“YANGI O‘ZBEKISTON” UNIVERSITETI HUZURIDAGI ILG‘OR TADQIQOTLAR INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.03/07.07.2025. FM/T.192.01 RAQAMLI ILMIY KENGASH

SHAROF RASHIDOV NOMIDAGI SAMARQAND DAVLAT UNIVERSITETI

RAHMATOV BEKZOD MANSUR O‘G‘LI

TASHQI MATERIYA MAYDONLARI MAVJUDLIGIDA QORA O‘RALARNING TERMODINAMIK XUSUSIYATLARI VA FUNDAMENTAL CHASTOTALAR

01.04.02 – Nazariy fizika

FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

Toshkent – 2026

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KIRISH (Falsafa doktori (PhD) dissertatsiya annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi va zarurligi. Bugungi Qora o‘ralar kuchli gravitatsion maydon rejimida umumiy nisbiylik nazariyasini tekshirish uchun eng muhim obyektlardan biri hisoblanadi. Ularning fazo-vaqt geometriyasi, gorizont tuzilishi, termodinamik xatti-harakati va zarralar dinamikasi kompakt obyektlar yaqinida sodir bo‘ladigan kuzatiladigan astrofizik jarayonlar bilan bevosita bog‘liqdir. Xususan, sinov zarralarining harakati, akkretsiya bilan bog‘liq hodisalar hamda rentgen nurlanishida kuzatiladigan kvazidavriy tebranishlar qora o‘ralar va ularni o‘rab turgan muhitning fizik xossalarini o‘rganishda muhim vosita bo‘lib xizmat qiladi.

So‘nggi astrofizik kuzatuvlar galaktikalarda qorong‘i moddaning taqsimoti kutilganidan ham xilma-xil bo‘lishi mumkinligini ko‘rsatmoqda. Qorong‘i modda miqdori juda kam bo‘lgan yoki qorong‘i modda komponenti aniq kuzatilmaydigan NGC 1052-DF2, NGC 1052-DF4, FCC 224 hamda qorong‘i modda yetishmaydigan spiral galaktikalar kabi bir qator tizimlar qorong‘i modda ta’siri astrofizik muhitga kuchli bog‘liq bo‘lishi mumkinligini ko‘rsatadi. Sonli modellashtirish natijalari ham bunday tizimlar tidal o‘zaro ta’sirlar va moddaning ajralib chiqishi kabi muhit bilan bog‘liq mexanizmlar natijasida yuzaga kelishi mumkinligini ko‘rsatmoqda. Ushbu natijalar kompakt obyektlar, ayniqsa qora o‘ralar atrofidagi mumkin bo‘lgan qorong‘i muhitlarni tavsiflovchi effektiv modellarni o‘rganishga asos bo‘ladi.

Umumiy nisbiylik nazariyasi doirasida mukammal suyuqlik ko‘rinishidagi qorong‘i modda va Chaplygin-simon qorong‘i suyuqlik qora o‘ra fizikasiga qorong‘i muhitlarning ta’sirini o‘rganish uchun foydali effektiv tavsiflarni beradi. Mukammal suyuqlik ko‘rinishidagi qorong‘i modda qora o‘ralar yaqinidagi qorong‘i modda effektlarini fenomenologik tarzda tavsiflash imkonini beradi, Chaplygin-simon qorong‘i suyuqlik esa qorong‘i moddaga o‘xshash va qorong‘i energiyaga o‘xshash xatti-harakatlar orasida o‘tuvchi muhit sifatida muhim ahamiyatga ega. Shu sababli bunday maydonlar bilan o‘ralgan qora o‘ralarni o‘rganish tashqi qorong‘i muhitlarning kuchli gravitatsion xossalarga qanday ta’sir qilishini tushunish uchun dolzarb hisoblanadi.

Mukammal suyuqlik ko‘rinishidagi qorong‘i modda va Chaplygin qorong‘i suyuqligining mavjudligi qora o‘ralar atrofidagi fazo-vaqt tuzilishini o‘zgartirishi, hodisalar gorizontiga ta’sir qilishi hamda Hawking temperaturasi, entropiya va issiqlik sig‘imi kabi termodinamik kattaliklarni modifikatsiya qilishi mumkin. Ushbu maydonlar qora o‘ralar atrofida harakatlanayotgan zarralarning Kepler, radial va vertikal epitsiklik chastotalariga ham ta’sir ko‘rsatadi. Mazkur fundamental chastotalar mikrokvazarlar va boshqa kompakt astrofizik tizimlarda kuzatiladigan kvazidavriy tebranishlar bilan chambarchas bog‘liq bo‘lganligi sababli, ularni tahlil qilish nazariy qora o‘ra modellarini kuzatuv ma’lumotlari bilan bog‘lashda muhim ahamiyatga ega.

Ushbu dissertatsiyaning zarurati qora o‘ralarni real astrofizik muhitlarda o‘rganish hamda qorong‘i modda va qorong‘i suyuqlik parametrlarining kuzatiladigan kuchli maydon signaturalariga qanday ta’sir qilishini aniqlash ehtiyoji

bilan belgilanadi. Xususan, zarralar dinamikasi, termodinamik xossalar, fundamental chastotalar va kvazi davriy tebranishlar (QPO) signaturalarini tadqiq etish umumiy nisbiylik nazariyasi doirasida qora o‘ra modellarini tekshirish va nazariy bashoratlarni astrofizik kuzatuvlar bilan taqqoslash imkonini beradi. Shunday qilib, dissertatsiya qora o‘ra termodinamikasi, orbital harakat va kvazidavriy tebranishlarga mukammal suyuqlik ko‘rinishidagi qorong‘i modda hamda Chaplygin qorong‘i suyuqligining ta’sirini o‘rganishga bag‘ishlangan bo‘lib, bu masalalar zamonaviy relyativistik astrofizika va kuchli gravitatsion maydon fizikasi muammolari qatoriga kiradi.

Mazkur dissertatsiya O‘zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PF-4947-sonli “O‘zbekiston Respublikasini yanada rivojlantirish bo‘yicha Harakatlar strategiyasi to‘g‘risida”gi Farmoni hamda 2017-yil 18-fevraldagi PQ-2789-sonli “Fanlar akademiyasi faoliyati, ilmiy-tadqiqot ishlarini tashkil etish, boshqarish va moliyalashtirishni yanada takomillashtirish chora-tadbirlari to‘g‘risida”gi Qarorida belgilangan vazifalarga mos keladi.

Tadqiqotning Respublika fan va texnikasini rivojlantirishning asosiy ustuvor yo‘nalishlariga muvofiqligi. Dissertatsiya ishi bo‘yicha tadqiotlar O‘zbekiston Respublikasi fan va texnikasining ustuvor yo‘nalishlariga muvofiq amalga oshirildi.

Muammoning o‘rganilganlik darajasi. Kerr, Reissner-Nordström, Hawking, Page, Born-Infeld va Maldacena ishlari nochiziqli elektrodinamika sharoitida qora o‘ralarni o‘rganishga muhim hissa qo‘shgan. Kerr va Reissner-Nordström metrikalari klassik yechimlarni beradi, Hawking va Page kvant effektlarni o‘rgangan, Born-Infeld va Maldacena esa muqobil nazariy yondashuvlarni taklif qilgan. Ushbu tadqiqotlar qora o‘ralarning elektromagnit xossalari va ularning koinotdagi o‘rni haqidagi tasavvurlarimizni chuqurlashtirishga yordam beradi.

Mamlakatimizda Bobomurat Ahmedov, Ahmadjon Abdujabbarov, Javlon Rayimbaev, Farrux Atamuratov, Sanjar Shaymatov va Bobir Toshmatovning qora o‘ralarga oid ishlari zamonaviy astrofizika va nazariy fizika sohalarida muhim o‘rin tutadi. Ularning tadqiqotlari qora o‘ralar atrofidagi nochiziqli elektromagnit maydonlarning ta’sirlarini o‘rganish orqali yangi fizik xossalarni ochib beradi va bu jarayonlarni astrofizik kuzatuvlar bilan bog‘laydi.

Mening dissertatsiyam nochiziqli elektrodinamika va mukammal suyuqlik ko‘rinishidagi qorong‘i materiyaning qora o‘ra geometriyasi, gravitatsion va elektromagnit maydonlari, termodinamik xossalari hamda zarrachalar dinamikasiga birgalikdagi ta’sirlariga qaratilgan. Shuningdek, unda tashqi materiya maydonlarining fundamental chastotalar va kvazidavriy tebranish belgilariga ta’siri nazariy bashoratlarni astrofizik kuzatuvlar bilan solishtirish maqsadida tadqiq etiladi.

Dissertatsiya mavzusini ushbu mavzuda dissertatsiya olib borilayotgan oliy o‘quv yurtlari va ilmiy-tadqiqot muassasalarining ilmiy ishlari bilan bog‘liqligi. Dissertatsiya Sharof Rashidov nomidagi Samarqand davlat universitetida ilmiy tadqiqot mavzusi va Fundamental va amaliy tadqiqotlar instituti ilmiy loyihalari:

FL-7923051796 “Sferik simmetrik gravitasion maydonlarda skalyar maydonlar dinamikasi va kvant nurlatkichlarni modellashtirish” (2024-2026) doirasida bajarildi.

Tadqiqotning maqsadi: Chaplygin qorong‘i suyuqligi va mukammal suyuqlik ko‘rinishidagi qorong‘i materiya mavjudligida qora o‘ralar atrofidagi astrofizik jarayonlarni ularning zarrachalar dinamikasi, fundamental chastotalari, termodinamik xossalari hamda QPO belgilarini o‘rganish orqali baholashdan iborat.

Tadqiqot vazifalari:

• Chaplygin qorong‘i suyuqligi (CDF) bilan o‘ralgan qora o‘ralarning fazo-vaqt xossalarini o‘rganish;

• CDF mavjudligida qora o‘ralar atrofidagi sinov zarrachalari dinamikasi va barqaror aylana orbitalarni tahlil qilish;

• Mukammal suyuqlik qorong‘i materiyasi (PFDM) bilan o‘ralgan qora o‘ralarning fundamental chastotalarini hisoblash va QPO belgilarini o‘rganish;

• PFDM mavjudligida nochiziqli elektrodinamika (NED) bilan bog‘langan umumiy nisbiylik nazaryasi (GR) doirasida yangi qora o‘ra yechimlarini olish;

• NED va PFDM maydonlaridagi qora o‘ralarning termodinamik xossalari hamda fazaviy o‘tish xatti-harakatini o‘rganish;

• QPO ma’lumotlari va Monte Carlo Markov zanjiri (MCMC) tahlili yordamida qora o‘ra va uni o‘rab turgan maydon parametrlariga cheklovlar qo‘yish.

Tadqiqot obyektlari Mukammal suyuqlik qorong‘i materyasi va nochiziqli elektrodinamika maydoni ta’siridagi qora o‘ralar, ularning geometrik va termodinamik xossalari, test zarrachalarining orbital harakati, akretsion disk, Monte Carlo Markov zanjiri simulyatsiyalari yordamida mukammal suyuqlik qorong‘i materyasi va nochiziqli elektrodinamika maydoni parametrlarini aniqlash hisoblanadi.

Tadqiqot predmeti bu mukammal suyuqlik qorong‘i materyasi va nochiziqli elektrodinamika maydoni modellarining qora o‘ralar, test zarrachalari, akretsion disk va soyalariga ta’siri, shuningdek, bu modellar orqali qora o‘ralarning geometrik va termodinamik xossalari o‘rganish hisoblanadi.

Tadqiqot metodi bu nazariy fizika va astrofizika usullari, umumiy nisbiylik nazariyasi matematik apparati, hisoblash matematikasi metodlari, matematik fizika va statistik fizikaning zamonaviy metodlaridan iborat.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

• birinchi marta PFDM mavjudligida NED bilan bog‘langan GR doirasida ikki turdagi yangi qora o‘ra yechimlari olindi;

• birinchi marta qora o‘raning issiqlik sig‘imi NED maydoni va qorong‘i materiya ta’sirida fazaviy o‘tishlarga uchrashi ko‘rsatildi;

• birinchi marta mikrokvazarlar va yulduz massali qora o‘ralardan olingan QPO ma’lumotlari asosida MCMC tahlili orqali qora o‘ralar, NED va qorong‘i maydonlar, ya’ni Chaplygin qorong‘i suyuqligi hamda mukammal suyuqlik ko‘rinishidagi qorong‘i materiya parametrlariga cheklovlar qo‘yildi.

Tadqiqotning amaliy natijalari quyidagilardan iborat: Kuzatilgan QPO ma’lumotlari asosida MCMC modellashtirishlari yordamida Chaplygin tipidagi qorong‘i suyuqlik bilan o‘ralgan qora o‘ralarning parametrlari cheklangan. XTE

J1550–564, GRO J1655–40, GRS 1915+105, Sgr A* va M82 X-1 obyektlari uchun parametrlar $B = 5.311 \times 10^{-5}$–9.340×10^{-5}, $q = 0.651$–0.953 hamda $r/M = 3.057$–4.496 oraliqlarida olingan.

Mukammal suyuqlik ko‘rinishidagi qorong‘i materiya bilan o‘ralgan magnit zaryadli qora o‘ralar uchun XTE J1550–564, GRS 1915+105, GRO J1655–40 va M82 X-1 obyektlarining kuzatilgan QPO ma’lumotlari parametrlarni $q/M = 0.0241$–0.0692 va $\lambda/M = 0.0203$–0.0281 oraliqlarida chekladi. Tegishli QPO orbital radiuslari $r/M = 6.0961$–7.3697 oraliqda aniqlangan.

NED va PFDM mavjudligida qora o‘ralarning issiqlik sig‘imi fazaviy o‘tish nuqtasida o‘z ishorasini o‘zgartirishi ko‘rsatildi. Bu natija termodinamik jihatdan nobarqaror va barqaror qora o‘ra konfiguratsiyalarini farqlash imkonini beradi. Shuning uchun olingan natijalardan QPO kuzatuvlari asosida qora o‘ra massasi, magnit zaryadi, qorong‘i suyuqlik parametrlari va PFDM parametrlarini baholashda amaliy foydalanish mumkin.

Tadqiqot natijalarining ishonchliligi matematik fizika, hisoblash matematikasi va relyativistik astrofizikaning yaxshi shakllangan usullaridan foydalanish orqali ta’minlangan. Barcha hisoblashlar umumiy nisbiylik va qora o‘ra fizikasi doirasida bajarilgan. Olingan natijalar fazo-vaqt geometriyasi, gorizont tuzilishi, termodinamik kattaliklar, zarrachalar harakati, fundamental chastotalar va kvazidavriy tebranish belgilarining analitik chiqarilishi hamda sonli hisob-kitoblariga asoslanadi. Natijalarning to‘g‘riligi ma’lum chegaraviy holatlar, ilgari chop etilgan natijalar va mavjud astrofizik kuzatuv ma’lumotlari bilan taqqoslash orqali qo‘shimcha ravishda tasdiqlangan. Dissertatsiya xulosalari ixcham obyektlar astrofizikasi va kuchli maydon gravitatsiyasining asosiy tamoyillariga mos keladi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati. Tadqiqotning ilmiy ahamiyati mukammal suyuqlik ko‘rinishidagi qorong‘i materiya, Chaplygin qorong‘i suyuqligi va nochiziqli elektrodinamikaning qora o‘ralar fizik xossalariga qanday ta’sir qilishini aniqlashtirishdan iborat. Xususan, dissertatsiyada ularning qora o‘ra geometriyasi, gorizont tuzilishi, termodinamik xatti-harakati, zarrachalar dinamikasi va fundamental chastotalarga ta’siri tadqiq etiladi. Ushbu natijalar tashqi materiya maydonlari bilan o‘ralgan qora o‘ralarni chuqurroq tushunishga hissa qo‘shadi hamda bunday muhitlar kuchli gravitatsion maydon jarayonlarini qanday modifikatsiya qilishi haqida nazariy tasavvur beradi.

Tadqiqotning amaliy ahamiyati shundaki, taklif etilgan qora o‘ra mimikerlari modeli yuqori energiyali astrofizik jarayonlarni tushuntirishda muhim rol o‘ynashi mumkin. Xususan, u qora o‘ralar soyasining o‘lchami va optik ko‘rinishi bo‘yicha yangi bashoratlarni beradi; bu bashoratlar Event Horizon Telescope kabi astrofizik kuzatuvlar orqali tekshirilishi mumkin. Ushbu natijalar astrofizik parametrlarni yanada aniqroq aniqlash va kelajakdagi kuzatuv ishlarini yo‘naltirishda qo‘llanishi mumkin. Bundan tashqari, PFDM va NED modellariga asoslangan nazariy yondashuvlar kosmik tizimlarning ishlash mexanizmlarini chuqurroq tushunishga hissa qo‘shadi. Tadqiqot natijalari yangi kosmik observatoriyalarni ishlab chiqish yoki mavjud texnologiyalarni takomillashtirish uchun ilmiy asos bo‘lib xizmat qilishi mumkin.

Amaliy ahamiyati esa astrofizik kuzatuvlar, masalan, Hodisalar Gorizonti Teleskopi orqali qora o‘ralarning soya o‘lchamlari va optik ko‘rinishiga oid yangi bashoratlarni yaratishga xizmat qiladi. Bu natijalar astrofizik parametrlarni aniqlash va yangi kuzatuvlarni amalga oshirishda qo‘llanilishi mumkin. Shuningdek, suyuqlik qorong‘i materyasi va nochiziqli elektrodinamika maydoni modellariga asoslangan nazariy yondashuvlar kosmik tizimlarning ishlash mexanizmlarini yanada chuqurroq tushunishga olib keladi. Tadqiqot natijalari yangi kosmik observatoriyalarni yaratish yoki mavjud texnologiyalarni takomillashtirish uchun ilmiy asos bo‘ladi.

Tadqiqot natijalarini qo‘llash. “Tashqi materiya maydonlari mavjudligida qora o‘ralarning fundamental chastotalari va termodinamik xossalari” nomli tadqiqotning natijalari quyidagi yo‘nalishlarda qo‘llanilgan:

- Ekzotik materiyaning ta’siridagi qora o‘ralarning asosiy tebranish chastotalari va termodinamik xossalarini o‘rganishda ushbu yondashuv bir qator mualliflar tomonidan tadqiqotlarda qo‘llanilgan: Li Xiang-Qian, Kim Yoonbai, Lee Bum-Hoon, Yan Hao-Peng, Yue Xiao-Jun. “Black holes immersed in modified Chaplygin-like dark fluid and cloud of strings: geodesics, shadows, and images”, Chinese Physics C, 2025. Javed Faisal, Waseem Arfa, Mustafa G., Fatima Ghulam, Alkarni Shalan. “Particle motion and thermal fluctuations of charged AdS black holes surrounded by exotic fluid with modified Chaplygin equation of state”, Physics of the Dark Universe, vol. 47, 2025. Turimov Bobur, Rahimov Ozodbek, Davlataliev Akbak, Tadjimuratov Pulat, Norkobilov Mukhiddin, Rahimova Shakhzoda. “Analysing geodesic motion in Bocharova–Bronnikov–Melnikov–Bekenstein spacetime”, Physics of the Dark Universe, vol. 48, 2025, 101855. Erdem Sucu, Izzet Sakallı. “Astrophysical reality of black hole thermodynamics and dynamics: Transformative influence of Hernquist dark matter distributions”, Physics of the Dark Universe, vol. 49, 2025, 102051.

- Zarrachalar va fotonlarning harakatiga oid olingan ilmiy natijalar 2025-yil 23-may kuni Shanxaydagi Fudan universiteti (Fudan University, FU) olimlari tomonidan ilmiy tadqiqotlarda foydalanilgan.

Tadqiqot natijalarining aprobatsiyasi. Tadqiqot natijalari 4 ta xalqaro va mahalliy ilmiy konferensiyalarda ma’ruza qilindiva tezis chop etilgan.

Tadqiqot natijalarini nashr etish. PhD tadqiqot natijalari O‘zbekiston Respublikasi Oliy ta’lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasi tomonidan tavsiya etilgan ilmiy jurnallarida chop etilgan 7 ta ilmiy maqolalarda taqdim etilgan.

