differentiation with respect to r.

In the second chapter, entitled "Black hole shadows in extended theories of gravity" the shadow structures of rotating and non-rotating black holes in various extended gravity models were analyzed. The effects of additional parameters, such as the Gauss-Bonnet coupling constant α, dark matter parameter β, nonlinear electrodynamics parameter b, and Yang-Mills charge, were thoroughly investigated. It was shown that these parameters significantly influence the shape, size, and distortion of the black hole shadow, thereby altering the observable features. For each case, shadow observables such as the shadow radius and distortion parameter were computed. Furthermore, a modified approach was used to estimate black hole parameters from shadow profiles. The results demonstrate that shadow characteristics can serve as effective probes for detecting deviations from general relativity and for constraining parameters of alternative gravitational models.

The black hole shadow is the dark region formed by photons that fall into the event horizon, contrasted by the bright photon ring created by photons that are scattered away to infinity due to strong gravitational lensing. This shadow provides crucial observational insights into the gravitational effects of black holes on the surroundings.

The shadow images of 4D EGB black holes for the various values of parameters are illustrated in Figure 5. It is clear from the plots that both GB and DM parameters cause a diminished area of shadow, while there can be some distortion in the larger values of spin parameters, as expected.

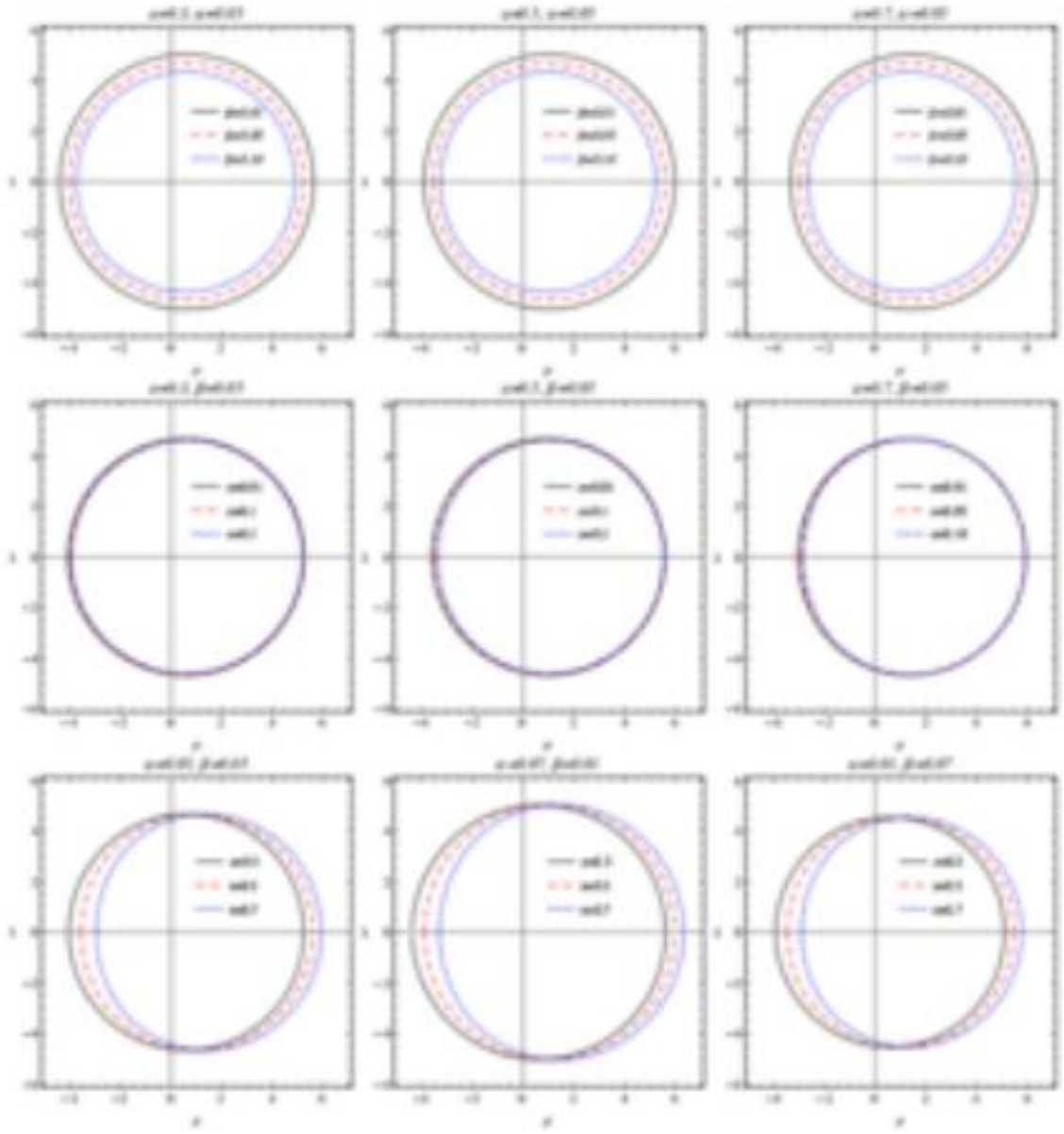


Figure 5: Shadow images of rotating 4D EGB BH for different cases.

