"YANGI O'ZBEKISTON" UNIVERSITETI HUZURIDAGI ILG'OR TADQIQOTLAR INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSc.03/07.07.2025 FM/T.192.01 RAQAMLI ILMIY KENGASH

ISLOM KARIMOV NOMIDAGI TOSHKENT DAVLAT TEXNIKA UNIVERSITETI
YANGI O'ZBEKISTON UNIVERSITETI

ALIMOVA ASALXON ALIBEK QIZI

MODIFIKATSIYALANGAN GRAVITATSIYA NAZARIYALARI DOIRASIDA QORA O'RALAR ATROFIDA OPTIK JARAYONLAR VA ZARRACHALAR DINAMIKASI

01.04.02 – Nazariy fizika
01.03.01 – Astronomiya

FIZIKA-MATEMATIKA FANLARI BO'YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFERATI

Toshkent – 2025

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertasiyasi avtoreferati mundarijasi

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Fizika-matematika fanlari bo‘yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O‘zbekiston Respublikasi Oliy ta’lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasida B2025.4.PhD/_____ raqami bilan ro‘yxatga olingan.

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KIRISH (Falsafa doktori (PhD) dissertatsiya annotatsiyasi)

Dissertatsiya mavzusining dolzarbligi

Qora tuynuklar va o'zgartirilgan gravitatsiya nazariyalarini o'rganish zamonaviy astrofizikaning dolzarb yo'nalishlaridan biridir. Event Horizon Telescope (EHT) tomonidan qora tuynuk soyalarining birinchi suratlari olinishi va LIGO-Virgo orqali gravitatsion to'lqinlarining aniqlanishi kabi inqilobiy kashfiyotlar bilan, Umumiy Nisbiylik nazariyasini (GR) kengaytiruvchi yoki o'zgartiruvchi muqobil gravitatsiya nazariyalarini tadqiq etish ehtiyoji ortmoqda. Ushbu nazariyalar GRdagi hal etilmagan muammolar - masalan, singulyarliklar, qorong'i moddalar va qorong'i energiyani hal qilish va kelajakdagi kuzatuvlar uchun sinovdan o'tkazilishi mumkin bo'lgan prognozlar berishni maqsad qilgan.

Ushbu dissertatsiyada qora tuynuklar atrofida zarrachalar va yorug'likning harakati kvant-tuzatilgan, asimptotik xavfsiz va Kalb Ramond gravitatsiyalarini o'z ichiga olgan modifikatsiyalangan gravitatsiyalar asosida o'rganiladi. Bunday tadqiqotlar yuqori dolzarblikka ega, chunki ular nazariy prognozlarni kuzatuv ma'lumotlari bilan bog'laydi va zich jismlar va ularning atrof-muhitini tushunishga yordam beradi. Spinli zarrachalar, plazmada tortishish linzasi va qora tuynuk termodinamikasini tahlil qilish akreatsiya disklarining harakati, kvazi-davriy tebranishlar (QPO) va qora tuynuk soyalarini talqin qilishda bevosita ahamiyatga ega.

Shuningdek, ushbu ishga bo'lgan talab, keyingi avlod EHT yangilanishlari, LISA va rentgen teleskoplari kabi kuchli maydon gravitatsiyasini ilg' or aniqlik bilan o'rganadigan kelajakdagi kuzatuv missiyalari bilan ta'kidlanadi. Qora tuynuk kuzatuvlari orqali o'zgartirilgan gravitatsiya modellarining parametrlarini cheklash orqali, ushbu tadqiqot GR dan chetlanishlarni aniqlash va yangi fizika yo'nalishlarini o'rganishga hissa qo'shadi. Shunday qilib, dissertatsiya zamonaviy astrofizik tadqiqotlar bilan uyg'un keladi va asosiy gravitatsiya fizikasini rivojlantirishga ko'maklashadi.