Dissertatsiyaning tuzilishi va hajmi. Dissertatsiya kirish, uchta bob, xulosa va foydalanilgan adabiyotlar ro‘yxatidan iborat bo‘lib, uning hajmi 120 betni tashkil etadi.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida dissertatsiya mavzusining dolzarbligi va zarurati asoslangan bo‘lib, tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo‘nalishlariga mosligi ko‘rsatilgan. Muammoning o‘rganilganlik darajasi keltirilgan. Tadqiqot maqsadi, vazifalari, obyekti va predmeti tavsiflangan. Tadqiqotning ilmiy yangiligi va amaliy natijalari bayon qilingan. Olingan natijalarning nazariy va amaliy

ahamiyati ochib berilgan. Tadqiqot natijalarining joriy qilinishi, nashr etilgan ishlar va dissertatsiya tuzilishi bo‘yicha ma’lumotlar keltirilgan.

Dissertatsiyaning birinchi “**Chaplyginga o'xshash qorong'u materiya bilan o'ralgan qora o‘ralar: fazo - vaqt xususiyatlari va kvazperiodik tebranish sinovlari**” deb nomlangan birinchi bobida keyingi boblar uchun asos bo‘ladigan tushunchalar va natijalar berilgan. Shuningdek, Chaplyginga o‘xshash holat tenglamalari ko‘rib chiqilgan. Qora o‘ra yechimi uchun sferik simmetrik metrika quyidagicha tanlanadi:

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (1)$$

Bu yerda $f(r)$ — radial koordinataga bog‘liq bo‘lgan metrik funksiya hisoblanadi. Chaplygin qorong‘i (mukammal) suyuqligi uchun holat tenglamasining nolinyar xatti-harakatini $p = -B/\rho$ shaklida ko‘rib chiqiladi. Bunda radial va tangensial bosimlar mos ravishda $p_r = -\rho$ va $2p_t = \rho - 3B/\rho$ tarzida aniqlanadi; bu yerda $B > 0$ bo‘lgan ijobiy doimiydir. Zichlik ρ va bosimlar radial masofaga bog‘liqdir. Suyuqlik uchun stress-energiya tenzori va unga mos Eynshteyn tenzori quyidagi ko‘rinishda qo‘llaniladi:

$$T_t^t = T_r^r = -\rho, T_\theta^\theta = T_\phi^\phi = \frac{1}{2}\left(\rho - \frac{3B}{\rho}\right), \quad (2)$$

va

$$G_t^t = G_r^r = \frac{1 - rf' - f}{r^2}, G_\theta^\theta = G_\phi^\phi = \frac{2f' + rf''}{2r}, \quad (3)$$

mos ravishda. 2 va 3 tenglamalaridan foydalanib, quyidagi tenglamalar tizimini hosil qilish mumkin:

$$f + rf' - 1 = -\rho r^2, \qquad \frac{1}{r}(2f' + rf'') = \rho - \frac{3B}{\rho}. \quad (4)$$

Yuqoridagi tenglamadan quyidagicha natijani bevosita olish mumkin:

$$\rho^2 = B + \frac{q^2}{r^6}, \quad (5)$$

bu yerda q— integrallash konstantasi bo‘lib, musbat normallash omili CDF intensivligini ifodalaydi. Geometrik birliklarda zichlikning birligi $[\rho] = cm^{-2}, [B] = cm^{-4}$ va $[q] = cm$ ga teng. XBS tizimida esa Chaplygin holat tenglamasidagi B parametrining birligi: $\mathrm{kg}^2/(\mathrm{m}^4\ \mathrm{s}^2)$.

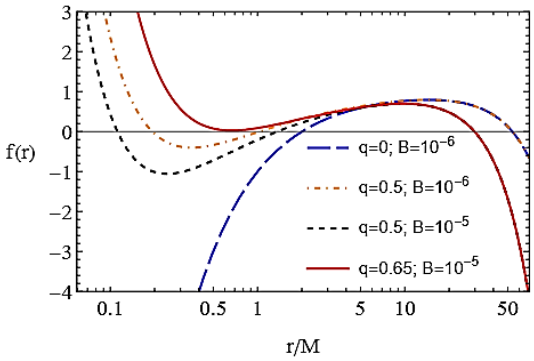
Oddiy algebraik tahlil shuni ko‘rsatadiki, $q \to 0$ bo‘lganda yoki katta masofalarda$\rho \simeq \sqrt{B}$ bo‘ladi, ya’ni zichlik asosan B ga bog‘liq bo‘ladi. Aksincha, $B \to 0$ bo‘lganda yoki kichik masofalarda $(r^6 \ll q^2/B)$ esa $\rho \simeq q/r^3$ ga yaqinlashadi, ya’ni zichlik kuchli radial bog‘liqlikka ega bo‘ladi.

Yuqoridagilarni hisobga olib 4 tenglamani yechish orqali quyidagicha hosil qilinadi:

$$f(r) = 1 - \frac{2M}{r} - \frac{r^2}{3}\sqrt{B + \frac{q^2}{r^6}} + \frac{q}{3r}\operatorname{arcsinh}\frac{q}{\sqrt{B}r^3}. \quad (6)$$

Bu yerda M - qora o‘ra massasi, q esa Chaplygin qorong‘i suyuqligining intensivligini ifodalovchi miqdor bo‘lib, ijobiy qiymatlar qabul qiladi. Chaplygin

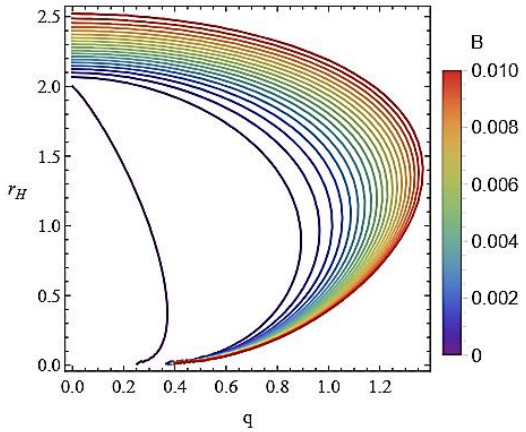
qorong‘i suyuqligi uchun holat tenglamasi parametri B ijobiy doimiy bo‘lib, $p = -B/\rho$ tenglama orqali aniqlanadi. Suyuqlik bilan birga harakatlanayotgan kuzatuvchi uchun ρ - energiya zichligini, p esa izotrop bosimni bildiradi. Agar $B = q = 0$ bo‘lganda fazo-vaqt metrikasi Schwarzschild holatiga o'tadi.



1-rasm. *Metrik funksiyasining radial masofaga bog‘liqligi q va B ning turli qiymatlarida*

Turli B va q parametr qiymatlarida metrik funksiyaning radial profillari 1-rasmda ko‘rsatilgan. q=0 bo‘lgan holatda ikki gorizont voqea gorizonti va kosmologik gorizont kuzatiladi. Biroq q noldan farqli bo‘lsa, ichki (Koshi) gorizonti paydo bo‘ladi. Shuningdek, q parametrining maksimal chegaraviy qiymati mavjud bo‘lib, bu qiymat oshirilsa Koshi va voqea gorizontlari yo‘qoladi, ammo kosmologik gorizont saqlanib qoladi.

Qora o‘ra atrofidagi geometriyaga holat tenglamasi parametrlari ta’sirini o‘rganish uchun biz voqea gorizontini ko‘rib chiqamiz. Bu gorizontni $f(r) = 0$ tenglamasi orqali tahlil qilish mumkin.



2-rasm. *Voqea gorizonti radiusining q parametriga bog‘liqligi B ning turli qiymatlari uchun.*

Voqea gorizonti radiusining q parametriga bog‘liqligi, B ning turli qiymatlarida, 2-rasmda ko‘rsatilgan. Grafikni chizishda barcha fizik kattaliklar qora o‘ra massasi bo‘yicha normallashtirildi va M = 1 deb olindi. Grafikdan ko‘rinib turibdiki, B parametri ortganda voqea gorizonti radiusi ham ortadi. Aksincha, q parametri oshganda voqea gorizonti kichrayadi, bu esa q parametri antigravitatsion ta’sirga ega ekanligini ko‘rsatadi.

Shuningdek, q ning yuqori chegaraviy qiymati mavjud bo‘lib, shu qiymatda voqea gorizonti va Koshi (ichki) gorizonti birlashadi. B ortganda bu yuqori chegaraviy qiymat oshadi, va voqea gorizontining minimal qiymati kamayadi. B = 0 holatda, q =0.25 dan kichik bo‘lgan qiymatlar uchun ichki gorizont mavjud emas.

Asosiy chastotalar. Endi o‘zgartirilgan Schwarzschild qora tuynugi atrofida sinov zarrachalarning asosiy (fundamental) chastotalari muhokama qilaylik. Bunda egizak cho‘qqili yuqori chastotali quasiperiodik tebranishlar uchun yuqori va past chastotalarning ehtimoliy qiymatlarini quyidagi modellar yordamida o‘rganish. Bu modellarda sinov zarrachalarining qora o‘ra atrofidagi harakatining fundamental chastotalari quyidagicha ifodalanadi: $\nu_{r,\theta,\phi} = c^3/(2\pi GM)\Omega_{r,\theta,\phi}$. Test zarracha ekvatorial tekislikdagi aylana orbitasi quyidagi uchta asosiy chastota bilan tavsiflanadi:

1. Orbital chastota (yoki Kepler chastotasi) ν_ϕ – bu orbitaning aylanish davriga teskari bo‘lgan chastota hisoblanadi.

2. Radial epitsiklik chastotaν_r – zarrachaning orbitasi atrofida radial (ya’ni qora o‘raka yaqinlashish va uzoqlashish yo‘nalishida) tebranish chastotasi.

3. Vertikal epitsiklik chastota ν_θ – zarrachaning vertikal (qutb yo‘nalishida) orbitaga nisbatan yuqoriga va pastga bo‘lgan tebranish chastotasi.

Ushbu uchta chastota faqat fazo-vaqt metrikasi va orbita radiusiga bog‘liq. Garchi ular erkin zarrachaning orbital harakati uchun aniqlangan bo‘lsa-da, bu chastotalar akretsion diskdagi suyuqlikning tebranish modlari bilan ham bevosita bog‘liq.

Sinov zarrachaning Kepler (orbital) chastotasini o‘rganish uchun geodezik tenglamadan foydalaniladi:

$$\ddot{x}^\alpha + \Gamma^\alpha_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = 0, \tag{7}$$

bu yerda $\Gamma^\alpha_{\mu\nu}$ - Kristoffel simvollari va $\dot{x} = \dot{t}(1,0,0,\Omega)$ - zarrachaning tezligi. Harakat tenglamasidan sinov zarrachalari uchun quyidagi tenglama olinadi:

$$\partial_r g_{tt} + \Omega^2\partial_r g_{\phi\phi} = 0, \tag{8}$$

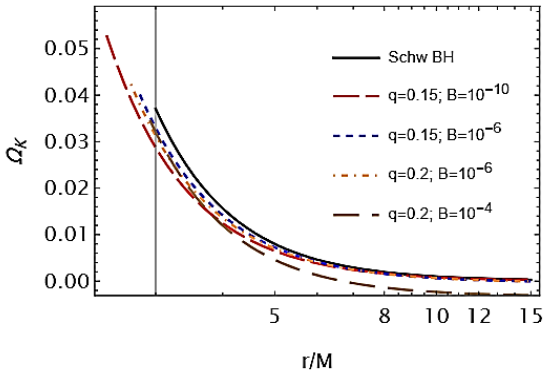
va ushbu tenglamaning Kepler chastotasi Ω_K uchun yechimi quyidagicha ifodalanadi:

$$\Omega_K^2 = \frac{M}{r^3} - \frac{1}{3r^3}\sqrt{Br^6 + q^2} - \frac{q}{6r^3}\operatorname{arcsinh}\frac{q}{\sqrt{B}r^3}. \tag{9}$$

Asosiy chastotalarning son qiymatini hisoblash uchun ular bilan bog‘langan birlik Gerts (Hz) sifatida quyidagi formula ishlatiladi:

$$\nu = \frac{1}{2\pi}\frac{c^3}{GM}\Omega, [\text{ Hz}], \tag{10}$$

bu yerda yorug‘lik tezligi $c = 3\cdot 10^8$ m/sec va tortishish doimiysi $G = 6.67\cdot 10^{-11}$ $\text{m}^3/(\text{kg}^2\cdot\text{sec})$ deb olingan.



3-rasm. *Rasmda Ω_K ning radial bog‘liqligi q va B ning turli qiymatlarida.*

Zarrachalarning Kepler orbitasidagi burchak impulsining radial bog‘liqligi 3-rasmda q va B parametrlari o‘zgarishi orqali ko‘rsatilgan. Shundan ko‘rinadiki, q parametri mavjud bo‘lgan holatda Kepler chastotasi Schwarzschild holatiga nisbatan ortadi. Aksincha, holat tenglamasi parametri bu chastotani kamaytiradi. Qorong‘i suyuqlikning intensivlik parametri chastotaga uzoq masofalarda sezilarli ta’sir ko‘rsatmaydi.

Qora o‘ra atrofida ekvatorial tekislikda harakatlanayotgan sinov zarrachalari barqaror orbitadan kichik siljish tufayli radial va vertikal yo‘nalishlarda tebranadi, ya’ni orbitalar $r_0 + \delta r$ va $\pi/2 + \delta\theta$ ko‘rinishida ifodalanadi.Radial va vertikal tebranish chastotalarini quyidagi garmonik osillatordan olingan tenglamalar yordamida hisoblash mumkin:

$$\frac{d^2\delta r}{dt^2} + \Omega_r^2 \delta r = 0, \frac{d^2\delta\theta}{dt^2} + \Omega_\theta^2 \delta\theta = 0, \tag{11}$$

bu yerda

$$\Omega_i^2 = -\frac{1}{2g_{ii}(u^t)^2}\,\partial_i^2 V_{\text{eff}}(r,\theta)\Big|_{\theta=\frac{\pi}{2}}, i = r, \theta \tag{12}$$

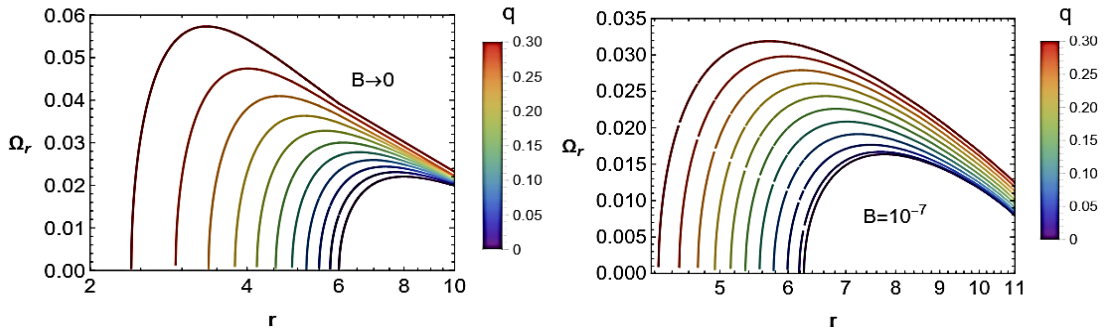
Yuqoridagi tenglamadan radial va vertikal burchak chastotalari hisoblanadi. Ba’zi algebraik amallarni bajargach, quyidagi ifodalar hosil bo‘ladi:

$$\Omega_r^2 = \Omega_{\text{K}}^2 \left\{ \frac{4\sqrt{q^2 + Br^6} - 12M + 2q\text{ArcSinh}\left(\frac{q}{\sqrt{B}r^3}\right)}{3r} \right.$$

$$+ \frac{\left(6M - 3r + \sqrt{q^2 + Br^6} - q\text{ArcSinh}\left(\frac{q}{\sqrt{B}r^3}\right)\right)}{6r\sqrt{q^2 + Br^6} - 18Mr + 3qr\text{ArcSinh}\left(\frac{q}{\sqrt{B}r^3}\right)}$$

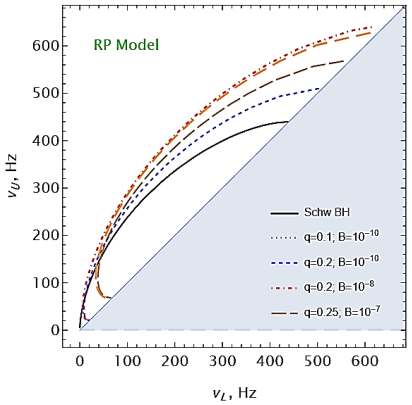
$$\left. \times \left(6Mr^3 + \frac{q^2 - 8Br^6}{\sqrt{q^2 + Br^6}} - q\text{ArcSinh}\left(\frac{q}{\sqrt{B}r^3}\right) \right) \right\}.$$

4-rasmda Ω_r ning radial masofa bo‘yicha grafigi o‘zgarmas B qiymatida keltirilgan. Grafikdan ko‘rinadiki, Ω_r parametri q oshgani sayin ortadi, holat tenglamasi parametri esa bu chastotani kamaytiradi.



4-rasm. *Ω_r ning radial masofaga bog'liqligi*

Stella va Vietri birinchi bo‘lib kichik massa yulduzlar bilan bog‘langan neytron yulduzlar uchun 0.2–1.25 kHz chastota oralig‘idagi kHz egizak cho‘qqili QPO larni tavsiflovchi nisbiylik pretsessiyasi (RP) modelini taqdim etgan. Ushbu model qora o‘rali ikkilik tizimlar uchun ham qo‘llaniladi va qora o‘ra atrofidagi akretsiya diskining quvvat-zichlik spektri asosida mikrokvazarlardagi markaziy qora o‘ra massasini va aylanish momentini aniq aniqlash imkonini beradi. RP modelida yuqori va pastki chastotalar akretsiya diskidagi zarrachalarning eng ichki barqaror orbita (ISCO) yaqinidagi radial, vertikal va orbital tebranish chastotalari bilan belgilanadi: $\nu_U = \nu_\phi$ va $\nu_L = \nu_\phi - \nu_r$.



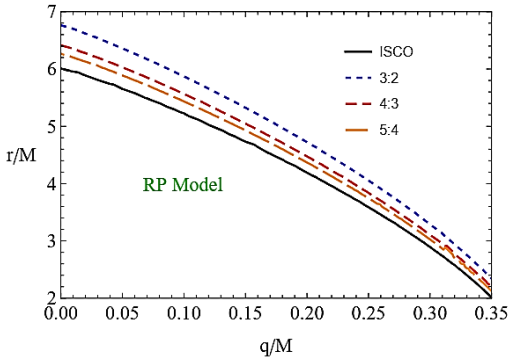
5-rasm. *Chaplygin tipidagi holat tenglamasiga ega bo‘lgan qorong‘i suyuqlik (CDF) bilan o‘ralgan Schwarzschild qora tuynugi atrofidagi yuqori va pastki chastotalar orasidagi bog‘liqlik q va B parametrlarining turli qiymatlari uchun ko‘rib chiqilgan.*

6-rasmda $\nu_U - \nu_L$ diagrammalari qora o‘ra metrikasi asosida egizak cho‘qqili QPO lar uchun RP model doirasida ko‘rsatilgan hamda bu natijalar Schwarzschild qora tuynugi atrofidagi QPO lar bilan taqqoslangan. Bu yerda markaziy qora o‘raning massasi test yulduz-massali qora o‘ra sifatida $M = 5M_\odot$ deb olingan. Diagrammadagi och moviy soya bilan belgilangan hudud egizak cho‘qqili QPO lar uchun “qabrzonasi”ni bildiradi, ya’ni bu sohada bunday QPO lar kuzatilmaydi.

Tahlillardan ko‘rinadiki, q mavjud bo‘lgan holatda past chastotali QPO lar mavjud emas. Holat tenglamasi parametri B ortgani sari yuqori va pastki chastotalar kilohertz diapazoniga yaqinlashib ortadi. Biroq q parametri oshganda bu chastotalar kamayadi. Bu esa B va q parametrlari orasida teskari bog‘liqlik mavjudligini

bildiradi: ular bir xil yuqori va pastki chastotalarni ta’minlaydi, ammo qarama-qarshi yo‘nalishda ta’sir qiladi.