The estimation of the properties of black holes can be achieved by utilizing shadow analysis. To proceed, it is necessary to establish the precise measurable variables that characterize the dimensions and configuration of the black hole's shadow. The parameters denoted as the area of the black hole shadow, denoted as A, and the oblateness, denoted as D, are closely linked to the deformation of the black hole shadow.

$$A = 2\int_{r_-}^{r_+} \left(\lambda(r)\frac{d\mu(r)}{dr}\right) dr \tag{27}$$

$$D = \frac{\Delta\mu}{\Delta\lambda} \tag{28}$$

where $r_\pm$ and r_- are radii of unstable circular orbits.

A/M^2	D	a/M	α/M^2	R_a
68	0.99	0.3868	0.0028	4.6524
67	0.98	0.5081	0.0342	4.6180
66	0.97	0.5819	0.0651	4.5835
65	0.96	0.6207	0.1016	4.5486
64	0.95	0.6404	0.1386	4.5135
63	0.94	0.6384	0.1810	4.4781

Table 1: Estimated values of BH parameters a/M and α/M^2 from shadow observables A/M^2 and D for the fixed value of $\beta = 0.05M$.

A/M^2	D	a/M	β/M	R_a
74	0.98	0.4926	0.0197	4.8533
72	0.97	0.5807	0.0241	4.7873
70	0.96	0.6399	0.0290	4.7203
68	0.96	0.6325	0.0369	4.6524
66	0.95	0.6763	0.0429	4.5835
64	0.95	0.6684	0.0522	4.5135

Table 2: Estimated values of BH parameters a/M and β/M from shadow observables A/M^2 and D for the fixed value of $\alpha = 0.1M^2$.

Estimates for different cases are described by contour plots of the BH parameters in Figure 6. The figures shown in this analysis demonstrate that the coordinates of the intersections serve as unique determinants for the two BH parameters in each respective case. The estimation method can be employed to ascertain the values of BH parameters based on the shadow of the black hole. We also present numerical results in table 1 and 2 to better demonstrate our findings. The obtained results possess the potential to serve as a valuable instrument for acquiring information about the BH through analysis of its shadow properties, which may be determined by observational measurements. Furthermore, the results can be used further to investigate the properties and behaviour of black holes, shedding light on their formation, evolution, and interactions with surrounding matter. The potential applications of this estimation method extend beyond astrophysics, as it could also contribute to our understanding of general relativity and the fundamental laws of physics.

The presence of the PFDM parameter β considerably affects the size and shape of the black hole shadow. As β increases, the shadow's size diminishes, indicating that PFDM reduces the apparent size of the black hole shadow. Non-

rotating black holes cast perfectly circular shadows, while rotating black holes exhibit distorted shadows that become more pronounced with increasing of parameters a and β. The distinct shadow characteristics due to dark matter provide a means to probe the distribution and properties of dark matter around black holes.

We observed that increasing a and β diminishing the shadow size while the oblateness grows. To estimate the parameters α and β for a rotating Gauss Bonnet black hole surrounded by dark matter, we plotted the observables A (shadow area) and D (oblateness) as functions of α and β for different values of a. Each plot represents curves with constant values of A or D. The unique intersection points of these curves provide specific values for the black hole parameters α and β.