Bundan tashqari, dissertatsiya O'zbekiston Respublikasidagi va xalqaro miqyosdagi strategik ilmiy ustuvorliklarga mos keladi. Ushbu dissertatsiya

- Fotonga traektoriyalar va qora tuynuk soyalariga bir jinsli va bir jinsli bo'lmagan plazma ta'sirini tahlil qilish;
- Nazariy prognozlarni Sgr A* uchun Event Horizon Telescope (EHT) kuzatuv ma'lumotlari bilan solishtirish;
- Oddiy neytral test zarrachalarining muntazam qora tuynuklar atrofidagi harakatini o'rganish;
- Aylana orbitaning energiyasi, burchak momenti va barqarorligi ifodalarini chiqarish;

- Epitsiklik tebranishlar, kvazi-davriy tebranishlar (QPO) va periastron precessiyasini tadqiq qilish;
- M87* va Sgr A* uchun EHT ma'lumotlari bilan qora tuynuk soyalarini solishtirish va model parametrlarini cheklash;

Tadqiqot ob'ektlari - bu turli modifikatsiyalangan gravitatsiya doiralari ichida qora tuynuklar yaqinidagi zarralar va maydonlarning harakati, optik xatti-harakatlari, termodinamik xususiyatlari va astrofizik ko’rsatkichlarini tavsiflovchi fizik hodisalar va nazariy modellar.

Tadqiqot predmeti - bu Mathisson-Papapetrou-Dixon tenglamalari orqali buriluvchi zarralar harakati, qora tuynuk soyasi, plazma muhitida gravitatsion linzalash va qora tuynuk xususiyatlariga oid kuzatuv chegaralari kabi masalalarni o'rganishda ishlatiladigan matematik modellar, analitik usullar va hisoblash texnikalari.

Tadqiqot metodlari - umumiy nisbiylik va modifikatsiyalangan gravitatsiya nazariyalariga asoslangan analitik va sonli usullar bo'lib, ular buriluvchi zarralar harakati uchun Mathisson-Papapetrou-Dixon tenglamalari, foton trayektoriyalari uchun geodezik tahlil va qora tuynuk tizimlari uchun termodinamik hisob-kitoblarni o’z ichiga oladi. Tadqiqot effektiv potensial tahlili, aylana orbitaning barqarorlik mezonlari bilan birga optik geometriya texnikalarini, soyani baholash va plazma muhitida gravitatsion linzalashni o'z ichiga oladi. Parametr bog’liqliklarini o’rganish va nazariy taxminlarni Event Horizon Telescope kabi kuzatuv ma'lumotlari bilan solishtirish uchun sonli simulyatsiyalar va grafik tahlil ishlatiladi.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

- Kvant tuzatmali qora tuynuklar atrofida spinli zarralar harakatining keng qamrovli tahlili o’tkazildi, bunda kvant gravitatsiya parametrlarining effektiv potensial, ISCO xususiyatlari va superluminal chegaralariga ta’siri ochib berildi.
- Reissner-Nordstrom qora tuynuklarning Kalb-Ramond gravitatsiyasidagi optik xususiyatlari bo'yicha yangi natijalar olingan, jumladan, soyalar radiusi va bir jinsli va bir jinsli bo'lmagan plazma muhitlarida gravitatsion linzalash effektlari.

Amaliy natijalar quyidagilardan iborat:

- Modifikatsiyalangan gravitatsiya ta'sirining qora tuynuk muhitlarida kuzatiladigan belgilari prognoz qilinishini ta'minlaydigan nazariy modellar va hisoblash vositalari ishlab chiqildi, Event Horizon Telescope ma'lumotlarini talqin qilishda qo'llanilishi mumkin.
- Qora tuynuk soyasi kuzatuvlariga asoslanib kvant tuzatmali va Lorentz simmetriyasi buzilgan qora tuynuklar uchun parametr chegaralari o'rnatildi, bu ularni astrofizik gravitatsiya nazariyalarini sinash uchun ishlatishga imkon beradi.

- Modifikatsiyalangan gravitatsiyada zarralar harakati, linzalash va termodinamik xususiyatlarni tahlil qilish uchun metodologik asos taqdim etildi, bu keyingi astrofizik modellashtirish va ta'lim maqsadlarida qo'llanishi mumkin.