Bu yerda biz RP modeli doirasida egizak cho‘qqili QPO sodir bo‘ladigan ISCO dan tashqaridagi orbitallarni o‘rganamiz. Bunda quyidagi tenglamalar to‘plami ishlatiladi: $m\nu_L(M,r,q) = n\nu_U(M,r,q)$, bu yerda$m$ va n - butun sonlar bo‘lib, ular yuqori ν_U va pastki ν_L chastotalar orasidagi rezonans nisbatlarini bildiradi. Bu usul orqali qora o‘ra atrofida mavjud bo‘lgan barqaror rezonans orbitallari aniqlanadi, bu esa QPO larni chuqurroq tushunishga yordam beradi.



6-rasm. *QPO radiusining q parametriga bog‘liqligi*

6-rasmda QPO orbitasining qorong‘i suyuqlik intensivligi parametri q ga bog‘liqligi ko‘rsatilgan. Kuzatishlardan ma’lum bo‘lishicha, q parametri ortgani sayin QPO radiusi kamayadi va ISCO ga yaqinlashadi. RP modeli doirasida bu orbitaning ISCO bilan deyarli ustma-ust tushishi kuzatiladi.

Dissertatsiyaning ikkinchi “**Nochiziqli elektrodinamika va mukammal suyuq qorong‘i materya bilan o‘ralgan qora o‘ralar: Fazoviy barqarorlik va termodinamika**” deb nomlangan bobida, nochiziqli elektrodinamika va mukammal suyuq qorong‘i modda bilan o‘ralgan qora o‘ra yechimi keltirigan. Shuningdek, berilgan qora o‘raning termodinamik xossalari muhokama qilingan.

Umumiy nisbiylik nazariyasi doirasida nochiziqli elektrodinamika maydoni va mukammal suyuq qorong‘i modda bilan bog‘langan, sferik simmetrik fazoda qaralgan qora o‘ra yechimiga qisqacha kirish beriladi, bu yerda ansats quyidagicha tanlangan.

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\Lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \qquad (13)$$

Singular bo‘lmagan qora o‘ra fazolarining kengaytmalari umumiy nisbiylik va nochiziqli elektrodinamika juftligida ko‘plab tadqiq qilingan. Biroq biz bu yerda mukammal suyuq qorong‘i modda deb ataluvchi ekzotik maydon ishtirokidagi GR + NED tizimida paydo bo‘ladigan yangi qora o‘ra yechimini ko‘rib chiqamiz. Mos keluvchi harakat tenglamasi quyidagicha:

$$\mathcal{S} = \frac{1}{16\pi}\int d^4x\sqrt{|g|}(\mathrm{R} - \mathcal{L}_{\mathrm{NED}} + \mathcal{L}_{\mathrm{PFDM}}) \qquad (14)$$

bu yerda R — Richchi skalyar, g — metrik tensor $g_{\mu\nu}$ determinantasi. NED maydoni uchun Lagrangian quyidagicha berilgan

$$\mathcal{L}_{\mathrm{NED}} = \frac{4\mu}{\alpha}\frac{(\alpha\,\mathrm{F})^{(\nu+3)/4}}{[1+(\alpha\mathrm{F})^{\nu/4}]^{1+\mu/\nu}} \qquad (15)$$

bu yerda α- bog‘lanish parametri, $F = F_{ab}\, F^{ab}$ esa elektromagnit maydonning Faradey tensori invariantidir. Materiya Lagrangiani mavjud bo‘lmagani uchun, Shvartsshild koordinatalarida energiya-impuls tensori faqat $\mathcal{L}_{\mathrm{NED}}$ ga bog‘liq bo‘ladi.

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu}^{(\mathrm{NED})} + T_{\mu\nu}^{(\mathrm{PFDM})}. \tag{16}$$

bu yerda ideal suyuqlik uchun energiya-impuls tensori

$$T_{\mu\nu}^{(\mathrm{PF})} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu.} \tag{17}$$

PFDM holatida esa faqat standart ortogonal bazaga ega bo‘lib,

$$T_{\mu\nu}^{(\mathrm{PFDM})} = \mathrm{diag}\left(-\epsilon, p_r, p_\theta, p_\phi\right). \tag{18}$$

bu yerda

$$-\epsilon = p_r = \frac{\lambda}{8\pi r^3}, \qquad p_\theta = p_\phi = -\frac{\lambda}{16\pi r^3}. \tag{19}$$

Nochiziqli elektrodinamika uchun energiya-impuls tensori

$$4\pi T_{\mu\nu}^{(\mathrm{NED})} = -2 \frac{\partial \mathcal{L}_{\mathrm{NED}}}{\partial F} F_\mu^\beta F_{\nu\beta} + \mathcal{L}_{\mathrm{NED}} g_{\mu\nu}. \tag{20}$$

NED maydonining noldan farqli diagonal komponentalari

$$T_0^0 = T_1^1 = -\frac{2\mu M q^\nu r^{\mu-3}}{(q^\nu + r^\nu)^{\frac{\mu+\nu}{\nu}}},$$

$$T_2^2 = T_3^3 = \frac{\mu M q^\nu r^{\mu-3}}{(q^\nu + r^\nu)^{\frac{\mu}{\nu}+2}} [(\nu+1) r^\nu - (\mu-1) q^\nu]. \tag{21}$$

Maydon tenglamasini yechish uchun avval ansatz

$$e^{\nu(r)} = e^{-\Lambda(r)} = f(r). \tag{22}$$

bo‘yicha Einstein tenzorining $$G_{tt}$$ komponentasini hisoblaymiz:

$$G_0^0 = G_1^1 = -\frac{1}{r}\frac{df(r)}{dr} - \frac{f(r)}{r^2} + \frac{1}{r^2},$$

$$G_2^2 = G_3^3 = -\frac{1}{2}\frac{d^2 f(r)}{dr^2} - \frac{1}{r}\frac{df(r)}{dr}.$$

Statik va sferik simmetrik fazo hamda magnit zaryadi q ga ega NED manbasi $F_{\mu\nu} = \left(\delta_\mu^\theta \delta_\nu^\phi - \delta_\nu^\theta \delta_\mu^\phi\right) q \sin\theta$ uchun f(r) uchun differensial tenglama hosil bo‘ladi:

$$\frac{1}{r}\frac{df(r)}{dr} + \frac{f(r)}{r^2} + \frac{1}{r^2} = \frac{2\mu M q^\nu r^{\mu-3}}{(q^\nu + r^\nu)^{\frac{\mu+\nu}{\nu}}} - \frac{\lambda}{8\pi r^3}. \tag{23}$$

Buning yechimi

$$f(r) = \frac{-2M r^\mu (q^\nu + r^\nu)^{-\frac{\mu}{\nu}} + \lambda \ln r + r}{r} + \frac{c_1}{r}. \tag{24}$$

Soddalashtirsak

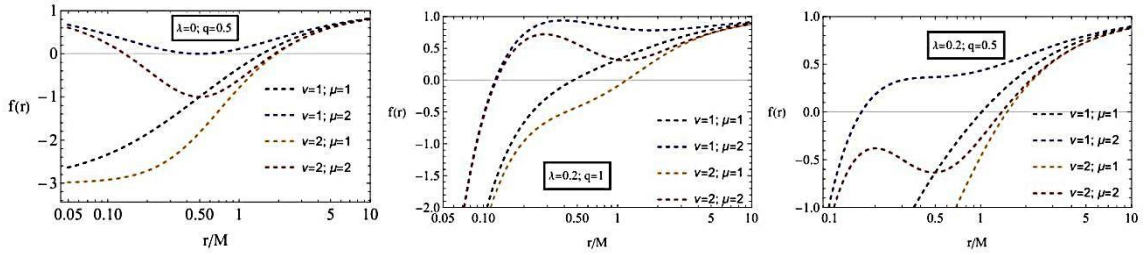
$$f(r) = 1 - \frac{2M}{r}\left(1 + \frac{q^\nu}{r^\nu}\right)^{-\frac{\mu}{\nu}} + \frac{\lambda \ln r}{r} + \frac{c_1}{r}. \tag{25}$$

Logarifmik haddagi ifoda o‘lchamsiz bo‘lishi lozim, shu bois $c_1 = -\lambda \ln |\lambda|$

deb olamiz va metrik funksiyaning yakuniy ko‘rinishi quyidagicha bo‘ladi:

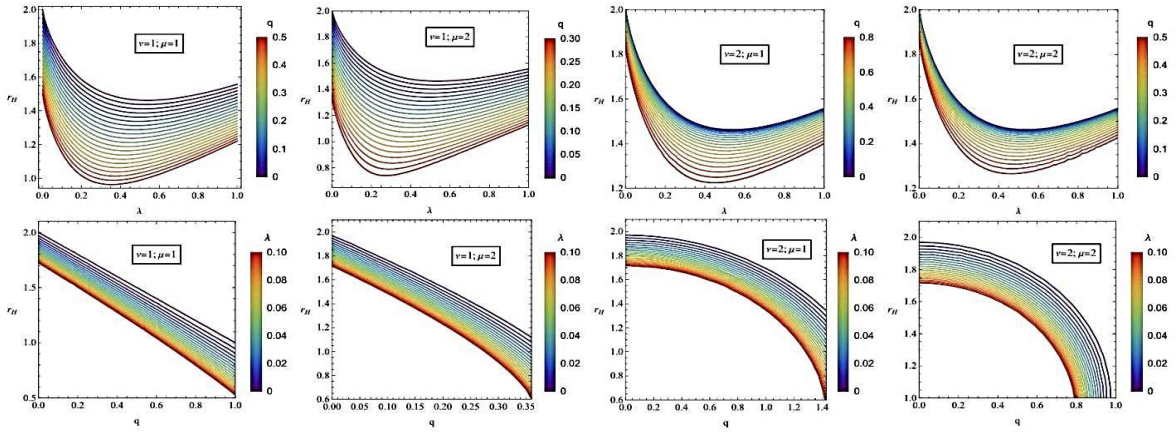
$$f(r) = 1 - \frac{2M}{r}\left(1 + \frac{q^{\nu}}{r^{\nu}}\right)^{-\frac{\mu}{\nu}} + \frac{\lambda}{r}\ln\frac{r}{|\lambda|}. \qquad (26)$$

Endi biz olingan qora o‘ra yechimining gorizont xossalarini grafik tarzda tahlil qilamiz.



7-rasm. *Metrika funksiyasining radial bog‘liqligi turli μ va ν qiymatlari uchun, λ va q ning doimiy qiymatlarida tahlil qilinadi.*

Qora o‘ra atrofidagi metrika funksiyasining turli parametrlar ta’sirida qanday o‘zgarishi 7-rasmda ko‘rsatilgan. NED parametrlarining ortishi fazovaqt egriligini kamaytiradi va qora o‘raning barqarorligini oshiradi. Zaryadning ortishi elektromagnit maydonini kuchaytiradi va hodisalar ufqining kengayishiga olib keladi. PFDM esa fazovaqtni cho‘zadi va qora o‘ra tashqarisidagi dinamik ta’sirlarni susaytiradi. Ushbu grafiklar parametrlar o‘rtasidagi o‘zaro ta’sirni ochib beradi va qora o‘ra atrofidagi fazovaqt tuzilmasi haqida muhim tasavvurlar beradi



8-rasm. *Hodisalar ufqi radiusi μ va ν ning berilgan qiymatlari uchun aniqlanadi*

Rasmda berilgan grafiklar to‘plami hodisalar ufqi radiusi r_H ning turli parametrlar - λ va q- ga bog‘liqligini, μ *va* ν parametrlarining har xil kombinatsiyalarida tasvirlaydi. Yuqori qator grafiklar r_H ning λ ga bog‘liqligini, pastki qator esa r_H ning q ga bog‘liqligini ko‘rsatadi. Har bir ustun esa μ *va* ν ning turli qiymatlariga mos keladi. Har bir grafikning o‘ng tomonidagi rangli panel uchinchi parametr - λ yoki q - qiymatini bildiradi.

Grafiklar shuni ko‘rsatadiki, hodisalar ufqi radiusi λ ning ma’lum bir qiymatida minimumga ega bo‘ladi. Bu minimum qiymat μ *va* ν qiymatlariga qarab o‘zgaradi. ν ortgani sayin boshlang‘ich hodisalar ufqi radiusi oshadi, ya’ni ν ortishi qora o‘ra hajmini kattalashtiradi. μ ning ortishi ham r_H ni oshiradi, biroq bu ta’si ν nikiga nisbatan kuchsizroq.

Grafiklardagi U-shaklidagi egri chiziqlar λ o‘zgarganida hodisalar ufqi radiusining minimal qiymatini aks ettiradi. Ushbu minimal qiymat μ *va* ν ning har xil qiymatlarida har xil λ da yuzaga chiqadi. Grafiklar shuni ko‘rsatadiki, λ ortishi avval r_H ni kamaytiradi, so‘ngra yana oshiradi.

Barcha grafiklarda kuzatiladiki, q ortganda hodisalar ufqi radiusi r_H umumiy holda kamayadi. Bu esa shuni anglatadiki, magnit zaryad q ortgani sari qora o‘raning hodisalar ufqi kichrayadi.

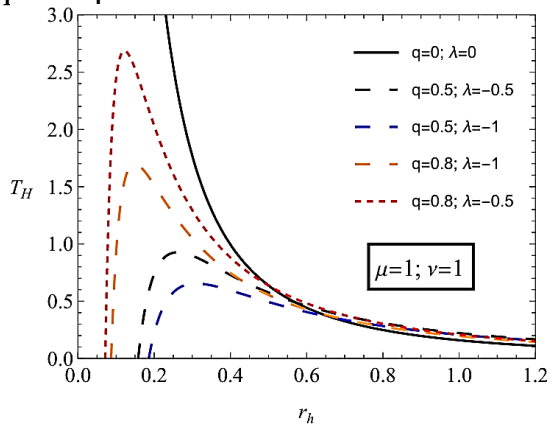
Exotik modda bilan o‘ralgan qora o‘ralarning termodinamikasi**.** $f(r_h)=0$ tenglamasini yechish orqali hodisalar ufqi radiusi r_h aniqlanadi va bu yordamida qora o‘raning massasi quyidagicha ifodalanadi:

$$M=\frac{1}{2}\left(1+\frac{q^{\nu}}{r_h^{\nu}}\right)^{\frac{\mu}{\nu}}\left(r_h+\lambda\ln\frac{r_h}{|\lambda|}\right). \tag{27}$$

Hawking harorati T_h esa sirt tortishish kuchi bilan bog‘liq bo‘lib, quyidagi tenglama orqali aniqlanadi:

$$T_h=\left.\frac{f'(r)}{4\pi}\right|_{r=r_h}=\frac{1}{4\pi}\left[\frac{2M}{r_h^2}\left(1+\frac{q^{\nu}}{r_h^{\nu}}\right)^{-\frac{\mu}{\nu}}+\frac{2M\mu q^{\nu}}{r_h^{2+\nu}}\left(1+\frac{q^{\nu}}{r_h^{\nu}}\right)^{-\frac{\mu}{\nu}-1}-\frac{\lambda}{r_h^2}\left(1+\ln\frac{r_h}{|\lambda|}\right)\right]. \tag{28}$$

9-rasmida ekzotik materiya bilan o‘ralgan va NED zaryadi bo‘lgan qora o‘raning harorati tasvirlangan. Grafikdan ko‘rinib turibdiki, qora o‘ra harorati T_h avval ortib, maksimum qiymatga yetadi, so‘ngra kamayish bosqichiga o‘tadi. Bundan tashqari, grafik maksimum qiymatning qorong‘i modda parametri λ kattaroq bo‘lganida yuqoriroq bo‘lishini ko‘rsatadi.



9-rasm. *Qora tuynuk harorati T_h va hodisa gorizonti radiusi r_h o‘rtasidagi bog‘liqlik, bunda qorong‘i modda parametri λ va qora tuynuk zaryadi q ning termal evolyutsiyaga ta’siri ko‘rsatilgan.*

Tadqiq qilinayotgan qora o‘ra uchun qora o‘ra termodinamikasining birinchi qonuni quyidagicha yoziladi:

$$dM=T_h dS_0+\Phi_h dq+\beta_h d\lambda. \tag{29}$$

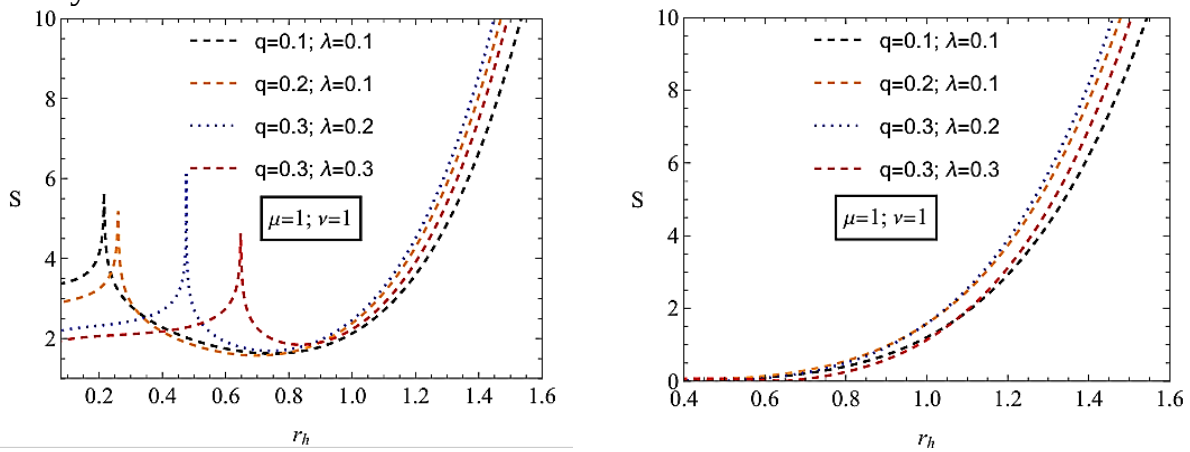
bu yerda S_0 - korreksiyalanmagan entropiya; q — magnit zaryad; λ —ekzotik materiya parametri. Entropiyaning korreksiyalanmagan formulasi S_0 quyidagicha aniqlanadi:

$$S_0 = \int \frac{1}{T_h} dM = \int \frac{1}{T_h} \frac{\partial M}{\partial r_h} dr_h. \qquad (30)$$

Endi biz muvozanat holatidagi korreksiyalangan entropiya S ni hisoblaymiz. Uning umumiy formulasi quyidagicha ifodalanadi:

$$S = S_0 - \beta \ln (S_0 T_h).$$

Bu yerda β — korreksiya parametri bo‘lib, faqat ikki xil qiymatga ega bo‘ladi. Agar β = 0 bo‘lsa, tenglama korreksiyalanmagan entropiyani bildiradi; β = 1/2 bo‘lganda esa, termal tebranishlar hisobga olingan korreksiyalangan entropiyani ifodalaydi.



10-rasm. *Entropiyaning qora o‘ra hodisalar ufqi radiusiga bog‘liqligi turli μ va ν qiymatlari uchun ko‘rib chiqiladi. Bu yerda β = 0 (korreksiyasiz holat) pastki panelda, β = 1/2 (korreksiya bilan) esa yuqori panelda tasvirlangan.*

10-rasmdan ko‘rinib turibdiki, ushbu grafik orqali berilgan qora o‘ra uchun korreksiyalangan entropiya bilan muvozanatdagi entropiyani solishtirish mumkin. Agar korreksiya parametri β mavjud bo‘lmasa, tizimning muvozanatdagi entropiyasi pastki panelda ko‘rsatilganidek chiziqli ortuvchi funksiyadir.

Agar korreksiya parametri β = 1/2 bo‘lsa, tizimning korreksiyalangan entropiyasi har doim musbat qiymatga ega bo‘ladi, bu esa tizim barqarorligi uchun muhimdir. Korreksiyalangan entropiya kritik hodisa radiusidan pastda kamayuvchi funksiyadir, biroq bu nuqtadan yuqorida u har doim ortib boradi. Shu sababli, yirik qora o‘ralar uchun korreksiya entropiyaga sezilarli ta’sir ko‘rsatmaydi. Biroq kichik qora o‘ra tizimlarida termal tebranishlar sezilarli bo‘lganligi sababli, korreksiya hadi tizim entropiyasiga muhim ta’sir ko‘rsatadi.