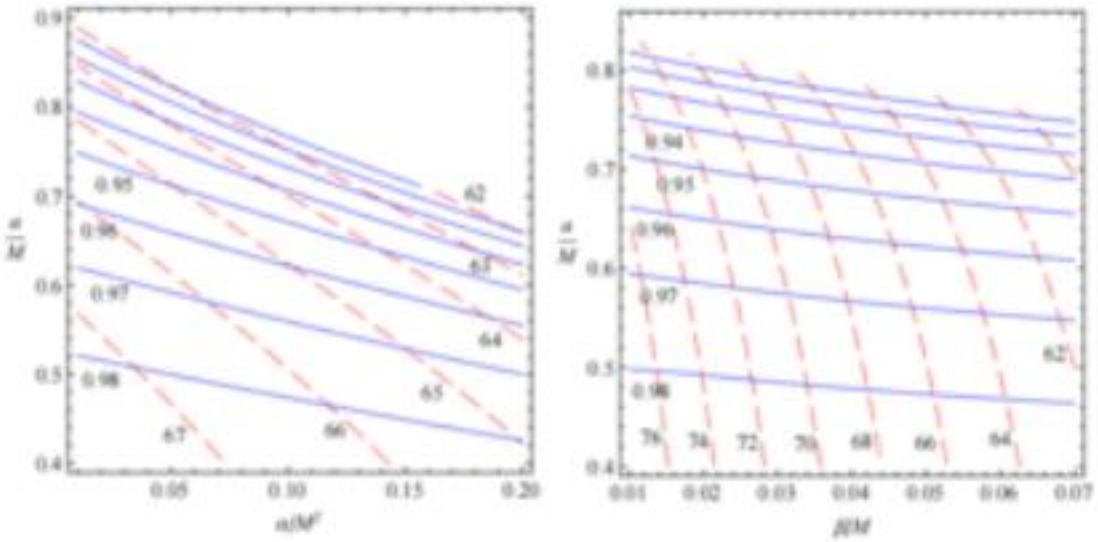


Figure 6: The contour plot illustrates the observables A/M^2 and D for the rotating 4D EGB black hole surrounded by PFDM in particular parameter spaces. The fixed values of parameters are $\beta = 0.05M, \alpha = 0.1M^2$. The dashed red lines indicate constant values of A, while the solid blue lines represent constant values of the oblateness parameter D for parameter estimation.

We also studied the shadow of non-rotating black holes in nonlinear electrodynamics. The dependencies of shadow radius on the black hole parameters q and b are shown in Fig. 7. From these plots, we observe that an increase in the electric charge q leads to a decrease in the black hole shadow radius. In contrast, the nonlinear electrodynamics parameter b has a noticeable impact on the shadow only in the small b regime ($0.1 \lesssim b \lesssim 1.5$), where nonlinear effects become significant; for larger values of b, its influence gradually diminishes as the system approaches the linear electrodynamics limit.

To demonstrate the influence of nonlinear electrodynamics on photon propagation, we present a comparative analysis of the black hole shadow radii calculated using the original spacetime metric and the effective geometry governing photon motion. We find that the shadow radius obtained within the effective geometry is systematically larger than that derived from the original metric. This indicates that nonlinear electrodynamic effects effectively enlarge the black hole shadow by modifying the photon trajectories through their interaction with the background electromagnetic field.

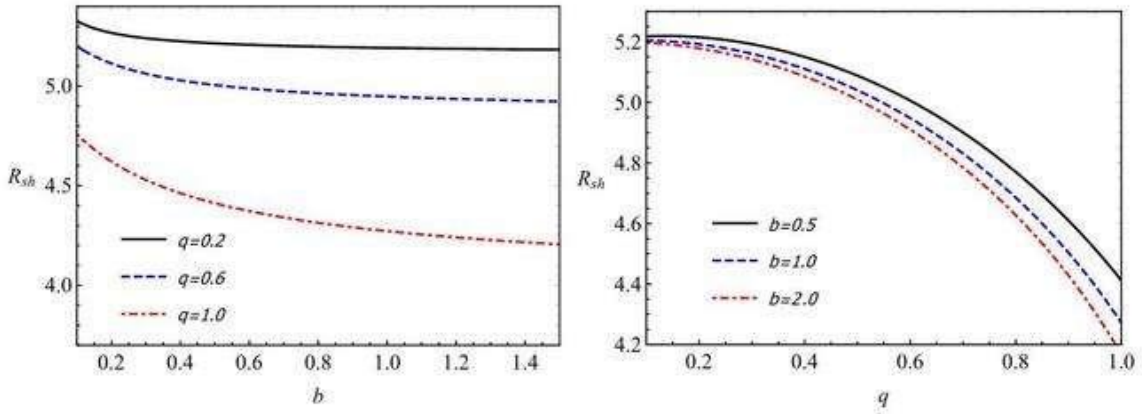


Figure 7: The shadow radii of BH in NED as functions of parameters b and q , with $M = 1$.

In the third chapter, entitled "Constraints on black hole parameters using EHT observations" black hole shadow-based constraints were obtained using observational data from the Event Horizon Telescope for the supermassive black holes M87* and Sgr A*. The analysis was performed within the framework of alternative gravity models, including charged black holes in nonlinear electrodynamics and 4D Einstein-Gauss-Bonnet black holes surrounded by perfect fluid dark matter and power-Yang-Mills fields. By comparing the theoretical shadow radii with the EHT data, parameter bounds were derived for each model.

We focus on obtaining the constraints of the supermassive BHs Sgr. A* and M87* based on recent observational data reported by the EHT Collaboration. First, we define the observable parameters that we use to describe the size and shape of the BH shadow. The angular diameter of the shadow image for an observer at a distance d from BH can be expressed as

$$\theta_d = 2\frac{R_a}{d}, \quad R_a = \sqrt{\frac{A}{\pi}}, \tag{29}$$

where R_a is the areal shadow radius.