Doimiy hajmda maxsus issiqlik sig‘imi C quyidagicha aniqlanadi:

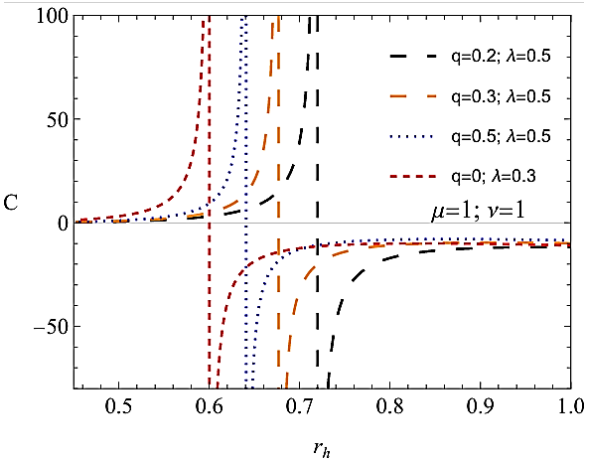
$$C = T_h \left(\frac{\partial S}{\partial T_h} \right). \qquad (31)$$

Entropiya S bilan Hawking harorati T_h orasidagi bog‘liqlikni aniqlash uchun bizga r_h ga bog‘liq ifodalar kerak bo‘ladi.

Qora o‘ra ikkinchi tartibli fazaviy o‘tish jarayonidan o‘tadi, bu esa qora o‘raning barqaror fazadan (C > 0) beqaror fazaga (C < 0) o‘tishini bildiradi.

Qora o‘raning issiqlik sig‘imi C ning hodisalar ufqi radiusiga bog‘liq o‘zgarishini 11-rasm tasvirlaydi va turli zaryad hamda ekzotik materiya parametri qiymatlarida qora o‘ra yechimi termodinamik barqarorligini ko‘rsatadi.

Grafiklardan ko‘rinib turibdiki, issiqlik sig‘imida uzilishlar mavjud — bu ikkinchi tartibli fazaviy o‘tishlar mavjudligini bildiradi. Ushbu o‘tishlar turli termodinamik fazalarni bir-biridan ajratadi: Kichik qora o‘ralar uchun (r_h uzilish nuqtasidan kichik), issiqlik sig‘imi manfiy bo‘lib, ular termal jihatdan beqaror hisoblanadi. Bu holat qora o‘ra Hawking nurlanishi orqali massasini yo‘qotadigan bug‘lanish bosqichiga mos keladi. Katta qora o‘ralar uchun (r_h o‘tish nuqtasidan katta), issiqlik sig‘imi musbat bo‘lib, termodinamik barqaror muvozanat fazasiga o‘tilganini bildiradi.



11-rasm. *Nochiziqli qora o‘raning issiqlik sig‘imi C ning ideal suyuqlik qorong‘i modda fonidagi o‘zgarishi quyidagicha tavsiflanadi.*

Qora o‘ra zaryadi ortgani sayin fazaviy o‘tish nuqtasi siljiydi, bu esa kuchliroq NED ta’siri qora o‘raning termodinamik tuzilmasiga ta’sir ko‘rsatishini bildiradi. Zaryad ortishi barqaror fazaning kengayishiga olib keladi va qora o‘ra bug‘lanishi ehtimolini kamaytiradi.

Ekzotik materiya parametrining ortishi esa barqarorlik xossalarini sezilarli darajada o‘zgartiradi. Uning yuqori qiymatlari fazaviy o‘tishni kattaroq hodisalar ufqiga siljitadi, bu esaEkzotik materiya mavjudligi katta qora o‘ralar uchun barqarorlikni qo‘llab-quvvatlashini anglatadi. Bu Ekzotik materiya muhiti bilan o‘ralgan qora o‘ralar vakuum yoki oddiy qorong‘i modda modellaridagilarga nisbatan bug‘lanishga kamroq moyil bo‘lishi mumkinligini ko‘rsatadi.

Astrofizik nuqtai nazardan, ushbu natijalar shuni ko‘rsatadiki, Ekzotik materiya ta’siri kuchli bo‘lgan muhitdagi qora o‘ralar uzoq umr ko‘rishga moyil bo‘lib, termodinamik barqaror fazaga o‘tish kattaroq hodisalar ufqida yuz beradi. Bundan tashqari, Ekzotik materiya qora o‘raning issiqlik jarayonini o‘zgartirib, akretsion disk dinamikasi, qora o‘ra birikmalari va termal tebranishlarga bog‘liq kuzatuv belgilariga ta’sir qilishi mumkin.

Yaxshi aniqlangan ikkinchi tartibli fazaviy o‘tishlar bunday qora o‘ralarning nooddiy barqarorlik xususiyatlariga ega ekanligini ko‘rsatadi va ularni klassik Schwarzschild yoki Reissner–Nordström qora o‘ralardan farqlaydi. Kelajakdagi kuzatuv sinovlari, masalan qora o‘ra soyasini o‘lchash yoki akretsion diskdan chiqqan rentgen spektrlari yordamida q va λ parametrlarini barqarorlik va bug‘lanish tezligiga ta’siri orqali aniqlash mumkin bo‘ladi.

Dissertatsiyaning uchinchi “**Magnit zaryadlangan qora o‘ralar va ideal**

suyuqlik qorong‘i modda muhitidagi radiatsiya jarayonlari, zarracha dinamikasi va kvaziperiodik tebranishlar” deb nomlanuvchi bobida, qora o‘ra uchun yangi yechim olingan. MCMC analizdan foydalanib, biz XTE J1550–564, GRS 1915+105, GRO J1655–40 va M82 X–1 kabi kuzatilgan qora o‘ra nomzodlari uchun massa, zaryad va ekzotik materiya parametrlariga cheklovlar qo‘yish, olingan natijalar ushbu modifikatsiyalangan gravitatsiya modellarining astrofizik ahamiyatini yanada chuqurroq tushunishga yangi yondashuvlarni taqdim etilishi tahlil qilingan.

Qorong‘i suyuqlik holat tenglamasiga ega qora o‘ra yechimlari. Umumiy nisbiylik doirasida, nochiziqli elektrodinamika va ekzotik materiyani o‘z ichiga olgan qora o‘ra yechimining qisqacha sharhi beriladi. Metrika sferik simmetriyaga ega qora o‘ra yechimini olish uchun maxsus ansatz yordamida cheklanadi. Ekzotik maydon — ideal suyuqlik qorong‘i modda ishtirokidagi GR + NED tizimiga e’tibor qaratamiz. Mos keluvchi harakat funksionali quyidagicha ifodalanadi:

$$\mathcal{L}_{\mathbf{NED}} = \boldsymbol{F e^{-\frac{q}{2M}(2q^2F)^{\frac{1}{4}}}}. \qquad (\mathbf{32})$$

Bu yerda q — magnit zaryadni, $\boldsymbol{F = F_{\mu\nu}F^{\mu\nu}/4}$ esa nochiziqli elektromagnit maydon invariantini bildiradi, bu yerda $\boldsymbol{F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu}$. Qora o‘ra yechimini olish uchun $\boldsymbol{F = q^2/(2r^4)}$ ifodasidan foydalaniladi.

$$\mathcal{L}_{\mathbf{NED}} = \frac{\boldsymbol{q^2}}{\boldsymbol{2r^4}}\boldsymbol{e^{-\frac{q^2}{2Mr}}}. \qquad (\mathbf{33})$$

Energiya - impuls tenzori 2 bobdagi ko‘rinishda olinadi va barcha hisoblashlardan so‘ng biz quyidagi yechimga ega bo‘lamiz:

$$\boldsymbol{f(r) = 1 - \frac{2Me^{-\frac{q^2}{2Mr}}}{r} + \frac{\lambda}{r}\ln\frac{r}{|\lambda|}}. \qquad (\mathbf{34})$$

MCMC analiz. Ushbu bo‘lim ifodalardagi massa va metrika parametrlariga cheklovlar qo‘yishga bag‘ishlangan. Buning uchun ikkita qora o‘ra nomzodi tanlandi. Qora o‘ralar turli massaviy toifalarga mansub: yulduzsimon va o‘rta massali. Masalan, biz mikrokvazar markazida joylashgan mashhur yulduzsimon qora o‘ralar — GRS 1915+105, GRO J1655-40 va XTE 1550-564 — hamda M82 X-1 manbaidan olingan QPO ma’lumotlarini ishlatdik.

Cheklovlarni olish uchun Python’ning *emcee* kutubxonasidan foydalanib, RP modelida MCMC tahlilini bajardik. Tanlangan taqsimot quyidagicha:

$$\boldsymbol{\mathcal{P}(\theta \mid \mathcal{D}, \mathcal{M}) = \frac{P(\mathcal{D} \mid \theta, \mathcal{M})\pi(\theta \mid \mathcal{M})}{P(\mathcal{D} \mid \mathcal{M})}}, \qquad (\mathbf{35})$$

bu yerda $\boldsymbol{\pi(\theta)}$ — prior funksiyasi, $\boldsymbol{P(\mathcal{D} \mid \theta, M)}$ esa o‘xshashlik funksiyasidir. Priorlarga mos ravishda normal (Gauss) taqsimot qabul qilindi:

$$\boldsymbol{\pi(\theta_i) \sim \exp\left[\frac{1}{2}\left(\frac{\theta_i - \theta_{0,i}}{\sigma_i}\right)^2\right], \theta_{\text{low},i} < \theta_i < \theta_{\text{high},i}}.$$

Bu yerda parametrlar $\theta_i = \{M, q, \lambda, r\}$, σ_i esa ularning og‘ishlari.

Oldingi bo‘limda hisoblangan orbital, vertikal va radial chastotalar asosida MCMC tahlili o‘tkazildi. O‘xshashlik funksiyasi Λ quyidagi ko‘rinishga ega:

$$\log \Lambda = \log \Lambda_{\mathrm{U}} + \log \Lambda_{\mathrm{L}}, \tag{36}$$

bu yerda yuqori chastotalar uchun o‘xshashlik funksiyasi

$$\log\ \Lambda_{\mathrm{U}} = -\frac{1}{2}\sum_{i} \frac{\left(\nu^{i}_{\phi,\mathrm{obs}} - \nu^{i}_{\phi,\mathrm{th}}\right)^{2}}{\left(\sigma^{i}_{\phi,\mathrm{obs}}\right)^{2}}, \tag{37}$$

quyi chastotalar uchun esa

$$\log \Lambda_{\mathrm{L}} = -\frac{1}{2}\sum_{i} \frac{\left(\nu^{i}_{\mathrm{L,obs}} - \nu^{i}_{\mathrm{L,th}}\right)^{2}}{\left(\sigma^{i}_{\mathrm{L,obs}}\right)^{2}}. \tag{38}$$

Shu tariqa, MCMC usuli yordamida massa M, magnit zaryad q, ekzotik materiya parametri $\boldsymbol{\lambda}$ va r kabi metrika o‘lchovlari bo‘yicha ehtimoliy cheklovlar olindi.

Bu yerda $\nu^{i}_{\phi,\,\mathrm{obs}}$ va $\nu^{i}_{\mathrm{L},\,\mathrm{obs}}$ — tadqiq qilinayotgan manbaga oid orbital/Kepler chastotalari (ν_{K}) va periastron pretsessiya chastotalari $\nu_{\mathrm{L}} = \nu_{\mathrm{K}} - \nu_{\mathrm{r}}$ ning to‘g‘ridan-to‘g‘ri kuzatuv natijalaridir. Boshqa tomondan, $\nu^{i}_{\phi,\,\mathrm{th}}, \nu^{i}_{\mathrm{L},\,\mathrm{th}}$ — ushbu chastotalarning nazariy hisoblangan qiymatlaridir.

Biz GRS~1915+105, GRO J1655–40, XTE J1550–564 va M82 X–1 kabi X-nurlanishli juftliklarda kuzatilgan QPO (kvaziperiodik tebranish) ma’lumotlarini tanladik. Bu obyektlar uchun aniq kuzatuv natijalari mavjud bo‘lib, ular 1-jadvalda keltirilgan.

Source [197, 208–212]	ν_{U} [Hz]	$\Delta\nu_{\mathrm{U}}$ [Hz]	ν_{L} [Hz]	$\Delta\nu_{\mathrm{L}}$ [Hz]	Mass [208] [$\mathrm{M}_{\odot}$]
XTE J1550-564	276	± 3	184	± 5	$12.4^{+2.0}_{-1.8}$
GRS 1915+105	168	± 3	113	± 5	$12.4^{+2.0}_{-1.8}$
GRO J1655-40	451	± 5	298	± 4	5.4 ± 0.3
M82 X-1	5.07	± 0.06	3.32	± 0.06	415 ± 63

1-jadval. *Mikrokvazarlarda va Galaktika markazida kuzatilgan juft tebranishli QPO chastotalari*

Keyin, magnit zaryadlangan qora o‘ralar uchun parametrlar (M, q, λ, r) bo‘yicha cheklovlar olish maqsadida MCMC simulyatsiyasini o‘tkazamiz. Bunda mavjud adabiyotlarda keltirilgan QPO ma’lumotlariga asoslanib, Gauss taqsimotli priorlar tanlanadi. Har bir parametr uchun Gauss taqsimotidan taxminan 10^5 ta namunaviy nuqta olingan. Bu yondashuv fizik jihatdan mumkin bo‘lgan parametrlar fazosini belgilangan chegaralar doirasida chuqur o‘rganish va eng yaxshi mos keluvchi (best-fit) parametr qiymatlarini aniqlash imkonini berdi.

P	XTE J1550-564		GRS 1915+105		GRO J1655-40		M82 X-1	
	μ	σ	μ	σ	μ	σ	μ	σ
$M/M_{\odot}$	6.9722	1.0458	11.5508	1.7326	4.2666	0.5546	372.77	44.64
q/M	0.0692	0.0103	0.0526	0.0079	0.024	0.0035	0.045	0.0067
λ/M	0.0225	0.0033	0.0241	0.0036	0.028	0.0042	0.0202	0.003
r/M	6.424	0.9634	6.3826	0.9573	6.4036	0.9605	6.5119	0.9768

2-jadval. *QPO ma'lumotlariga asoslangan magnit zaryadlangan qora o'ralar uchun Gauss priorlari.*

P	XTE J1550-564	GRS 1915+105	GRO J1655-40	M82 X-1
$M/M_{\odot}$	$6.9792^{+0.7642}_{-1.0218}$	$9.9321^{+1.1902}_{-1.0328}$	$4.8654^{+0.4159}_{-0.3854}$	$419.671^{+34.424}_{-32.544}$
q/M	$0.0692^{+0.0103}_{-0.0102}$	$0.0525^{+0.0078}_{-0.0078}$	$0.0241\ ^{+0.0036}_{-0.0036}$	$0.045^{+0.0066}_{-0.0067}$
λ/M	$0.0225\ ^{+0.0033}_{-0.0033}$	$0.024^{+0.0036}_{-0.0036}$	$0.0281^{+0.0042}_{-0.0042}$	$0.0203^{+0.003}_{-0.003}$
r/M	$6.4191^{+0.8775}_{-0.8479}$	$6.0961^{+0.884}_{-0.8433}$	$7.1962^{+0.8014}_{-0.7762}$	$7.3697^{+0.8113}_{-0.7783}$

3-jadval. *XTE J1550-564, GRS1915+105, GROJ1655-40 va M82 X-1 obyektlari uchun M, q, λ va r parametrlarining eng mos (best fit) qiymatlari.*

Priorlar belgilab olingach, mavjud ma'lumotlardan foydalanib, magnit zaryadlangan qora o'ra fazovakati uchun {M, q, $\boldsymbol{\lambda}$, r} parametrlarining ehtimoliy diapazonini aniqlash maqsadida MCMC simulyatsiyasini bajaramiz. Gauss taqsimotli priorni hisobga olib, har bir parametr uchun 10^5 ta namunaviy nuqta tanlab olinadi. Bu yondashuv belgilangan chegaralar doirasida fizik jihatdan mumkin bo'lgan ko'p o'lchovli parametr fazosini o'rganish hamda ma'lumotlarga eng yaxshi mos keluvchi parametr qiymatlarini aniqlash imkonini beradi.

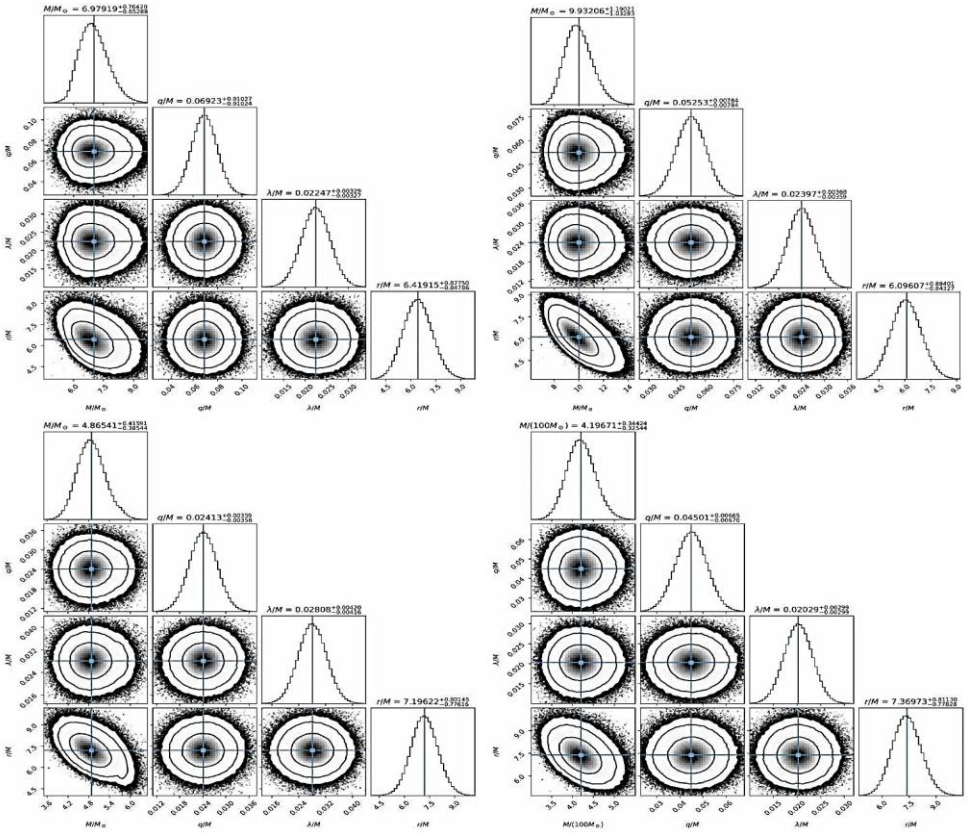
Magnit zaryadlangan 4 ta qora o'ra (uchta yulduzsimon massali va bitta o'rta massali nomzod) uchun {M, q, λ, r} parametrlarining MCMC tahlili natijalari 16-rasmda ko'rsatilgan. Ushbu qora o'ralar nochiziqli elektrodinamika maydonidan hosil bo'lgan magnit zaryadga va ideal suyuqlik qorong'i modda bilan o'ralgan muhitga ega. Maqsad — mikrokvazarlardagi QPO kuzatuv ma'lumotlarini modifikatsiyalangan Schwarzschild metrikasiga asoslangan nazariy bashoratlar bilan solishtirishdir.

Rasmda kontur grafiklari va posterior taqsimotlar keltirilgan bo'lib, ular har bir parametr uchun statistik ishonchlilik darajalarini ko'rsatadi. MCMC algoritmi QPO modellaridan olingan asosiy chastotalarga tayanib tuzilgan o'xshashlik funksiyasidan namuna oladi.

Har bir kichik grafik parametrlar o'rtasidagi bog'liqlikni tasvirlaydi. Diagonal elementlar — alohida parametrlar uchun marjinal (ya'ni boshqa parametrlar bo'yicha integrallashgan) ehtimollik taqsimotlarini ko'rsatadi; off-diagonal (diagonaldan tashqari) grafiklar esa ikki parametr o'rtasidagi ishonchlilik konturlarini (M, q, $\boldsymbol{\lambda}$, r va hokazo) ifodalaydi. Ushbu konturlar odatda 68% (3$\boldsymbol{\sigma}$), 95% (2$\boldsymbol{\sigma}$) va 99% (1$\boldsymbol{\sigma}$) ishonchlilik intervalini o'z ichiga oladi. Kontur grafiklaridagi

soya qilingan hududlar shu ishonchlilik darajalarini aks ettiradi.

Magnit zaryad q ortgani sari massa M ning posterior taqsimoti biroz pastroqqa siljiydi — bu magnit zaryadlangan qora o‘ralar QPO tahlilida Schwarzschild qora o‘ralariga qaraganda nisbatan yengil ko‘rinishi mumkinligini anglatadi. Shuningdek, λ ham orbital chastotalarga ta’sir ko‘rsatadi va qora o‘ra massasi bahosini biroz o‘zgartiradi. Konturlar parametrlar o‘rtasida kuchli degeneratsiyalar mavjudligini ham ko‘rsatadi; masalan, λ ortishi q ning kamayishini kompensatsiyalab, QPO chastotalarini baribir tushuntirib bera oladi.



12-rasm. *RP modelida to‘rt o‘lchamli MCMC tahlili yordamida QPO ma’lumotlari asosida quyidagi obyektlar uchun QPO orbitasiga tegishli magnit zaryadlangan qora o‘ra parametrlariga qo‘yilgan cheklovlar: yulduzsimon massali qora o‘ralar — XTE J1550-564 (yuqori chap), GRS~1915+105 (yuqori o‘ng), GRO J1655-40 (pastki chap) va o‘rta massali qora o‘ra — M82 X-1 (pastki o‘ng).*

Natijalar shuni ko‘rsatadiki, GRS 1915+105, GRO J1655-40 va XTE 1550-564 kabi mikrokvazarlardagi qora o‘ralar uchun q va λ ning eng mos keluvchi qiymatlari fizik jihatdan mumkin bo‘lgan oraliqlarda qoladi. Bu cheklovlar kuchli tortishish maydonlarida nochiziqli elektrodinamika va PFDM ta’sirining kuzatuvga asoslangan bilvosita dalillarini beradi.

XULOSA

“Tashqi materiya maydonlari mavjudligida qora o‘ralarning termodinamik xossalari va fundamental chastotalari” mavzusidagi falsafa doktori (PhD) dissertatsiyasi bo‘yicha olib borilgan tadqiqotlar asosida quyidagi xulosalar taqdim etildi:

1. Chaplygin-simon qorong‘i suyuqlik qora o‘ralarning hodisalar gorizonti tuzilishini sezilarli darajada o‘zgartirishi ko‘rsatib berildi. Xususan, holat tenglamasi parametri hodisalar gorizontining kattalashishiga olib kelishi, suyuqlik intensivligi parametri esa gorizont yuzasini kamaytirishi aniqlandi.

2. Qora o‘ralarni o‘rab turgan mukammal suyuqlik ko‘rinishidagi qorong‘i materiya ularning termodinamik xossalariga sezilarli ta’sir ko‘rsatishi aniqlandi. PFDM hissasi ortishi bilan Hoking haroratining maksimal qiymati vakuum holatiga nisbatan yuqoriroq bo‘lishi, fazaviy o‘tish nuqtasi esa hodisalar gorizontining kichik radiuslaridan kattaroq radiuslar tomonga siljishi ko‘rsatildi. Shuningdek, massasi katta qora o‘ralarning termodinamik xatti-harakati PFDM hissasiga nisbatan sezgirroq ekani aniqlandi.

3. Chaplygin-simon suyuqlik intensivligi 3:2 chastotalar nisbatiga mos keluvchi QPO orbital radiuslarining ichkariga siljishiga sabab bo‘lishi aniqlandi. Bu esa qorong‘i suyuqlik hissasi kuchaygan sari QPO signallari gravitatsion maydon kuchliroq bo‘lgan, qora o‘raga yaqinroq orbitalarda hosil bo‘lishini ko‘rsatadi.

4. XTE J1550–564, GRO J1655–40, GRS 1915+105 va M82 X-1 obyektlari uchun MCMC asosidagi QPO tahlili PFDM parametrlarining $q \in [0.024, 0.069]$ va $\lambda \in [0.020, 0.028]$ oraliqlarida mustaqil kuzatuv natijalari bilan mos keluvchi massa qiymatlarini chegaralash imkonini berishi ko‘rsatildi. Bu QPO kuzatuvlari qora o‘ra zaryadi va PFDM parametrlarini chegaralashda samarali vosita bo‘lib xizmat qilishi mumkinligini tasdiqlaydi.

SCIENTIFIC COUNCIL DSc. 03/07.07.2025. FM/T.192.01 ON AWARD OF SCIENTIFIC DEGREE AT INSTITUTE FOR ADVANCED STUDIES AT "NEW UZBEKISTAN" UNIVERSITY

SAMARKAND STATE UNIVERSITY NAMED AFTER SHAROF RASHIDOV

RAKHMATOV BEKZOD

FUNDAMENTAL FREQUENCIES AND THERMODYNAMICAL PROPERTIES OF BLACK HOLES IN THE PRESENCE OF EXTERNAL MATTER FIELDS

01.04.02 – Theoretical Physics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) IN PHYSICS AND MATHEMATICS

Tashkent – 2026

The theme of dissertation of the Doctor of Philosophy (PhD) on physical and mathematical sciences is registered by Supreme Attestation Commission of Higher Education, Science and Innovations of Republic of Uzbekistan under B2025.2.PhD/FM1324.

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The defense of the dissertation will be held on "16" iyul 2026 at 16.00 on the meeting of the Scientific Council No. DSc.03/07.07.2025.FM/T.192.01 at the Institute for Advanced Studies at "New Uzbekistan" university (Address: 100007, Tashkent city, Mirzo Ulughbek district, Movarounnahr Street 1, Institute for Advanced Studies at "New Uzbekistan" university, phone: +99871 202-41-11; e-mail: info@newuu.uz)

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INTRODUCTION (presentation abstract)

Topicality and demand of the dissertation. Black holes are among the most important objects for testing general relativity in the strong gravitational-field regime. Their spacetime geometry, horizon structure, thermodynamic behavior, and particle dynamics are directly connected with observable astrophysical processes occurring near compact objects. In particular, the motion of test particles, accretion-related phenomena, and quasi-periodic oscillations observed in

X-ray emissions provide important tools for studying the physical properties of black holes and their surrounding environments.

Recent astrophysical observations have shown that the distribution of dark matter in galaxies can be more diverse than expected. Several systems with very low dark matter content or without a clearly detectable dark matter component, including

NGC 1052-DF2, NGC 1052-DF4, FCC 224, and dark matter deficient spiral galaxies, indicate that dark matter effects may strongly depend on the astrophysical environment. Numerical simulations also suggest that such systems can appear due to environmental mechanisms such as tidal interactions and stripping. These results motivate the study of effective models describing possible dark environments around compact objects, especially black holes.

Within general relativity, perfect fluid dark matter and Chaplygin-like dark fluid provide useful effective descriptions for investigating how dark environments may influence black hole physics. Perfect fluid dark matter can phenomenologically describe dark matter effects near black holes, while Chaplygin-like dark fluid is important because it can represent a medium with properties interpolating between dark-matter-like and dark-energy-like behavior. Therefore, studying black holes surrounded by these fields is relevant for understanding how external dark environments modify strong-field gravitational properties.

The presence of perfect fluid dark matter and Chaplygin dark fluid can change the spacetime structure around black holes, affect the event horizon, and modify thermodynamic quantities such as Hawking temperature, entropy, and heat capacity. These fields can also influence the Keplerian, radial, and vertical epicyclic frequencies of particles moving around black holes. Since these fundamental frequencies are closely related to quasi-periodic oscillations observed in microquasars and other compact astrophysical systems, their analysis is important for connecting theoretical black hole models with observational data.

The demand for this dissertation is determined by the need to study black holes in realistic astrophysical environments and to clarify how dark matter and dark fluid parameters affect observable strong-field signatures. In particular, the investigation of particle dynamics, thermodynamic properties, fundamental frequencies, and QPO signatures makes it possible to test black hole models within general relativity and compare theoretical predictions with astrophysical observations. Thus, the dissertation is topical because it addresses the influence of perfect fluid dark matter and Chaplygin dark fluid on black hole thermodynamics, orbital motion, and quasi-periodic oscillations, which are among the important problems of modern

relativistic astrophysics and strong-field gravity.This dissertation corresponds to the tasks set by the following state regulatory documents: Decree of the President of the Republic of Uzbekistan No. UP-4947 "On the Strategy of Actions for the Further Development of the Republic of Uzbekistan" dated February 07, 2017, and Decree of the President of the Republic of Uzbekistan No. PP-2789 "On Measures for Further Improvement of Academy of Sciences, organization, management, and financing of research activities from 18.02.2017.

Compliance of the research with the main priority areas for the development of science and technology in the Republic. The research conducted for the dissertation was carried out in accordance with the priority areas of science and technology of the Republic of Uzbekistan.

Degree of study the problem. The works of Kerr, Reissner-Nordström, Hawking, Page, Born-Infeld, and Maldacena have made significant contributions to the study of black holes under the conditions of nonlinear electrodynamics. The Kerr and Reissner-Nordström metrics provide classical solutions, while Hawking and Page explored quantum effects. Born-Infeld and Maldacena proposed alternative theoretical approaches. These studies help deepen our understanding of the electromagnetic properties of black holes and their role in the universe.

In our country, the works of Bobomurat Ahmedov, Ahmadjon Abdujabbarov, Javlon Rayimbaev, Farrux Atamuratov, and Bobir Toshmatov on black holes in NED models hold an important place in modern astrophysics and theoretical physics. Their research reveals new physical properties by studying the effects of nonlinear electromagnetic fields around black holes and connects these processes with astrophysical observations.

My dissertation focuses on the combined effects of nonlinear electrodynamics and perfect fluid dark matter on black hole geometry, gravitational and electromagnetic fields, thermodynamic properties, and particle dynamics. It also investigates how external matter fields affect fundamental frequencies and quasi-periodic oscillation signatures, with the aim of comparing theoretical predictions with astrophysical observations.

Connection of the dissertation topic with the scientific research of the higher educational/research institutions, where the dissertation was carried out. The dissertation was carried out at Samarkand State University named after Sharof Rashidov within the framework of the scientific research topic and scientific projects of the Institute of Fundamental and Applied Research: FL-7923051796 “Modeling the dynamics of scalar fields and quantum emitters in spherically symmetric gravitational fields” (2024–2026).

The aim of the research of the dissertation: Aim of the work is to assess astrophysical processes around black holes by studying their particle dynamics, fundamental frequencies, thermodynamic properties, and QPO signatures in the presence of Chaplygin dark fluid and perfect fluid dark matter.

The tasks of the research:

• to study the spacetime properties of black holes surrounded by CDF;

• to analyze the dynamics of test particles and stable circular orbits around black holes in the presence of CDF;

• to calculate the fundamental frequencies and study QPO signatures of black holes surrounded by dark fluid matter;

• to obtain new black hole solutions in GR coupled to NED in the presence of PFDM;

• to study the thermodynamic properties and phase-transition behavior of black holes in NED and PFDM fields;

• to constrain black hole and surrounding-field parameters using QPO data and MCMC analysis.

The object of the research are black holes influenced by PFDM and NED, including their geometric and thermodynamic properties, the orbital motion of test particles, accretion disk and radiation characteristics, as well as the determination of PFDM and NED parameters using MCMC simulations.

The subject of the research is the influence of PFDM and NED models on black holes, test particles, accretion disks, and shadows, as well as the study of the geometric and thermodynamic properties of black holes within these models.

The methods of the research theoretical and analytical methods; methods of general relativity; numerical modeling; thermodynamic analysis; orbital dynamics analysis; statistical and MCMC methods.

The scientific novelty of the research is the following:

• for the first time, two types new black hole solutions are obtained in GR coupled to NED in the presence of PFDM;

• for the first time, we have shown that the heat capacity of the black hole undergoes phase transitions due to the NED field and dark matter;

• for the first time, we constrain the parameters of black holes, NED, and dark fields, namely Chaplygin dark fluid and perfect fluid dark matter, using QPO data from microquasars and stellar-mass black holes through MCMC analysis.

The practical results of the research are the following:

The parameters of black holes surrounded by Chaplygin-like dark fluid were constrained using MCMC simulations based on observed QPO data. For XTE J1550–564, GRO J1655–40, GRS 1915+105, Sgr A*, and M82 X-1, the parameters were obtained in the ranges B = 5.311 × 10-5–9.340 × 10-5, q = 0.651–0.953, and r/M = 3.057–4.496.

For magnetically charged black holes surrounded by perfect fluid dark matter, the observed QPO data of XTE J1550–564, GRS 1915+105, GRO J1655–40, and M82 X-1 constrained the parameters within q/M = 0.0241–0.0692 and λ/M = 0.0203–0.0281. The corresponding QPO orbital radii were found in the range r/M = 6.0961–7.3697.

It has been shown that the heat capacity of black holes in the presence of NED and PFDM changes its sign at the phase-transition point. This result allows one to distinguish thermodynamically unstable and stable black hole configurations. Therefore, the obtained results can be practically used to estimate black hole mass, magnetic charge, dark-fluid parameters, and PFDM parameters from QPO

observations.

The reliability of the research results is ensured by the use of well-established methods of mathematical physics, computational mathematics, and relativistic astrophysics. All calculations are carried out within the framework of general relativity and black hole physics. The obtained results are based on analytical derivations and numerical computations of the spacetime geometry, horizon structure, thermodynamic quantities, particle motion, fundamental frequencies, and quasi-periodic oscillation signatures. The validity of the results is further supported by comparison with known limiting cases, previously published results, and available astrophysical observational data. The conclusions of the dissertation are consistent with the basic principles of compact object astrophysics and strong-field gravity.

Scientific and practical significance of the research

The scientific significance of the research lies in clarifying how perfect fluid dark matter, Chaplygin dark fluid, and nonlinear electrodynamics affect the physical properties of black holes. In particular, the dissertation investigates their influence on black hole geometry, horizon structure, thermodynamic behavior, particle dynamics, and fundamental frequencies. These results contribute to a deeper understanding of black holes surrounded by external matter fields and provide theoretical insight into how such environments can modify strong gravitational-field processes.

The practical significance of the research is that the proposed model for black hole mimickers can play a crucial role in explaining high-energy astrophysical processes. In particular, it offers new predictions regarding the shadow size and optical appearance of black holes, which can be tested through astrophysical observations such as those conducted by the Event Horizon Telescope. These results can be applied to determine astrophysical parameters more accurately and guide future observational efforts. Additionally, the theoretical approaches based on PFDM and NED models contribute to a deeper understanding of the functioning mechanisms of cosmic systems. The outcomes of this research can serve as a scientific foundation for the development of new space observatories or the improvement of existing technologies.

Applications of the research results. The results of the study titled fundamental frequencies and thermodynamical properties of black holes in the presence of external matter fields have been applied as follows:

In the investigation of the fundamental frequencies and thermodynamical properties of black holes influenced by exotic matter, this approach has been utilized by several authors ("Li, Xiang-Qian and Kim, Yoonbai and Lee, Bum-Hoon and Yan, Hao-Peng and Yue, Xiao-Jun. Black holes immersed in modified Chaplygin-like dark fluid and cloud of strings: geodesics, shadows, and images. Chinese Physics C, 2025", "Javed, Faisal and Waseem, Arfa and Mustafa, G. and Fatima, Ghulam and Alkarni, Shalan". Particle motion and thermal fluctuations of charged AdS black holes surrounded by exotic fluid with modified Chaplygin equation of state. Phys. Dark Univ. 47, 2025" and "Turimov, Bobur and Rahimov, Ozodbek and

Davlataliev, Akbak and Tadjimuratov, Pulat and Norkobilov, Mukhiddin and Rahimova, Shakhzoda. Analysing geodesic motion in Bocharova–Bronnikov–Melnikov–Bekenstein spacetime. 2025. Phys.Dark Univ. 48 (2025) 101855"). "Analysing geodesic motion in Bocharova–Bronnikov–Melnikov–Bekenstein spacetime", Physics of the Dark Universe, vol. 48, 2025, 101855. Erdem Sucu, Izzet Sakallı. "Astrophysical reality of black hole thermodynamics and dynamics: Transformative influence of Hernquist dark matter distributions", Physics of the Dark Universe, vol. 49, 2025, 102051.

Scientific results obtained on the motion of particles and photons have been used by scientists from Fudan University (FU) in Shanghai (FU, China, May 23, 2025);

Approbation of the research results. The research results were reported and discussed at 4 international and 3 local scientific conferences.

Publication of research results were presented in 7 scientific articles published in scientific journals recommended by the Higher Attestation Commission under the Ministry of Higher Education, Science and Innovation of the Republic of Uzbekistan.

Volume and structure of the dissertation. Consists of an introduction, three chapters, a conclusion, and a list of references all in 119 pages.

MAIN CONTENT OF THE DISSERTATION

In the introduction, the relevance and necessity of the dissertation topic are substantiated, and its alignment with the priority areas of the development of science and technology in the republic is demonstrated. The degree of study of the problem is presented. The objectives, tasks, object, and subject of the research are described. The scientific novelty and practical results of the study are outlined. The theoretical and practical significance of the obtained results is explained. Information is provided regarding the implementation of the research results, published works, and the structure of the dissertation.

The first chapter of the dissertation, titled "Black Holes Surrounded by Chaplygin-like Dark Matter: Spacetime Properties and Quasiperiodic Oscillation Tests," presents the fundamental concepts and results that serve as a basis for the subsequent chapters. Additionally, Chaplygin-like equations of state are reviewed. For the black hole solution, a spherically symmetric metric is chosen as follows:

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \qquad (1)$$

where $f(r)$ is the metric function depending on the radial coordinate r. The nonlinear behavior of the equation of state for Chaplygin dark (perfect) fluid is considered in the form of a $p = -B/\rho$ model, with radial and tangential pressures $p_r = -\rho$ and $2p_t = \rho - 3B/\rho$, where B is a constant being positive. Here, the density ρ and pressure are radial dependent. The stress energy and Einstein tensors for the fluid are used in the following form:

$$T_t^t = T_r^r = -\rho, T_\theta^\theta = T_\phi^\phi = \frac{1}{2}\left(\rho - \frac{3B}{\rho}\right) \qquad (2)$$

and

$$G_t^t = G_r^r = \frac{1 - rf' - f}{r^2}, G_\theta^\theta = G_\phi^\phi = \frac{2f' + rf''}{2r} \tag{3}$$

respectively. Using Eqs. (2) and (3) one can get the system of equations

$$f + rf' - 1 = -\rho r^2, \frac{1}{r}(2f' + rf'') = \rho - \frac{3B}{\rho} \tag{4}$$

From the above equation, one can immediately get,

$$\rho^2 = B + \frac{q^2}{r^6} \tag{5}$$

where q is an integration constant, and a positive normalization factor represents the CDF's intensity. In the geometrized units, the unit of density $[\rho] = cm^{-2}, [B] = cm^{-4}$ and $[q] = cm$. The unit of the parameter of the Chaplygin equation of state B is $\mathrm{kg}^2/(\mathrm{m}^4\ \mathrm{s}^2)$ in the SI system.

Simple algebraic analysis shows that $\rho \simeq \sqrt{B}$ when $q \to 0$ or at large distances, while when $B \to 0$ or at small distances $(r^6 \ll q^2/B)\rho \simeq q/r^3$. The function derived from [?] is obtained by solving Eq. (4) as

$$f(r) = 1 - \frac{2M}{r} - \frac{r^2}{3}\sqrt{B + \frac{q^2}{r^6}} + \frac{q}{3r}\operatorname{arcsinh}\frac{q}{\sqrt{B}r^3} \tag{6}$$

where M is BH mass, the quantity q represents CDF's intensity and takes positive values. The CDF's EoS parameter, a positive constant B, is calculated using the EoS $p = -B/\rho$. For a comoving observer with the fluid, ρ and p represent the energy density and isotropic pressure, respectively. The spacetime metric (6) tuns to the Schwarzschild case at $B = q = 0$.