We used the following observational data to obtain constraints. The mass of M87* and the distance from Earth can be considered as $M = 6.5 \times 10^9 M_\odot$ and $d = 16.8Mpc$, respectively. For simplicity, we do not take into account the uncertainties of the mass and distance measurements of the supermassive BHs . The angular diameter of the image of the supermassive BH M87* is $\theta_d = 42 \pm 3\mu as$ in the $1-\sigma$ confidential level. The angular diameter of the shadow of the supermassive BHSgrA* is $\theta_d = 48.7 \pm 7\mu as$. The mass of SgrA* and the distance from the solar system be considered as $M \simeq 4 \times 10^6 M_\odot$ and $d \simeq 8kpc$, respectively.

We first consider supermassive BH Sgr A* as 4D EGB BH in PFDM to compare our theoretical results of shadow analysis with the shadow image of Sgr A* from EHT data. In the context of our research, we use a simplified approach by disregarding the potential uncertainties associated with the measurements of the masses and distances of the supermassive black hole. Our results are subject to some uncertainty due to the uncertainty in the angular diameter of Sgr A* from EHT observations. However, this analysis allows us to accurately determine how

BH and DM parameters affect on the observable parameter. In Figure 8, the constraints from Sgr A* are described for different cases. The fitted density plots of the angular diameter of BH lay inside the measured angular diameter of Sgr A*, $\theta_d = 48.7 \pm 7\mu as$. Therefore, we can claim that our theoretical results of 4D EGB BH in PFDM can be aligned with the supermassive black hole at the center of the Milky Way galaxy.