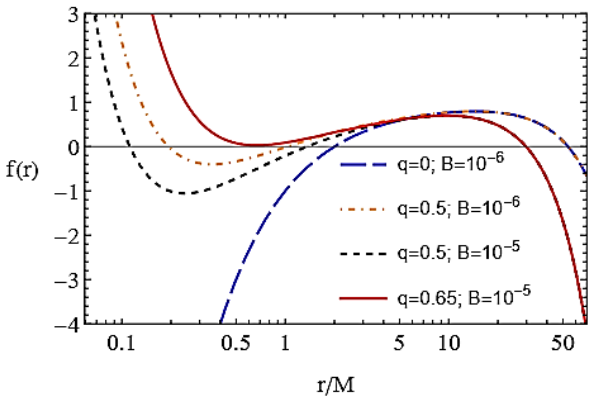


***Figure 1**: Radial dependence of lapse function $f(r)$ for varying values of q and*

The radial profiles of the metric function $f(r)$ for different values of B and q parameters are shown in Fig. 1. At $q = 0$, we can observe two horizons: event horizon and cosmological horizons, while the non-vanishing q parameter causes the appearance of the inner (Cauchy) horizon. Also, there is an upper limit in the values of q where Cauchy and event horizons disappear, but not the cosmological one.

To study the effect of the EoS parameters on the geometry around the BH, we consider the event horizon, which can be analyzed using the equation $f(r) = 0$.

The dependence of the event horizon radius from the parameter q for various values of B is shown in Fig. 2. Plotting the figure, we normalized all quantities to the BH mass and took $M = 1$. It is observed from the figure that the radius increases with the increase in the value of B. The event horizon shrinks as the parameter q increases, implying the parameter behaves with an antigravitating effect. There is an upper value for q where the event horizon and Cauchy horizons coincide. An increase in B causes an increase in the upper value and a decrease in the minimum value of the event horizon. In the $B = 0$ case, there is no inner horizon for the values of q that are less than about 0.25 .

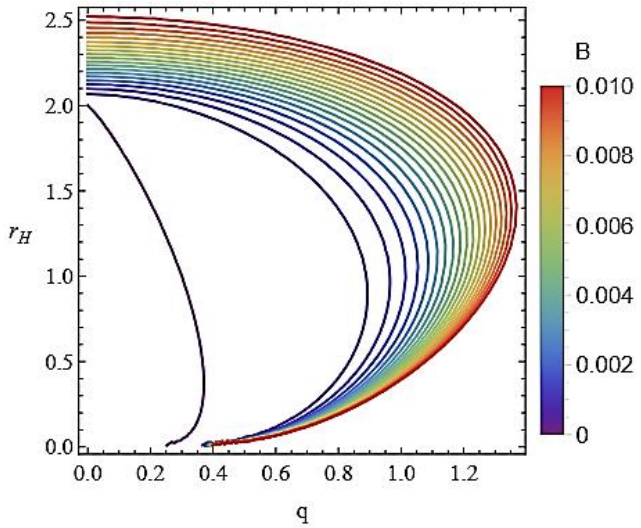


***Figure 2**: The event horizon radius as a function of q for different values of B.*

Fundamental frequencies. This section discusses the fundamental frequencies of test particles around our modified Schwarzschild BH of metric (1) are discussed.

The plan is to study the possible values of the upper and lower frequencies by the following models for twin peak HF QPOs described by the fundamental frequencies of test particles around compact gravitating objects $\left(\nu_{r,\theta,\phi} = c^3/(2\pi GM)\Omega_{r,\theta,\phi}\right)$. The equatorial circular orbits of a test particle are characterized by three fundamental frequencies:

1. Orbital frequency (or Keplerian frequency) ν_ϕ, which is the inverse of the orbital period.
2. Radial epicyclic frequency ν_r, which is the frequency of radial oscillations around the mean orbit.
3. The vertical epicyclic frequency ν_θ is the frequency of vertical oscillations around the mean orbit.

These three frequencies depend only on the space-time metric and the orbit radius. Although they are defined as the characteristic frequencies for the orbital motion of a free particle, there is a direct relation between these frequencies and those of the oscillation modes of the fluid of an accretion flow.

To study the Keplerian frequency of a test particle, one uses the geodesic equation,

$$\ddot{x}^{\alpha} + \Gamma^{\alpha}_{\mu\nu}\dot{x}^{\mu}\dot{x}^{\nu} = 0 \tag{7}$$

here, $\Gamma^{\alpha}_{\mu\nu}$ are the Christoffel symbols and $\dot{x} = \dot{t}(1,0,0,\Omega)$, the equation of motion, the test particles get the form

$$\partial_r g_{tt} + \Omega^2 \partial_r g_{\phi\phi} = 0 \tag{8}$$

and the solution of the above equation for the Keplerian frequency, Ω_K, can be expressed as

$$\Omega_K^2 = \frac{M}{r^3} - \frac{1}{3r^3}\sqrt{Br^6 + q^2} - \frac{q}{6r^3}\operatorname{arcsinh}\frac{q}{\sqrt{B}r^3} \tag{9}$$

To estimate the value of the fundamental frequencies, the unit of Hz is associated with them as,

$$\nu = \frac{1}{2\pi}\frac{c^3}{GM}\Omega, [\text{ Hz}] \tag{10}$$

Note that the values of the speed of light in vacuum and the gravitational constant are $c = 3 \cdot 10^8$ m/sec and $G = 6.67 \cdot 10^{-11}$ m^3/(kg^2 $\cdot$ sec), respectively.

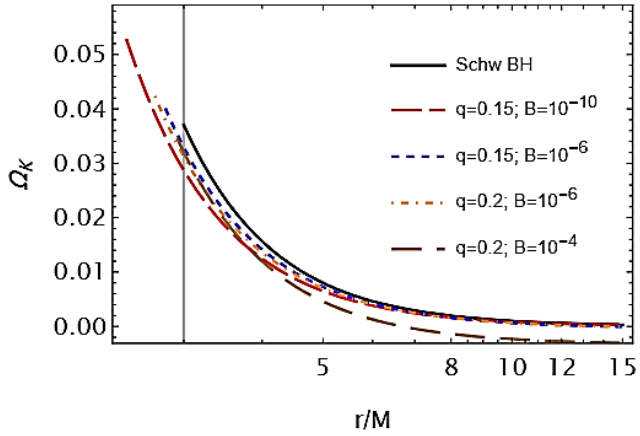


Figure 3: *The figure shows the radial dependence of f(r) for different values of q and B.*

The radial dependence of angular momentum of particles in Keplerian orbits is shown in Fig. 3 by varying the parameters q and B. One can observe that the Keplerian frequency increases in the presence of the parameter q compared to the Schwarzschil case. In contrast, the EoS parameter causes it to decrease. The effect of the intensity parameter of the dark fluid on the frequency is negligible at large distances.

The test particles moving in the equatorial plane around BH in stable orbits oscillate along radial and vertical axes because of the small displacement from their stable orbits as $r_0 + \delta r$ and $\pi/2 + \delta\theta$

The radial and vertical frequencies can be calculated by the following harmonic oscillator equations as:

$$\frac{d^2\delta r}{dt^2} + \Omega_r^2\delta r = 0, \frac{d^2\delta\theta}{dt^2} + \Omega_\theta^2\delta\theta = 0, \tag{11}$$

where

$$\Omega_i^2 = -\frac{1}{2g_{ii}(u^t)^2}\partial_i^2 V_{\text{eff}}(r,\theta)\Big|_{\theta=\pi/2}, i = r,\theta \tag{12}$$

are the radial and vertical angular frequencies, respectively. Some algebra leads to

$$\Omega_r^2 = \Omega_{\text{K}}^2\left\{\frac{4\sqrt{q^2+Br^6}-12M+2q\text{ArcSinh}\left(\frac{q}{\sqrt{B}r^3}\right)}{3r}\right.$$

$$+\frac{\left(6M-3r+\sqrt{q^2+Br^6}-q\text{ArcSinh}\left(\frac{q}{\sqrt{B}r^3}\right)\right)}{6r\sqrt{q^2+Br^6}-18Mr+3qr\text{ArcSinh}\left(\frac{q}{\sqrt{B}r^3}\right)}$$

$$\left.\times\left(6Mr^3+\frac{q^2-8Br^6}{\sqrt{q^2+Br^6}}-q\text{ArcSinh}\left(\frac{q}{\sqrt{B}r^3}\right)\right)\right\}.$$

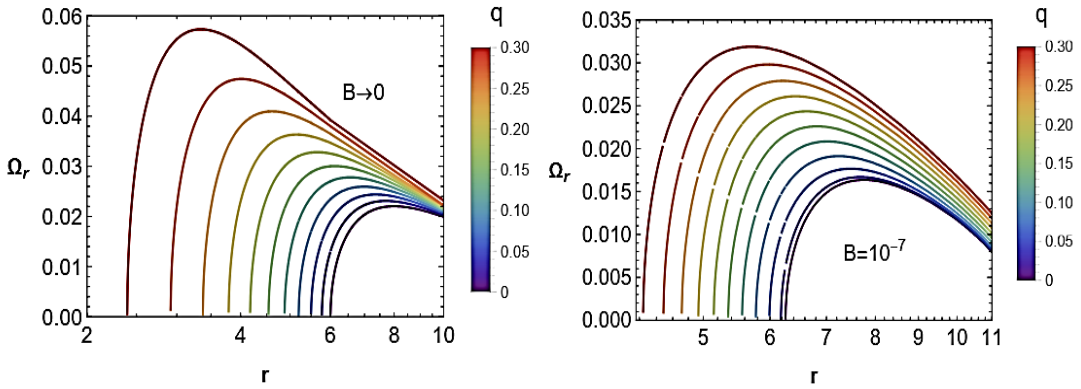


***Figure 4**: The figure shows the radial dependence Ω_r(for fixed B).*

Fig. 4 represents the radial plots of Ω_r for a fixed value of B. The Ω_r graph shows that it increases with an increase in q, while the EoS parameter causes a decrease in it.

Stella and Vietri first presented the relativistic precession (RP) model for kHz twin peak QPOs related to the frequency range $0.2-1.25$kHz from neutron stars detected in LMXRBs. This model also applies to BH s in binary systems and accurately determines the mass and spin of central BH in microquasars by employing data from the power-density spectrum of the BH accretion disk. Corresponds upper and lower frequencies in the RP model are described by the frequencies of the radial, vertical, and orbital oscillations of test particles in the accretion disc near their ISCO $\nu_U = \nu_\phi$ and $\nu_L = \nu_\phi - \nu_r$;

In Fig. 5, the $\nu_U - \nu_L$ diagrams are shown for twin peak QPOs around the BH of metric (1) in RP, ER2, ER3, ER4, and WD models with plots being compared to QPOs around Schwarzschild BH. Here, the mass of the central BH is $M = 5M_\odot$ as a test stellar-mass BH . The light-blue shaded area in these diagrams represents the

"graveyard" for twin-peak QPOs, where no twin-peaked QPOs are noticed. One can observe from the analyses that no LF QPOs exist in the presence of q. An increase in the EoS parameter B causes an increase in the upper and lower frequencies near the kilohertz band. However, the frequencies decrease with the q parameter. That means that there are inverse relations between the parameters B and q, providing the same upper and lower frequencies.

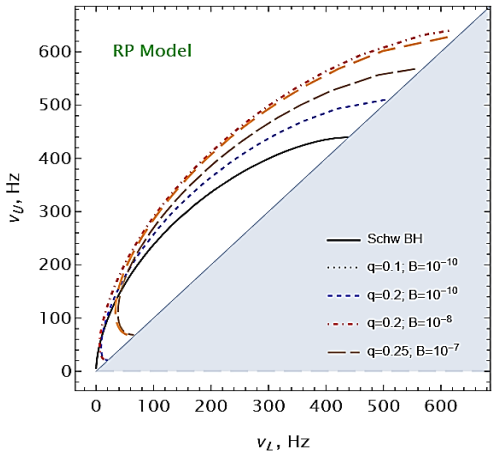


***Figure 5**: The relationship between the upper and lower frequencies around a Schwarzschild BH surrounded by CDF with Chaplygin-like EoS for the different parameter values q and B. Diagrams are shown in the RP, ER, and WD models from top left to bottom right.*

We study the outer orbits of ISCO where a twin peak QPO occurs in the RP, ER2-4, and WD models using the set of equations $m\nu_L(M,r,q) = n\nu_U(M,r,q)$ where m and n are integers.

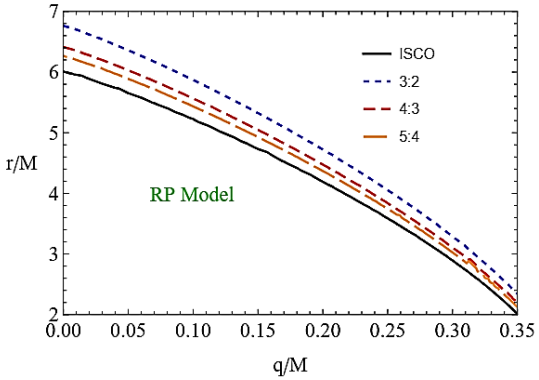


***Figure 6**: QPO radius as a function of q.*

Fig. 6 presents the dependence of QPO orbits from the dark fluid intensity parameter. It is observed that the parameter q increases the QPO radius decrease and comes close to the ISCO. According to the RP, ER3, and ER4 models, the orbit almost coincides with the ISCO.

In the second chapter of the dissertation, titled *"***Black Holes Surrounded by NED and PFDM: Spacetime Stability and Thermodynamics,** a black hole solution surrounded by NED and PFDM is provided. In addition, the thermodynamic properties of the given black hole are discussed.

Within the framework of General Relativity, a brief introduction is given to the spherically symmetric black hole solution coupled with a nonlinear electrodynamics field and perfect fluid dark matter. In this context, the ansatz is chosen as follows.

$$ds^2 = -e^{\nu(r)}dt^2 + e^{\Lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (13)$$

Nonsingular (regular) extensions of black hole spacetimes have been extensively studied within the framework of GR coupled with NES. However, in this work, we focus on a new black hole solution arising in a GR + NED system in the presence of an exotic field known as PFDM. The corresponding action is given by:

$$\mathcal{S} = \frac{1}{16\pi}\int d^4x\sqrt{|g|}(\mathrm{R} - \mathcal{L}_{\mathrm{NED}} + \mathcal{L}_{\mathrm{PFDM}}). \quad (14)$$

Here, R is the Ricci scalar and ggg is the determinant of the metric tensor. The Lagrangian for the NED field is given by:

$$\mathcal{L}_{\mathrm{NED}} = \frac{4\mu}{\alpha}\frac{(\alpha\,\mathrm{F})^{(\nu+3)/4}}{[1+(\alpha\mathrm{F})^{\nu/4}]^{1+\mu/\nu}}\,, \quad (15)$$

with the coupling parameter α and the Faraday tensor of the electromagnetic field $\mathrm{F} = \mathrm{F}_{ab}\,\mathrm{F}^{ab}$. Since there is no matter Lagrangian, the energy-momentum tensor in Schwarzschild coordinates is due solely to $\mathcal{L}_{\mathrm{NED}}$ as

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = T_{\mu\nu}^{(\mathrm{NED})} + T_{\mu\nu}^{(\mathrm{PFDM})}, \quad (16)$$

where the perfect fluid energy-momentum tensor is

$$T_{\mu\nu}^{(\mathrm{PF})} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, \quad (17)$$

which, in the case of PFDM, has only the standard orthogonal basis

$$T_{\mu\nu}^{(\mathrm{PFDM})} = \mathrm{diag}\big(-\epsilon, p_r, p_\theta, p_\phi\big), \quad (18)$$

with

$$-\epsilon = p_r = \frac{\lambda}{8\pi r^3}, p_\theta = p_\phi = -\frac{\lambda}{16\pi r^3}\,. \quad (19)$$

The energy-momentum tensor for the NED reads as,

$$4\pi T_{\mu\nu}^{(\mathrm{NED})} = -2\frac{\partial\mathcal{L}_{\mathrm{NED}}}{\partial F}F_\mu^\beta F_{\nu\beta} + \mathcal{L}_{\mathrm{NED}}g_{\mu\nu}. \quad (20)$$

The non-zero diagonal components of the tensor for the NED field are

$$T_0^0 = T_1^1 = -\frac{2\mu M q^\nu r^{\mu-3}}{(q^\nu + r^\nu)^{\frac{\mu+\nu}{\nu}}},$$

$$T_2^2 = T_3^3 = \frac{\mu M q^\nu r^{\mu-3}}{(q^\nu + r^\nu)^{\frac{\mu}{\nu}+2}}[(\nu+1)r^\nu - (\mu-1)q^\nu]. \quad (21)$$

To solve this field equation, first, we find the component G_{tt} of the Einstein tensor using the following ansatz

$$e^{\nu(r)} = e^{-\Lambda(r)} = f(r), \quad (22)$$

and we get

$$G_0^0 = G_1^1 = -\frac{1}{r}\frac{df(r)}{dr} - \frac{f(r)}{r^2} + \frac{1}{r^2},$$
$$G_2^2 = G_3^3 = -\frac{1}{2}\frac{d^2f(r)}{dr^2} - \frac{1}{r}\frac{df(r)}{dr}.$$

For a static and spherically symmetric spacetime with line element and NED source given by a magnetic charge q in which $F_{\mu\nu} = \left(\delta_\mu^\theta \delta_\nu^\phi - \delta_\nu^\theta \delta_\mu^\phi\right) q \sin\,\theta$.

$$\frac{1}{r}\frac{df(r)}{dr} + \frac{f(r)}{r^2} + \frac{1}{r^2} = \frac{2\mu M q^\nu r^{\mu-3}}{(q^\nu + r^\nu)^{\frac{\mu+\nu}{\nu}}} - \frac{\lambda}{8\pi r^3}. \tag{23}$$

The solution of this is,

$$f(r) = \frac{-2Mr^\mu (q^\nu + r^\nu)^{-\frac{\mu}{\nu}} + \lambda \ln r + r}{r} + \frac{c_1}{r}. \tag{24}$$

After simplifying,

$$f(r) = 1 - \frac{2M}{r}\left(1 + \frac{q^\nu}{r^\nu}\right)^{-\frac{\mu}{\nu}} + \frac{\lambda \ln r}{r} + \frac{c_1}{r}. \tag{25}$$

Noting that a quantity in the logarithmic term should be dimensionless. Therefore, we chose the constant as $c_1 = -\lambda \ln\,|\lambda|$, and we get the following form for the metric function:

$$f(r) = 1 - \frac{2M}{r}\left(1 + \frac{q^\nu}{r^\nu}\right)^{-\frac{\mu}{\nu}} + \frac{\lambda}{r}\ln\frac{r}{|\lambda|}. \tag{26}$$

Now, we analyze the horizon properties of the obtained black hole solution graphically.

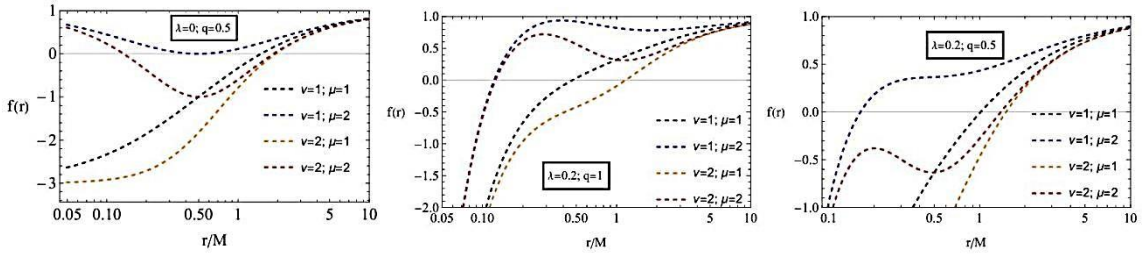


***Figure 7**: Radial dependence of the metric function $f(r)$ for various values of q and λ and fixed values of μ and ν.*

Figure 7 shows how the metric function $f(r)$ around the black hole changes under different parameter influences. Increasing the NED parameters reduces spacetime curvature, making the black hole more stable. A higher charge strengthens the electromagnetic field, expanding the event horizon. PFDM causes spacetime to extend and diminishes dynamic effects outside the black hole. These graphs highlight the interplay of these factors, providing insights into the structure of spacetime surrounding the black hole.