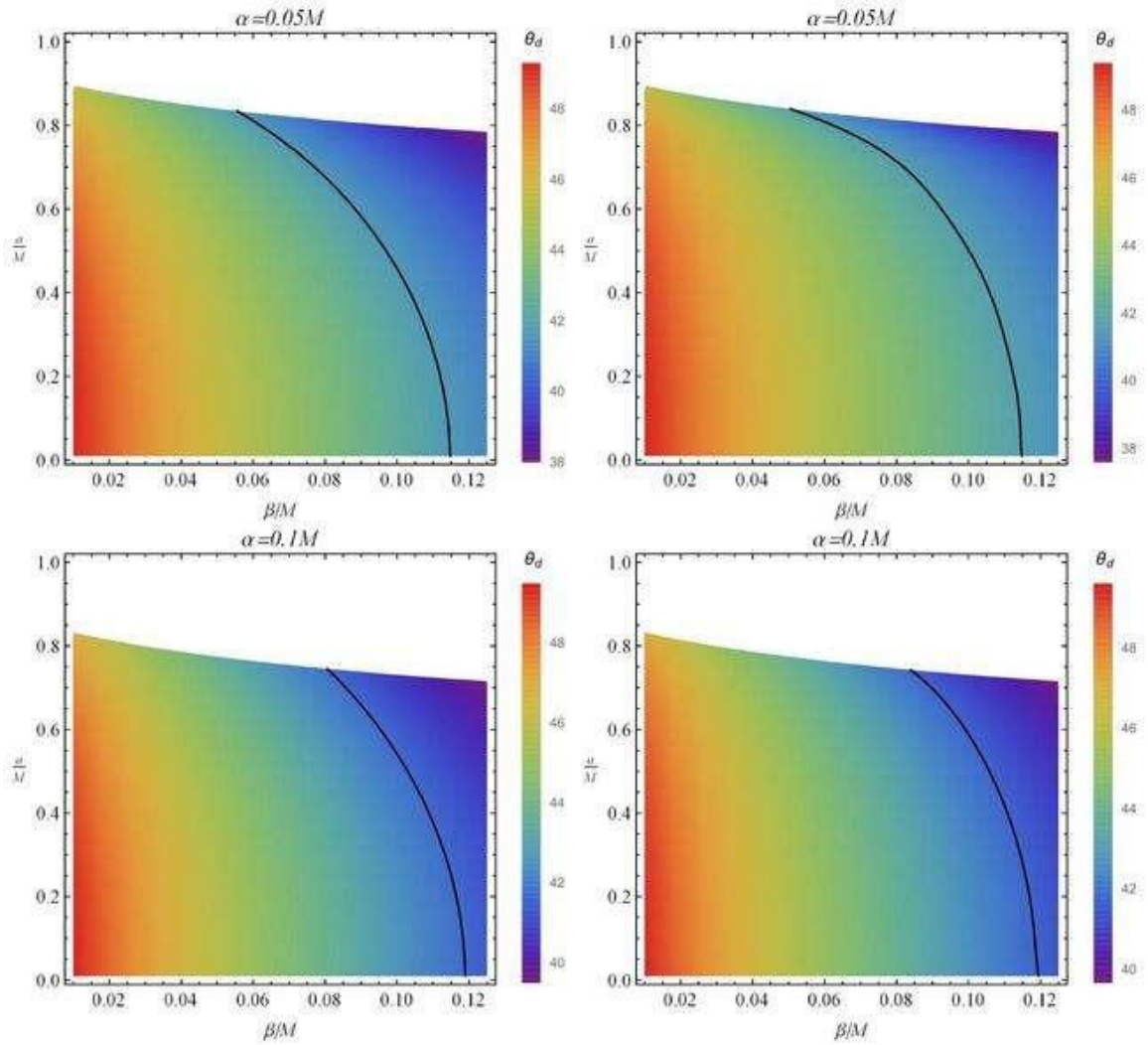


Figure 8: Angular diameter θ_d for a rotating 4D GB black hole surrounded by PFDM in (a/M, β/M^2) parameter space at inclination angles 20° (left) and 50° (right).

Today in black hole physics, there are a bunch of black hole solutions for different modified/alternative theories of gravity. As we continue to investigate new gravity models, their viability must be evaluated using observational data and one of the observations of black holes is shadow images by EHT. In this research, we get constraints on parameters of 4D EGB BH and PFDM based on EHT observations. We can conclude that this constraining method can be useful to study black hole solutions in novel gravity models. Furthermore, we can also investigate the dark matter effect on black hole physics by this approach.

Novel charged black holes in nonlinear electrodynamics. The spacetime is characterized by two dimensionless parameters: the charge parameter q, associated with the NED field, and the nonlinear coupling parameter b, which encodes

deviations from standard Maxwell electrodynamics. For each point in the (b, q) parameter space, the angular diameter of the black hole shadow, denoted by θ_d, is computed from the radius of the unstable circular photon orbit, taking into account the effective geometry governing photon propagation in NED.

Figure 9 shows contour plots of the angular diameter θ_d in the (b, q) plane for M87* (left panel) and Sgr A* (right panel). The color scale represents the magnitude of θ_d , while the solid black curves correspond to the lower observational bounds inferred from the EHT measurements. Regions of the parameter space lying below these curves are excluded, as they predict shadow sizes smaller than those allowed by observations.

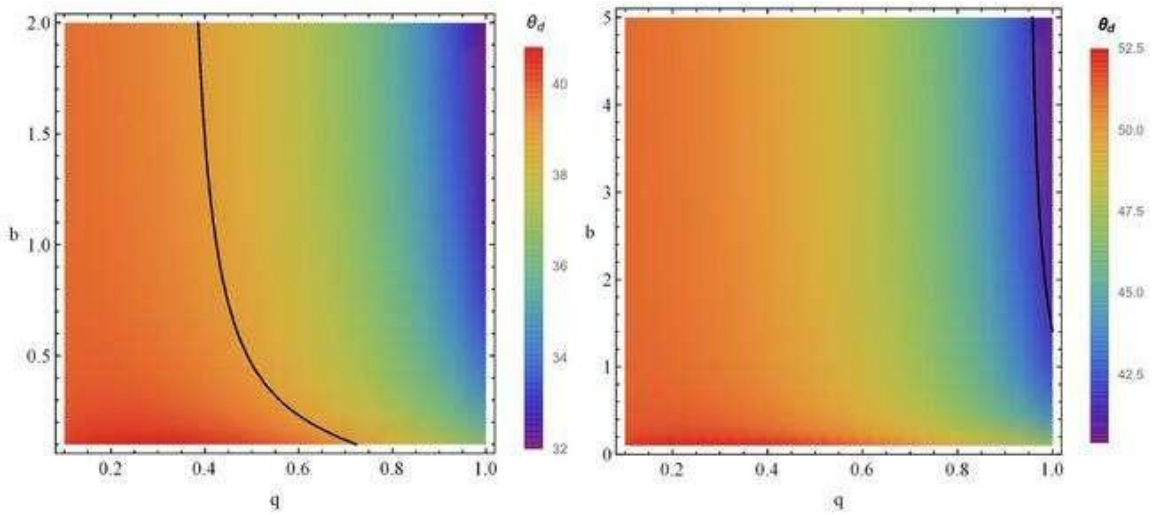


Figure 9: Observable angular diameter of novel charged black hole in NED in the parameter space of *b*-*q* for M87* (left) and Sgr A* (right). The black curves describe the lower borders of the measured angular diameter of the SMBH M87* and Sgr A* reported by the EHT.

From the left panel corresponding to $M87^*$, it is evident that the angular diameter decreases monotonically with increasing charge parameter q for fixed values of b. This behavior can be understood as a consequence of the effective weakening of the gravitational attraction near the black hole due to the presence of the NED charge, which leads to a reduction in the photon sphere radius and, consequently, a smaller shadow. The nonlinear parameter b further modulates this effect, with larger values of b enhancing the deviation from the Schwarzschild limit and thus tightening the constraints on q.

A similar qualitative behavior is observed for $SgrA^*$, as shown in the right panel of Fig. 9. However, the corresponding constraints are comparatively weaker, reflecting the larger observational uncertainties associated with the shadow measurement of Sgr A*. Nevertheless, the exclusion curve still imposes an upper bound on the charge parameter across a broad range of b, ruling out configurations with sufficiently large q that would otherwise yield an unacceptably small shadow.

CONCLUSIONS

The following conclusions have been presented on the basis of research carried out on the dissertation.

1. For the first time, it has been obtained and analyzed rotating black hole metrics in extended 4D Einstein-Gauss-Bonnet gravity (EGB), demonstrating deviations from classical Kerr-Newman metrics.

2. For the first time, the photon orbits, shadow boundaries, and distortion parameters have been investigated for black holes in nonlinear electrodynamics (NED), an extended 4D EGB gravity scenarios. It has been shown that black hole shadows in alternative theories exhibit measurable differences in size, distortion, and photon ring structure compared to general relativity (GR).

3. Parameter estimation plots have been obtained for the 4D EGB black holes through shadow observables (area and oblateness), providing an effective method for connecting black hole properties with observable quantities.

4. It has been found that the presence of NED increase the shadow radius of charged black holes compared to the standard Einstein–Maxwell case, providing an observable signature of NED corrections.

5. It has been established that perfect fluid dark matter (PFDM) surrounding black holes significantly reduces shadow radii, providing an observational signature to constrain dark matter density profiles.

6. For the first time the constraints on black hole parameters in NED, in extended 4D EGB gravity have been found using M87* and Sgr A* shadow observations from EHT.

7. Specific observational signatures (shadow asymmetry) have been identified to distinguish between GR and alternative theories with next-generation telescopes (ngEHT).

НАУЧНЫЙ СОВЕТ DSc.03/07.07.2025. FM/T.192.01
ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ИНСТИТУТЕ ПЕРСПЕКТИВНЫХ ИССЛЕДОВАНИЙ УНИВЕРСИТЕТА "НОВЫЙ УЗБЕКИСТАН"

АСТРОНОМИЧЕСКИЙ ИНСТИТУТ

САРИКУЛОВ ФУРКАТ ШЕРАЛИ УГЛИ

ТЕСТИРОВАНИЕ ПРИРОДЫ ЧЁРНЫХ ДЫР В АЛЬТЕРНАТИВНЫХ ТЕОРИЯХ ЧЕРЕЗ АСТРОФИЗИЧЕСКИЕ НАБЛЮДЕНИЯ

01.03.01 – Астрономия

АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD) ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