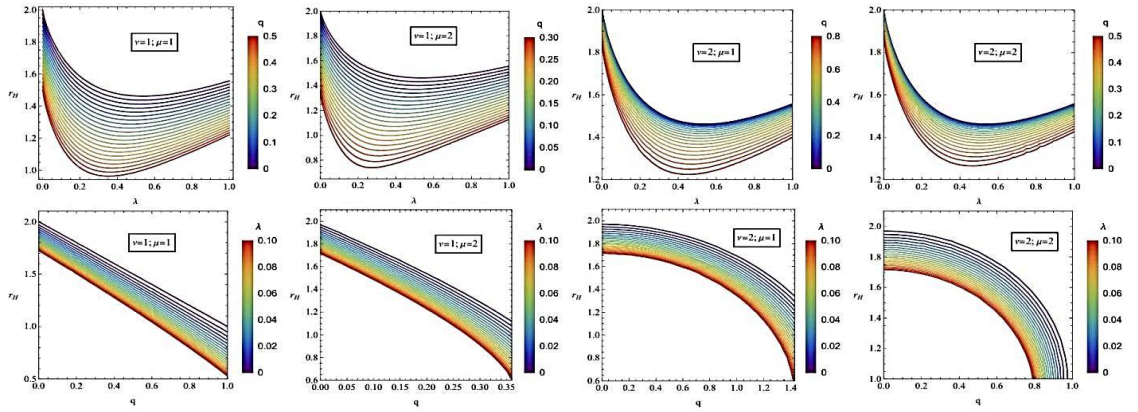


***Figure** 8: The Event horizon radius for q and λ.*

The given set of graphs in Fig.8 represents the radius of the event horizon r_H versus different parameters (and q) for various combinations of the parameters ν and μ. The upper row shows r_H versus λ, and the bottom row shows r_H versus q. Each column corresponds to different values of ν and μ. The color panel on the right side of each graph shows the value of the third parameter, λ and q. The radius of the event horizon has a minimum value of a certain value of λ. The curves show all graphs. This minimum value varies according to the values of ν and μ. As the value increases, the initial value of the event horizon radius, r_H, increases on the graphs. It shows that increasing ν causes the black hole to grow larger. An increase in the value of μ also increases the value of r_H, but this effect is weaker than the impact of ν. The U-shaped curves show the radius of the minimum event horizon as λ changes. This minimum value is reached at a certain value of λ, which varies for different parameters ν and μ. The graphs show that an increase in λ first decreases and then increases the radius of the event horizon. All graphs show that with increasing q, the radius of the event horizon r_H generally decreases. This means that an increase in magnetic charge shrinks the radius of the black hole's event horizon.

Thermodynamics of black holes surrounded by PFDM. Solving the equation $f(r_h) = 0$, one can obtain the event horizon radius r_h, with which the mass of the black hole can be expressed as

$$M = \frac{1}{2}\left(1 + \frac{q^\nu}{r_h^\nu}\right)^{\frac{\mu}{\nu}}\left(r_h + \lambda \ln \frac{r_h}{|\lambda|}\right). \tag{27}$$

The Hawking temperature T_h is related to the surface gravity by:

$$T_h = \frac{f'(r)}{4\pi}\bigg|_{r=r_h} = \frac{1}{4\pi}\left[\frac{2M}{r_h^2}\left(1 + \frac{q^\nu}{r_h^\nu}\right)^{-\frac{\mu}{\nu}} + \frac{2M\mu q^\nu}{r_h^{2+\nu}} \times \left(1 + \frac{q^\nu}{r_h^\nu}\right)^{-\frac{\mu}{\nu}-1} - \frac{\lambda}{r_h^2}\left(1 + \ln \frac{r_h}{|\lambda|}\right)\right] \tag{28}$$

In Fig. 9, we plot the temperature of the black hole with NED charge surrounded by PFDM. We can see that the black hole temperature increases and reaches a maximum before having a phase of decrease. Furthermore, this figure

shows us that this maximum increases for higher values of the dark matter parameter λ. Let us notice that the case " $\lambda = 0$ " corresponds to the temperature of a black hole without dark matter. We write the first law of black hole thermodynamics from the black hole studied here and then find the entropy before computing the corrected entropy. The first law is expressed as

$$dM = T_h dS_0 + \Phi_h dq + \beta_h d\alpha, \tag{29}$$

where S_0 represents the entropy at the equilibrium without considering thermal fluctuation or the uncorrected entropy. Let us notice that S_0, the magnetic charge q, and the dark matter parameter α form a complete set of extensible variables.

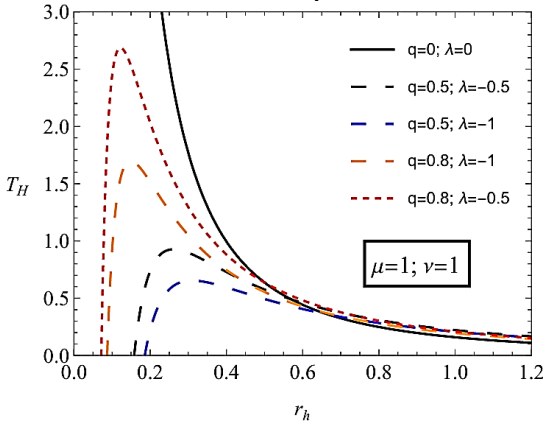


***Figure 9**: The black hole temperature T_h as a function of the event horizon radius r_h, showing the effects of the dark matter parameter λ and the black hole charge q on thermal evolution*

T_h is the Hawking temperature at the horizon and Φ_h is the potential. β_h is the conjugating quantity of the dark matter parameter λ. Now, in the first expression of (29), we can find the formula of the uncorrected entropy S_0, given by

$$S_0 = \int \frac{1}{T_h} dM = \int \frac{1}{T_h} \frac{\partial M}{\partial r_h} dr_h. \tag{30}$$

Now, we will compute the corrected entropy at the equilibrium S, for which the general formula is expressed as

$$S = S_0 - \beta \ln (S_0 T_h)$$

Here, β is the corrected parameter with only two values. If $\beta = 0$, describes the uncorrected entropy, and for $\beta = 1/2$.

From the plot Fig.10, one can compare the corrected and equilibrium entropy for a given black hole. In the absence of the correction parameter β, the equilibrium entropy of the system is a linearly increasing function, as shown in the bottom. For the correction parameter $\beta = 1/2$, the corrected entropy of the system always has a positive value, which is important for the system. The corrected entropy is a decreasing function below the critical horizon radius, but above this point is always an increasing function. Therefore, for a larger black hole, the corrected entropy of the system is not significant as expected. However, for a smaller black hole system, the correction term significantly affects the system's entropy due to small thermal fluctuations.

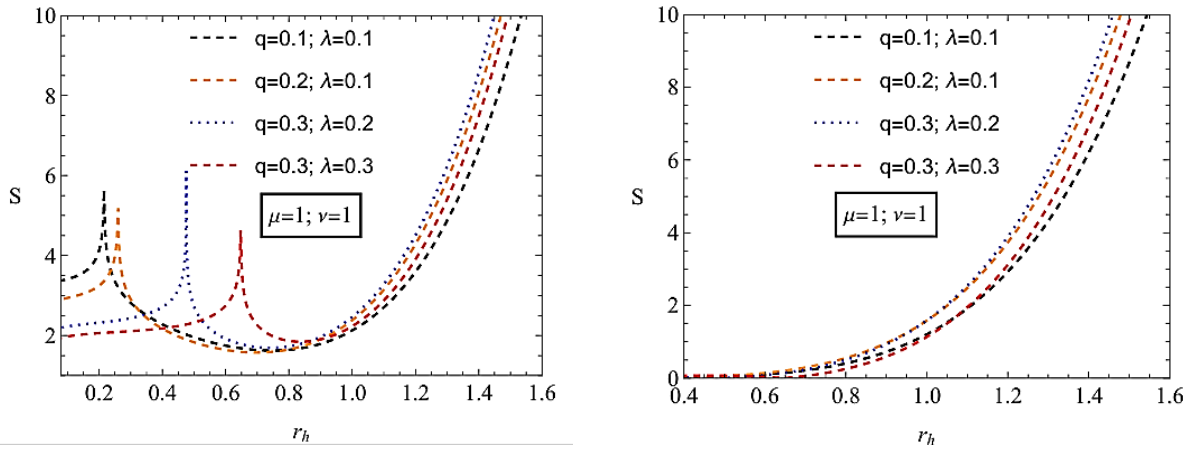


***Figure 10**: Variation of entropy S with black hole horizon radius r for different values of q and λ. Here $\beta = 0$ (no correction) is the bottom panel and $\beta = 1/2$ (with correction) is the top panel.*

The specific heat C at constant volume is given by:

$$C = T_h \left(\frac{\partial S}{\partial T_h}\right). \tag{31}$$

A black hole undergoes a second-order phase transition in phase transitions that lead the black hole to move from the stable phase $(C > 0)$ to the unstable phase $(C < 0)$.

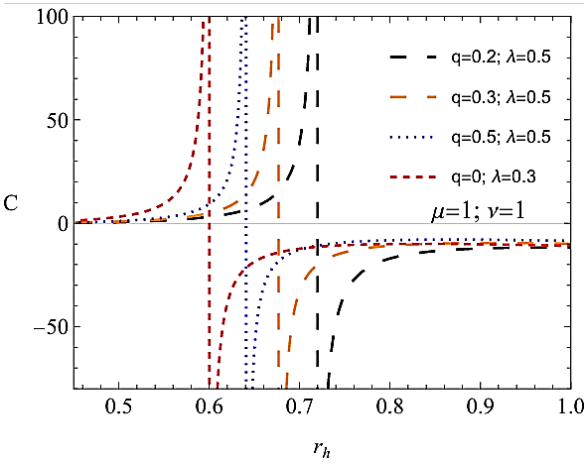


Figure 11: Change of the heat capacity C of the non-linear black hole in the background of perfect fluid dark matter

Figure 11 presents the variation of the black hole heat capacity C as a function of the horizon radius, demonstrating the thermodynamic stability of the black hole solution under different values of the charge and the PFDM parameter. One can observe from the plots the presence of discontinuities in the heat capacity, which indicate second-order phase transitions. These transitions separate different thermodynamic phases:

The heat capacity is negative for small black holes (r_h below the discontinuity point), implying that they are thermally unstable. This corresponds to an evaporating phase, in which the black hole loses mass via Hawking radiation. The heat capacity for large black holes (r_h above the transition point) becomes positive,

signaling a stable equilibrium phase where the black hole can maintain thermal stability.

Increasing the black hole charge shifts the phase transition point, suggesting that stronger NED effects influence the black hole's thermodynamic structure. The increase in charge leads to a more extended stable phase, reducing the likelihood of black hole evaporation. Increasing the PFDM parameter significantly alters the stability properties. A higher value of it shifts the phase transition to a larger event horizon, implying that the presence of PFDM supports stability for larger black holes. This suggests that black holes embedded in PFDM environments are more resistant to evaporation than those in vacuum or standard dark matter models. From an astrophysical perspective, these findings suggest that black holes in environments with significant PFDM influence may have longer lifetimes as the transition to a stable thermodynamic phase occurs at a larger horizon radius. Moreover, PFDM modifies the black hole's thermal response, potentially affecting accretion disk dynamics, black hole mergers, and observational signatures related to thermal fluctuations.

A well-defined second-order phase transition strengthens the argument that such black holes exhibit non-trivial stability properties, distinguishing them from classical Schwarzschild or Reissner-Nordström black holes. Future observational tests, such as those involving black hole shadow measurements or X-ray spectra from accretion disks, could constrain the parameters q and λ based on their effects on stability and evaporation rates.

In the third chapter of the dissertation, titled **"Magnetically Charged BHs and Radiation Processes, Particle Dynamics, and QPOs in a PFDM Environment"**, a new black hole solution is derived. Additionally, the radiative properties of accretion disks surrounding the black hole are investigated based on the Novikov–Thorne model. Using MCMC analysis, constraints are placed on the mass, charge, and exotic matter parameters for observed black hole candidates such as XTE J1550–564, GRS 1915+105, GRO J1655–40, and M82 X–1. The obtained results offer new approaches to better understand the astrophysical significance of these modified gravity models.

This section provides a concise overview of a black hole solution in GR that incorporates both NED and PFDM. The metric is constrained using a specific ansatz to derive a spherically symmetric black hole solution.

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\,\theta d\phi^2) \tag{31}$$

Nonsingular extensions of black hole spacetimes have been widely considered in GR coupled to NED. Here, we focus on GR coupled with NED in the presence of an exotic field called the PFDM field, with the action

$$\mathcal{L}_{\mathrm{NED}} = Fe^{-\frac{q}{2M}(2q^2F)^{\frac{1}{4}}} \tag{32}$$

where q and $F = F_{\mu\nu}F^{\mu\nu}/4$ refer to the magnetic charge and the invariant of the non-linear electromagnetic field with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. To obtain the BH solution, one can use $F = q^2/(2r^4)$ and the corresponding Lagrangian of a NED

field in the following form

$$\mathcal{L}_{\mathrm{NED}} = \frac{q^2}{2r^4} e^{-\frac{q^2}{2Mr}} \tag{33}$$

The energy–momentum tensor is taken in the form presented in chapter 2, and after all calculations, we obtain the following solution:

$$f(r) = 1 - \frac{2Me^{-\frac{q^2}{2Mr}}}{r} + \frac{\lambda}{r} \ln \frac{r}{|\lambda|} \tag{34}$$

MCMC Analysis. This section is dedicated to placing constraints on the mass and metric parameters appearing in the expressions. To this end, two black hole candidates were selected. These black holes belong to different mass categories: stellar-mass and intermediate-mass. For example, we used QPO data from well-known stellar-mass black holes located at the centers of microquasars — GRS 1915+105, GRO J1655-40, and XTE 1550-564 — as well as data from M82 X-1.

To obtain these constraints, we performed an MCMC analysis in the RP model using Python's emcee library. The chosen posterior distribution is given as:

$$\mathcal{P}(\theta \mid \mathcal{D}, \mathcal{M}) = \frac{P(\mathcal{D} \mid \theta, \mathcal{M})\pi(\theta \mid \mathcal{M})}{P(\mathcal{D} \mid \mathcal{M})}, \tag{35}$$

where $\pi(\theta)$ denotes the prior distribution and $P(\mathcal{D} \mid \theta, M)$ is the likelihood function.

We consider priors to have a normal distribution known as Gaussian distribution within suitable boundaries (see Tables 2 and 3), i.e., $\pi(\theta_i) \sim \exp\left[\frac{1}{2}\left(\frac{\theta_i - \theta_{0,i}}{\sigma_i}\right)^2\right], \theta_{\mathrm{low},i} < \theta_i < \theta_{\mathrm{high},i}$. Here, the parameters are $\theta_i = \{M, q, \lambda, r\}$ and σ_i are their deviations. We provide MCMC analysis using the orbital, vertical, and radial frequencies calculated in the previous section. The likelihood function Λ has form,

$$\log \Lambda = \log \Lambda_{\mathrm{U}} + \log \Lambda_{\mathrm{L}} \tag{36}$$

where $\log \Lambda_{\mathrm{U}}$ denotes the likelihood of the upper frequencies, as

$$\log \Lambda_{\mathrm{U}} = -\frac{1}{2} \sum_i \frac{\left(\nu^i_{\phi,\mathrm{obs}} - \nu^i_{\phi,\mathrm{th}}\right)^2}{\left(\sigma^i_{\phi,\mathrm{obs}}\right)^2}, \tag{37}$$

where $\log \Lambda_{\mathrm{L}}$ is the probability (likelihood) of the lower frequency data as a form

$$\log \Lambda_{\mathrm{L}} = -\frac{1}{2} \sum_i \frac{\left(\nu^i_{\mathrm{L,obs}} - \nu^i_{\mathrm{L,th}}\right)^2}{\left(\sigma^i_{\mathrm{L,obs}}\right)^2} \tag{38}$$

Here $\nu^i_{\phi,\,\mathrm{obs}}, \nu^i_{\mathrm{L},\,\mathrm{obs}}$ are direct observational results of the orbital/Keplerian frequencies (ν_{K}), periastron precession frequencies $\nu_{\mathrm{L}} = \nu_{\mathrm{K}} - \nu_{\mathrm{r}}$ for the source of our interest. On the other side, $\nu^i_{\phi,\,\mathrm{th}}, \nu^i_{\mathrm{L},\,\mathrm{th}}$ are the respective theoretical estimations. We chose data from QPOs observed in X-ray binaries: GRS 1915+105, GRO J1655-40, XTE J1550-564, and M82 X-1 with well-reported observational results, as presented in Table 3.1.

Source [197, 208–212]	ν_U [Hz]	$\Delta\nu_U$ [Hz]	ν_L [Hz]	$\Delta\nu_L$ [Hz]	Mass [208] [$M_\odot$]
XTE J1550-564	276	± 3	184	± 5	$12.4^{+2.0}_{-1.8}$
GRS 1915+105	168	± 3	113	± 5	$12.4^{+2.0}_{-1.8}$
GRO J1655-40	451	± 5	298	± 4	5.4 ± 0.3
M82 X-1	5.07	± 0.06	3.32	± 0.06	415 ± 63

Table 1. Twin-Peak QPO Frequencies Observed in Microquasars and the Galactic Center

Next, we perform the MCMC simulation to constrain the parameters (M, q, λ, r) for magnetically charged BHs. We use Gaussian priors based on parameter values from existing literature on QPO data processing. We sampled approximately 10^5 points from a prior Gaussian distribution for each parameter. This allowed us to explore the physically possible parameter space within set boundaries and identify the best-fitting parameter values.

P	XTE J1550-564		GRS 1915+105		GRO J1655-40		M82 X-1	
	μ	σ	μ	σ	μ	σ	μ	σ
$M/M_\odot$	6.9722	1.0458	11.5508	1.7326	4.2666	0.5546	372.77	44.64
q/M	0.0692	0.0103	0.0526	0.0079	0.024	0.0035	0.045	0.0067
λ/M	0.0225	0.0033	0.0241	0.0036	0.028	0.0042	0.0202	0.003
r/M	6.424	0.9634	6.3826	0.9573	6.4036	0.9605	6.5119	0.9768

Table 2: The Gaussian priors (is the mean value and σ the variance) of magnetically charged BHs from QPOs.

P	XTE J1550-564	GRS 1915+105	GRO J1655-40	M82 X-1
$M/M_\odot$	$6.9792^{+0.7642}_{-1.0218}$	$9.9321^{+1.1902}_{-1.0328}$	$4.8654^{+0.4159}_{-0.3854}$	$419.671^{+34.424}_{-32.544}$
q/M	$0.0692^{+0.0103}_{-0.0102}$	$0.0525^{+0.0078}_{-0.0078}$	$0.0241^{+0.0036}_{-0.0036}$	$0.045^{+0.0066}_{-0.0067}$
λ/M	$0.0225^{+0.0033}_{-0.0033}$	$0.024^{+0.0036}_{-0.0036}$	$0.0281^{+0.0042}_{-0.0042}$	$0.0203^{+0.003}_{-0.003}$
r/M	$6.4191^{+0.8775}_{-0.8479}$	$6.0961^{+0.884}_{-0.8433}$	$7.1962^{+0.8014}_{-0.7762}$	$7.3697^{+0.8113}_{-0.7783}$

Table 3: Best fit values of M, q, λ, and r parameters for XTE J1550-564, GRS 1915+105, GRO J1655-40 and M82 X-1

Having established the priors, we use the available data to perform an MCMC simulation to determine the plausible range of the parameters $\{M, q, \lambda, r\}$ for magnetically charged BH spacetime. Considering the Gaussian prior distribution, we sample 1×10^5 points for every parameter. This approach allows us to investigate the physically allowed multidimensional parameter space within defined

limits and obtain the parameter values that best match the data.

In Fig.12, we show the results of the MCMC analysis of the parameters (M, q, λ, r) by using the QPO data from 4 different magnetically charged BHs (three candidates of stellar-mass BH and one intermediate-mass BH) with magnetic charged sourced by NED field and surrounded by PFDM. The goal is to compare observational data from QPOs in microquasars with theoretical predictions based on the modified Schwarzschild metric.

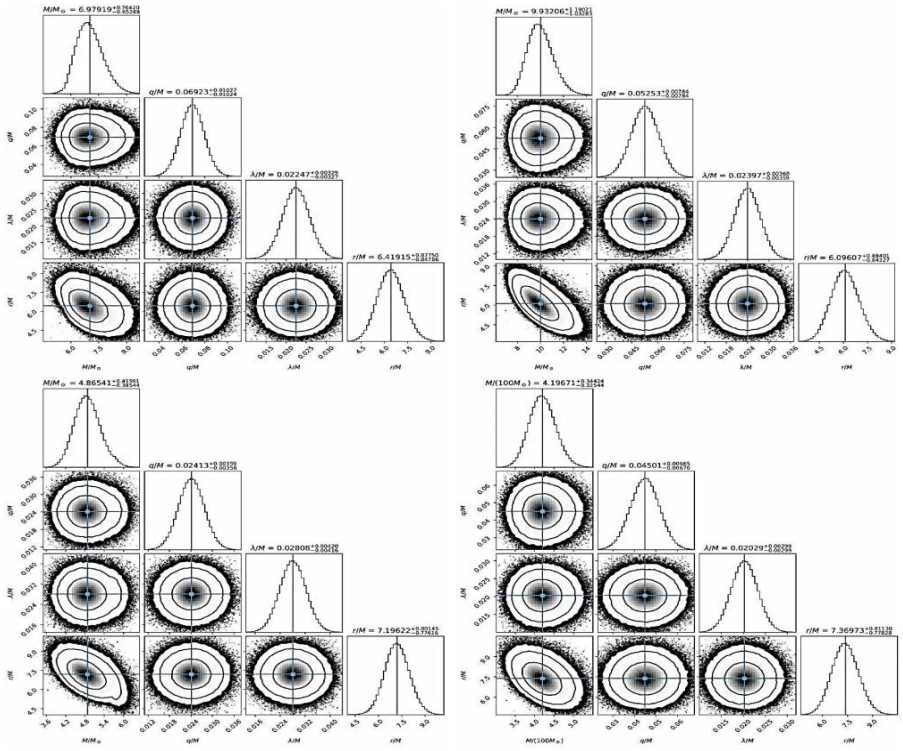


Figure 12: *Constraints on magnetically charged BH parameters of the QPO orbit from a fourdimensional MCMC analysis using the QPO data for the stellar-mass BHs XTE J1550-564 (top left), GRS 1915+105 (top right), and GRO J1655-40 (bottom left) and intermediate-mass BHs M82 X-1 (bottom right) in the RP model.*

The figure consists of contour plots and posterior distributions that indicate the statistical confidence levels for each parameter. The MCMC algorithm samples the likelihood function, which is constructed using the fundamental frequencies from the QPO models.

Each subplot presents different parameter correlations. The diagonal elements show the marginalized probability distributions for individual parameters, while the off-diagonal plots display 2D confidence contours (e.g., M vs. q, q vs. λ, etc.), typically including $68\%(3\sigma)$,$95\%(2\sigma)$, and $99\%(1\sigma)$ confidence intervals. These contours provide insight into how well the observational data constrain the parameters. The shaded regions on the contour plots reflect these confidence levels. Increasing q tends to shift the posterior mass distribution toward slightly lower values, indicating that magnetically charged black holes may appear less massive in QPO analysis than their Schwarzschild counterparts. Similarly, λ affects the orbital

frequencies, causing minor shifts in the estimated black hole mass. The contours also reveal strong degeneracies between parameters; for instance, an increase in λ can compensate for a lower q in determining QPO frequencies.

The results suggest that for black holes in microquasars such as GRS 1915+105, GRO J1655-40, and XTE 1550-564, the best-fit values of q and λ remain within physically viable ranges. These constraints provide indirect observational evidence for the effects of nonlinear electrodynamics and PFDM in strong gravity regimes.

CONCLUSIONS

The following conclusions were presented based on research carried out on the topic of "Fundamental frequencies and thermodynamical properties of black holes in the presence of external matter fields matter" for the Doctor of Philosophy (PhD) dissertation:

1. It has been shown that the Chaplygin-like dark fluid significantly modifies the event-horizon structure of black holes. In particular, the equation-of-state parameter leads to an enlargement of the event horizon, whereas the fluid-strength parameter reduces the horizon area.

2. It has been established that perfect fluid dark matter surrounding black holes considerably affects their thermodynamical properties. As the PFDM contribution increases, the maximum value of the Hawking temperature becomes higher than in the vacuum case, while the phase-transition point shifts from smaller to larger event-horizon radii. It has also been shown that the thermodynamical behavior of more massive black holes is more sensitive to the PFDM contribution.

3. It has been obtained that the Chaplygin-like fluid strength causes an inward shift of the orbital radii corresponding to QPOs with a 3:2 frequency ratio. This indicates that, as the dark-fluid contribution becomes stronger, QPO signals are generated at orbits closer to the black hole, where the gravitational field is stronger.

4. It has been demonstrated that the MCMC-based QPO analysis of XTE J1550–564, GRO J1655–40, GRS 1915+105, and M82 X-1 provides constrained mass values consistent with independent observational results within the PFDM parameter ranges $q \in [0.024, 0.069]$ and $\lambda \in [0.020, 0.028]$. This confirms that QPO observations can serve as an effective tool for constraining the black hole charge and PFDM parameters.

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РИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ИНСТИТУТЕ
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РАХМАТОВ БЕКЗОД МАНСУР УГЛИ

ФУНДАМЕНТАЛЬНЫЕ ЧАСТОТЫ И ТЕРМОДИНАМИЧЕСКИЕ СВОЙСТВА ЧЕРНЫХ ДЫР ПРИ НАЛИЧИИ ВНЕШНИХ ПОЛЕЙ МАТЕРИИ

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АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD) ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ

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ВВЕДЕНИЕ (Аннотация к представлению)

Актуальность и востребованность диссертации. Черные дыры являются одними из важнейших объектов для проверки общей теории относительности в режиме сильного гравитационного поля. Их пространственно-временная геометрия, структура горизонта, термодинамическое поведение и динамика частиц непосредственно связаны с наблюдаемыми астрофизическими процессами, происходящими вблизи компактных объектов. В частности, движение пробных частиц, явления, связанные с аккрецией, а также квазипериодические осцилляции, наблюдаемые в рентгеновском излучении, являются важными инструментами для изучения физических свойств черных дыр и окружающей их среды.

Недавние астрофизические наблюдения показали, что распределение темной материи в галактиках может быть более разнообразным, чем ожидалось. Ряд систем с очень низким содержанием темной материи или без явно обнаруживаемой компоненты темной материи, включая NGC 1052-DF2, NGC 1052-DF4, FCC 224, а также спиральные галактики с дефицитом темной материи, указывают на то, что эффекты темной материи могут существенно зависеть от астрофизической среды. Численное моделирование также показывает, что такие системы могут возникать вследствие механизмов среды, таких как приливные взаимодействия и стриппинг. Эти результаты мотивируют изучение эффективных моделей, описывающих возможные среды из темного вещества/энергии вокруг компактных объектов, особенно черных дыр.

В рамках общей теории относительности идеальная жидкая темная материя и темная жидкость типа Чаплыгина дают полезные эффективные описания для исследования того, как среды из темного вещества/энергии могут влиять на физику черных дыр. Идеальная жидкая темная материя может феноменологически описывать эффекты темной материи вблизи черных дыр, тогда как темная жидкость типа Чаплыгина важна тем, что может представлять среду со свойствами, интерполирующими между поведением, подобным темной материи, и поведением, подобным темной энергии. Поэтому изучение черных дыр, окруженных такими полями, является актуальным для понимания того, как внешние «темные» среды изменяют гравитационные свойства в режиме сильного поля.

Присутствие идеальной жидкой темной материи и темной жидкости Чаплыгина может изменять структуру пространства-времени вокруг черных дыр, влиять на горизонт событий и модифицировать такие термодинамические величины, как температура Хокинга, энтропия и теплоемкость. Эти поля также могут влиять на кеплеровские, радиальные и вертикальные эпициклические частоты частиц, движущихся вокруг черных дыр. Поскольку эти фундаментальные частоты тесно связаны с квазипериодическими осцилляциями, наблюдаемыми в микроквазарах и других компактных астрофизических системах, их анализ имеет важное значение для связи теоретических моделей черных дыр с наблюдательными данными.

Востребованность данной диссертации определяется необходимостью изучения черных дыр в реалистичных астрофизических средах и уточнения того, как параметры темной материи и темной жидкости влияют на наблюдаемые сигнатуры сильного поля. В частности, исследование динамики частиц, термодинамических свойств, фундаментальных частот и QPO-сигнатур позволяет проверять модели черных дыр в рамках общей теории относительности и сопоставлять теоретические предсказания с астрофизическими наблюдениями. Таким образом, диссертация является актуальной, поскольку рассматривает влияние идеальной жидкой темной материи и темной жидкости Чаплыгина на термодинамику черных дыр, орбитальное движение и квазипериодические осцилляции, что относится к числу важных проблем современной релятивистской астрофизики и гравитации сильного поля. Настоящая диссертация соответствует задачам, определенным следующими государственными нормативными документами: Указом Президента Республики Узбекистан № УП-4947 “О Стратегии действий по дальнейшему развитию Республики Узбекистан” от 7 февраля 2017 года, а также Постановлением Президента Республики Узбекистан № ПП-2789 “О мерах по дальнейшему совершенствованию деятельности Академии наук, организации, управления и финансирования научно-исследовательской деятельности” от 18.02.2017.

Соответствие исследования основным приоритетным направлениям развития науки и технологий в Республике

Исследование, выполненное в рамках диссертации, проведено в соответствии с приоритетными направлениями развития науки и технологий Республики Узбекистан.

Степень изученности проблемы

Работы Керра, Рейсснера–Нордстрёма, Хокинга, Пейджа, Борна–Инфельда и Малдасены внесли значительный вклад в изучение черных дыр в условиях нелинейной электродинамики. Метрики Керра и Рейсснера–Нордстрёма дают классические решения, тогда как Хокинг и Пейдж исследовали квантовые эффекты. Борн–Инфельд и Малдасена предложили альтернативные теоретические подходы. Эти исследования способствуют углублению нашего понимания электромагнитных свойств черных дыр и их роли во Вселенной.

В нашей стране работы Бобомурода Ахмедова, Ахмаджона Абдужаббарова, Жавлона Раимбаева, Фарруха Атамуратова и Бобира Тошматова по черным дырам в моделях NED занимают важное место в современной астрофизике и теоретической физике. Их исследования раскрывают новые физические свойства путем изучения влияния нелинейных электромагнитных полей вокруг черных дыр и связывают эти процессы с астрофизическими наблюдениями.

Моя диссертация посвящена совместному влиянию нелинейной электродинамики и идеальной жидкой темной материи на геометрию черных дыр, гравитационные и электромагнитные поля, термодинамические свойства и динамику частиц. В ней также исследуется влияние внешних полей материи

на фундаментальные частоты и сигнатуры квазипериодических осцилляций с целью сопоставления теоретических предсказаний с астрофизическими наблюдениями.

Связь темы диссертации с научно-исследовательскими работами высшего образовательного или научно-исследовательского учреждения, где выполнена диссертация

Диссертация выполнена в Самаркандском государственном университете имени Шарофа Рашидова в рамках научно-исследовательской темы и научных проектов Института фундаментальных и прикладных исследований: FL-7923051796 “Моделирование динамики скалярных полей и квантовых излучателей в сферически-симметричных гравитационных полях” (2024–2026).

Цель диссертационного исследования

Цель работы состоит в оценке астрофизических процессов вокруг черных дыр путем изучения их динамики частиц, фундаментальных частот, термодинамических свойств и QPO-сигнатур в присутствии темной жидкости Чаплыгина и идеальной жидкой темной материи.

Задачи исследования

- изучить пространственно-временные свойства черных дыр, окруженных CDF;
- проанализировать динамику пробных частиц и устойчивые круговые орбиты вокруг черных дыр в присутствии CDF;
- вычислить фундаментальные частоты и изучить QPO-сигнатуры черных дыр, окруженных темной жидкой материей;
- получить новые решения черных дыр в рамках общей теории относительности, связанной с NED, в присутствии PFDM;
- изучить термодинамические свойства и поведение фазовых переходов черных дыр в полях NED и PFDM;
- ограничить параметры черных дыр и окружающих полей с использованием QPO-данных и MCMC-анализа.

Объект исследования

Объектом исследования являются черные дыры, находящиеся под влиянием PFDM и NED, включая их геометрические и термодинамические свойства, орбитальное движение пробных частиц, характеристики аккреционного диска и излучения, а также определение параметров PFDM и NED с использованием MCMC-моделирования.

Предмет исследования

Предметом исследования является влияние моделей PFDM и NED на черные дыры, пробные частицы, аккреционные диски и тени, а также изучение геометрических и термодинамических свойств черных дыр в рамках этих моделей.

Методы исследования

Методы исследования включают теоретические и аналитические методы, методы общей теории относительности, численное моделирование, термодинамический анализ, анализ орбитальной динамики, а также статистические методы и методы MCMC.

Научная новизна исследования

- впервые получены два типа новых решений черных дыр в рамках общей теории относительности, связанной с NED, в присутствии PFDM;
- впервые показано, что теплоемкость черной дыры испытывает фазовые переходы вследствие влияния поля NED и темной материи;
- впервые с использованием QPO-данных микроквазаров и черных дыр звездных масс посредством MCMC-анализа ограничены параметры черных дыр, NED и темных полей, а именно темной жидкости Чаплыгина и идеальной жидкой темной материи.

Практические результаты исследования

Параметры черных дыр, окруженных темной жидкостью типа Чаплыгина, были ограничены с использованием MCMC-моделирования на основе наблюдаемых QPO-данных. Для XTE J1550–564, GRO J1655–40, GRS 1915+105, Sgr A* и M82 X-1 параметры были получены в диапазонах $B = 5.311 \times 10^{-5}$–9.340×10^{-5}, $q = 0.651$–0.953 и $r/M = 3.057$–4.496.

Для магнитно заряженных черных дыр, окруженных идеальной жидкой темной материей, наблюдаемые QPO-данные объектов XTE J1550–564, GRS 1915+105, GRO J1655–40 и M82 X-1 ограничили параметры в пределах $q/M = 0.0241$–0.0692 и $\lambda/M = 0.0203$–0.0281. Соответствующие QPO-орбитальные радиусы были найдены в диапазоне $r/M = 6.0961$–7.3697.

Показано, что теплоемкость черных дыр в присутствии NED и PFDM меняет знак в точке фазового перехода. Этот результат позволяет различать термодинамически неустойчивые и устойчивые конфигурации черных дыр. Поэтому, полученные результаты могут быть практически использованы для оценки массы черной дыры, магнитного заряда, параметров темной жидкости и параметров PFDM на основе QPO-наблюдений.

Достоверность результатов исследования

Достоверность результатов исследования обеспечивается применением хорошо зарекомендовавших себя методов математической физики, вычислительной математики и релятивистской астрофизики. Все расчеты выполнены в рамках общей теории относительности и физики черных дыр. Полученные результаты основаны на аналитических выводах и численных вычислениях пространственно-временной геометрии, структуры горизонта, термодинамических величин, движения частиц, фундаментальных частот и сигнатур квазипериодических осцилляций. Обоснованность результатов дополнительно подтверждается сравнением с известными предельными случаями, ранее опубликованными результатами и доступными астрофизическими наблюдательными данными. Выводы диссертации согласуются с основными принципами астрофизики компактных объектов и гравитации сильного поля.

Научная и практическая значимость исследования

Научная значимость исследования заключается в уточнении того, как идеальная жидкая темная материя, темная жидкость Чаплыгина и нелинейная электродинамика влияют на физические свойства черных дыр. В частности, в диссертации исследуется их влияние на геометрию черных дыр, структуру горизонта, термодинамическое поведение, динамику частиц и фундаментальные частоты. Эти результаты способствуют более глубокому пониманию черных дыр, окруженных внешними полями материи, и дают теоретическое представление о том, как такие среды могут модифицировать процессы в сильном гравитационном поле.

Практическая значимость исследования состоит в том, что предложенная модель мимикеров черных дыр может играть важную роль в объяснении высокоэнергетических астрофизических процессов. В частности, она дает новые предсказания относительно размера тени и оптического изображения черных дыр, которые могут быть проверены с помощью астрофизических наблюдений, таких как наблюдения Event Horizon Telescope. Эти результаты могут применяться для более точного определения астрофизических параметров и направления будущих наблюдательных исследований. Кроме того, теоретические подходы, основанные на моделях PFDM и NED, способствуют более глубокому пониманию механизмов функционирования космических систем. Результаты данного исследования могут служить научной основой для разработки новых космических обсерваторий или совершенствования существующих технологий.

Применение результатов исследования

Результаты исследования на тему “Фундаментальные частоты и термодинамические свойства черных дыр в присутствии внешних полей материи” были применены следующим образом:

При исследовании фундаментальных частот и термодинамических свойств черных дыр, находящихся под влиянием экзотической материи, данный подход был использован рядом авторов: “Li, Xiang-Qian and Kim, Yoonbai and Lee, Bum-Hoon and Yan, Hao-Peng and Yue, Xiao-Jun. Black holes immersed in modified Chaplygin-like dark fluid and cloud of strings: geodesics, shadows, and images. Chinese Physics C, 2025”, “Javed, Faisal and Waseem, Arfa and Mustafa, G. and Fatima, Ghulam and Alkarni, Shalan. Particle motion and thermal fluctuations of charged AdS black holes surrounded by exotic fluid with modified Chaplygin equation of state. Phys. Dark Univ. 47, 2025” и “Turimov, Bobur and Rahimov, Ozodbek and Davlataliev, Akbak and Tadjimuratov, Pulat and Norkobilov, Mukhiddin and Rahimova, Shakhzoda. Analysing geodesic motion in Bocharova–Bronnikov–Melnikov–Bekenstein spacetime. 2025. Phys. Dark Univ. 48 (2025) 101855”. Также данный подход использован в работах “Analysing geodesic motion in Bocharova–Bronnikov–Melnikov–Bekenstein spacetime”, Physics of the Dark Universe, vol. 48, 2025, 101855, и Erdem Sucu, Izzet Sakallı. “Astrophysical reality of black hole thermodynamics and dynamics: Transformative influence of Hernquist dark matter distributions”, Physics of the Dark Universe, vol. 49, 2025, 102051.

Научные результаты, полученные по движению частиц и фотонов, были использованы учеными Фуданьского университета (FU) в Шанхае (FU, Китай, 23 мая 2025 года).

Апробация результатов исследования

Результаты исследования были представлены и обсуждены на 4 международных и 3 местных научных конференциях.

Публикация результатов исследования

Результаты исследования представлены в 7 научных статьях, опубликованных в научных журналах, рекомендованных Высшей аттестационной комиссией при Министерстве высшего образования, науки и инноваций Республики Узбекистан.

Объем и структура диссертации

Диссертация состоит из введения, трех глав, заключения и списка литературы, общий объем составляет 119 страниц.

ВЫВОДЫ

На основе исследований, выполненных по теме диссертации доктора философии (PhD) “Фундаментальные частоты и термодинамические свойства чёрных дыр в присутствии внешних полей материи”, были представлены следующие выводы:

1. Показано, что тёмная жидкость типа Чаплыгина существенно изменяет структуру горизонта событий чёрных дыр. В частности, параметр уравнения состояния приводит к увеличению горизонта событий, тогда как параметр интенсивности жидкости уменьшает площадь горизонта.
2. Установлено, что тёмная материя в виде идеальной жидкости (PFDM), окружающая чёрные дыры, оказывает значительное влияние на их термодинамические свойства. При увеличении вклада PFDM максимальное значение температуры Хокинга становится выше, чем в вакуумном случае, а точка фазового перехода смещается от меньших к большим радиусам горизонта событий. Также показано, что термодинамическое поведение более массивных чёрных дыр является более чувствительным к вкладу PFDM.
3. Получено, что интенсивность чаплыгиноподобной жидкости вызывает смещение внутрь орбитальных радиусов, соответствующих QPO с отношением частот 3:2. Это указывает на то, что при усилении вклада тёмной жидкости QPO-сигналы формируются на орбитах, расположенных ближе к чёрной дыре, где гравитационное поле является более сильным.
4. Продемонстрировано, что MCMC-анализ QPO для объектов XTE J1550–564, GRO J1655–40, GRS 1915+105 и M82 X-1 позволяет получить ограниченные значения масс, согласующиеся с независимыми наблюдательными результатами, в диапазонах параметров PFDM $q \in [0.024, 0.069]$ и $\lambda \in [0.020, 0.028]$. Это подтверждает, что наблюдения QPO могут служить эффективным инструментом для получения ограничений на заряд чёрной дыры и параметров PFDM.

E’LON QILINGAN ISHLAR RO‘YXATI
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ
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II bo‘lim (part II; II часть)

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