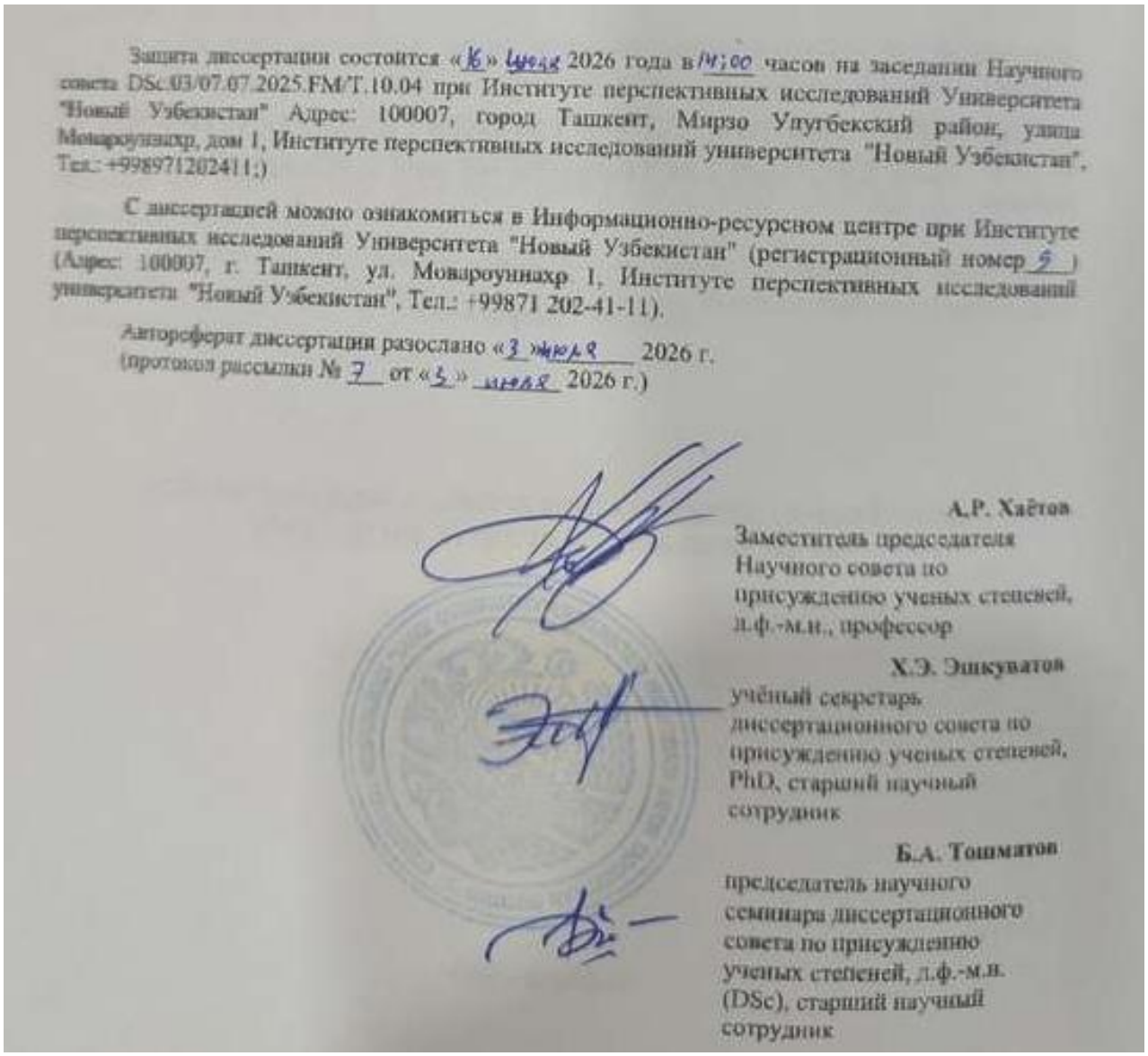
Toshkent - 2026

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Академии наук Республики Узбекистан за номером B2025.4.PhD/FM1412.

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Цель исследования. Целью настоящей работы является проверка альтернативных теорий гравитации на основе анализа теней чёрных дыр и получения ограничений, вытекающих из наблюдений Телескопа горизонта событий (EHT).

Задачи исследования. Для достижения поставленной цели в диссертации решаются следующие задачи:

- вывести новые решения чёрных дыр в нелинейной электродинамике (NED) и 4D теории Эйнштейна–Гаусса–Бонне (EGB);
- расширить статические решения на вращающиеся случаи с использованием алгоритма Ньюмана–Джаниса и проанализировать структуру горизонтов;
- изучить движение фотонов и эффективные потенциалы для определения неустойчивых круговых орбит и границ тени;
- вычислить тени чёрных дыр для различных гравитационных моделей и сравнить их с наблюдениями EHT;
- оценить параметры чёрных дыр (спин, заряд, константы связи) по наблюдаемым характеристикам тени (площадь, сплюснутость);
- сопоставить теоретические предсказания с данными EHT для ограничения отклонений от общей теории относительности.

Объекты исследования. Статические чёрные дыры в нелинейной электродинамике; 4D чёрные дыры Эйнштейна–Гаусса–Бонне, окружённые идеальной жидкой тёмной материей (PFDM); чёрные дыры в полях Power–Yang–Mills (PYM) в рамках модифицированной гравитации; сверхмассивные чёрные дыры M87* и Sgr A* как наблюдательные тестовые объекты.

Предмет исследования. Метрики и структура горизонтов чёрных дыр в альтернативных теориях гравитации; фотонные сферы и неустойчивые круговые орбиты в модифицированных пространствах-временах; морфология тени чёрных дыр и наблюдаемые параметры; методы оценки параметров с использованием данных EHT; ограничения параметров модифицированной гравитации на основе астрофизических наблюдений; сравнительный анализ различных альтернативных сценариев гравитации.

Методы исследования: аналитическое моделирование уравнений нелинейной электродинамики и модифицированных гравитационных теорий; численные методы реализации алгоритма Ньюмана–Джаниса для получения вращающихся решений чёрных дыр; формализм Гамильтона–Якоби для вычисления эффективных потенциалов движения пробных частиц; стандартные методы реконструкции тени чёрной дыры; сравнительный анализ различных гравитационных моделей.

Научная новизна исследования:

впервые получены ранее неизвестные вращающиеся решения чёрных дыр в 4D теории Эйнштейна–Гаусса–Бонне, связанной с идеальной жидкой тёмной материей и полем Power–Yang–Mills, выходящие за рамки классических метрик Керра–Ньюмана;

получены оценки параметров для чёрных дыр Эйнштейна–Гаусса–Бонне, окружённых тёмной материей PFDM;

выявлены уникальные наблюдательные сигнатуры PFDM в тенях чёрных дыр, показывающие возможность ограничения плотности тёмной материи через высокоточные изображения.

Практические результаты:

впервые получены количественные ограничения на спин чёрной дыры (a), заряд NED (q) и константу Гаусса–Бонне (α) с использованием данных EHT, что даёт проверяемые предсказания для будущих наблюдений;

модифицированы открытые численные коды для моделирования теней и оценки параметров в альтернативных теориях гравитации, обеспечивая возможность проверки со стороны научного сообщества;

предсказаны специфические искажения теней для ngEHT, что позволяет выделять наблюдательные цели для различения ОТО и альтернативных теорий.

Достоверность результатов исследования обеспечивается применением современных, проверенных методов математической физики, вычислительной математики и релятивистской астрофизики. Все полученные решения чёрных дыр удовлетворяют фундаментальным полевым уравнениям соответствующих модифицированных теорий гравитации, что обеспечивает математическую строгость и согласованность с установленными рамками. Критические результаты (радиусы теней, искажения) вычислены независимыми аналитическими и численными методами, совпадение которых подтверждает надёжность. Параметрические ограничения согласуются с эмпирическими данными Event Horizon Telescope (M87*, Sgr A*), а отклонения от предсказаний ОТО находятся в пределах текущих наблюдательных неопределённостей. Предположения (например, асимптотическая плоскость, конкретные профили тёмной материи) и их влияние явно обсуждены, что определяет область применимости.

Научная и практическая значимость результатов исследования:

полученные результаты обеспечивают строгую проверку альтернативных моделей гравитации (например, 4D EGB, NED) путём сравнения их решений чёрных дыр с наблюдательными данными, что позволяет подтверждать или ограничивать расширения общей теории относительности;

показано, что тени чёрных дыр и их динамика могут выявлять наличие и распределение тёмной материи, предоставляя новый астрофизический инструмент её изучения;

получены конкретные наблюдательно проверяемые предсказания (формы теней) для текущих (EHT) и будущих (ngEHT) инструментов, что помогает в поиске отклонений от ОТО;

разработаны открытые аналитические и численные методы для анализа теней чёрных дыр и оценки параметров в модифицированной гравитации, что способствует более широким исследованиям и совместной проверке результатов.

Практическое применение результатов исследования.

Результаты исследования динамики частиц в окрестностях чёрных дыр в рамках модифицированных теорий гравитации нашли следующее применение:

Теоретические результаты и методы, опубликованные в статье
Furkat Sarikulov, Farruh Atamurotov, Ahmadjon Abdujabbarov, Bobomurat Ahmedov, “Shadow of the Kerr-like black hole” // The European Physical Journal C, Vol. 82 (2022), No. 9, 771 (Web of Science: IF = 4.2), DOI: [10.1140/epjc/s10052-022-10711-4]
и представленные в докторской (PhD) диссертации Ф. Сарикулова, были использованы в рамках исследовательских программ, поддержанных Фуданьским университетом (письмо проф. Козимо Бамби).

Апробация результатов исследования. Основные результаты диссертации были обсуждены на одной международной и одной республиканских конференциях.

Публикации по теме исследования. По результатам выполненной работы опубликовано 14 научных трудов, все из которых представлены в рецензируемых научных журналах.

Объём и структура диссертации. Диссертация состоит из введения, трёх глав, заключения и списка литературы и занимает 83 страниц.

ВЫВОДЫ

Основные выводы, полученные в результате проведённого исследования:

1. Впервые были получены и проанализированы метрики вращающихся чёрных дыр в расширенной 4D теории Эйнштейна–Гаусса–Бонне (EGB), демонстрирующие отклонения от классической метрики Керра–Ньюмана.
2. Впервые исследованы фотонные орбиты, границы тени и параметры искажения для чёрных дыр в нелинейной электродинамике (NED), в рамках расширенной 4D EGB-гравитации. Показано, что тени чёрных дыр в альтернативных теориях демонстрируют измеримые различия в размере, искажении и структуре фотонного кольца по сравнению с общей теорией относительности (ОТО).
3. Получены графики оценки параметров для чёрных дыр 4D EGB на основе наблюдаемых характеристик тени (площадь и сплюснутость), что предоставляет эффективный метод связи свойств чёрных дыр с наблюдаемыми величинами.
4. Обнаружено, что наличие NED увеличивает радиус тени заряженных чёрных дыр по сравнению со стандартным случаем Эйнштейна–Максвелла, что даёт наблюдаемый сигнал корректировок NED.
5. Установлено, что идеальная жидкая тёмная материя (PFDM), окружающая чёрные дыры, существенно уменьшает радиусы теней, предоставляя наблюдательный способ ограничения профилей плотности тёмной материи.

6. Впервые получены ограничения на параметры чёрных дыр в NED и расширенной 4D EGB гравитации на основе наблюдений теней M87* и Sgr A* с помощью EHT.

7. Выявлены специфические наблюдательные сигнатуры (асимметрия тени), позволяющие различать ОТО и альтернативные теории гравитации с использованием телескопов следующего поколения (ngEHT).

E’LON QILINGAN ISHLAR RO’YXATI
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