Tadqiqot natijalarining ishonchliligi:

Umumiy nisbiylik, modifikatsiyalangan gravitatsiya nazariyalari va spinli zarralar harakati uchun Mathisson-Papapetrou-Dixon formalizmidan foydalangan holda mustahkam nazariy model ta'minlanadi. Matematik modellar va analitik hosilalar qat'iy fizik prinsiplariga asoslangan, sonli simulyatsiyalar esa tekshirilgan hisoblash usullari yordamida amalga oshirilgan. Olingan natijalar Schwarzschild va Kerr yechimlari kabi ma'lum cheklovli hollarga mos keladi va Event Horizon Telescope hamda boshqa astrofizik ma'lumotlar bilan solishtirilgan, bu ularning fizik ishonchliligi va qo'llanilishini tasdiqlaydi.

Tadqiqot natijalarining ilmiy va amaliy ahamiyati quyidagilardan iborat:

Modifikatsiyalangan gravitatsiya nazariyalari doirasida qora tuynuklarning zarralar harakati, optik effektlari va termodinamik xususiyatlari bo'yicha nazariy tushunchalarni kengaytiradi, alternativ gravitatsiya modellari rivojlanishiga hissa qo'shadi. Kuzatuv belgilari - qora tuynuk soyasi, gravitatsion linzalash va orbitaviy dinamikani prognoz qilish uchun analitik va sonli vositalar taqdim etadi, ular joriy va kelajak astronomik ma'lumotlar bilan to'g'ridan to'g'ri solishtirilishi mumkin.

Kvant tuzatmali, Lorentz simmetriyasi buzilgan va vektor maydoni bilan modifikatsiyalangan qora tuynuklar uchun parametr chegaralarini beradi, bu ularni astrofizik kuzatuvlarda gravitatsiya nazariyalarini amaliy sinovlari uchun ishlatishga imkon beradi.

Ekstremal gravitatsion muhitlarni modellashtirish uchun keyingi astrofizik tadqiqotlar va oliy ta'limda qo'llaniladigan metodologik asosni yaratadi, shu bilan birga ilmiy tadqiqotlar va nazariy astrofizika mutaxassislarini tayyorlashni qo'llab-quvvatlaydi.

Tadqiqot natijalarining qo'llanilishi.

Modifikatsiyalangan gravitatsiya nazariyalari doirasida qora tuynuklar atrofidagi optik jarayonlar va zarralar dinamikasini o'rganish natijalari quyidagicha qo'llanildi: Nazariy tadqiqot natijalari va usullari, 'A. Alimova, Z. Turakhonov, F. Atamurotov, A. Abdujabbarov, Shadow and weak gravitational lensing of RN-like BH in plasma, Physics of the Dark Universe, Vol. 47, 101749 (2025), https://doi.org/10.1016/j.dark.2024.101749" ilmiy maqolasida nashr etilgan va Alimova Asalkhonning falsafa doktori (PhD) dissertatsiyasida taqdim etilgan, Fudan universiteti tomonidan qo'llab-quvvatlangan dasturlar doirasida ishlatilgan (Prof. Cosimo Bambi tomonidan xat bilan tasdiqlangan).

Tadqiqot natijalarining tasdiqlanishi

Dissertatsiya natijalari 3 ta xalqaro konferensiyada muhokama qilingan.

Tadqiqot natijalarining nashri

Dissertatsiyaning asosiy natijalari ilmiy jurnallar va konferensiya materiallarida e’lon qilingan. Dissertatsiya mavzusiga oid jami 11 ta ilmiy ish chop etilgan bo‘lib, shulardan 6 tasi O‘zbekiston Respublikasi Oliy attestatsiya komissiyasi tomonidan PhD dissertatsiyalarining asosiy ilmiy natijalarini e’lon qilish uchun tavsiya etilgan xalqaro, taqrizdan o‘tgan ilmiy jurnallarda nashr qilingan.

DISSERTATSIYANING ASOSIY MAZMUNI

Kirish qismida mavzuning dolzarbligi va zarurati, tadqiqotning respublikaning fan va texnologiya rivojlanishining ustuvor yo'nalishlariga muvofiqligi, muammoning o'rganilganlik darajasi, dissertatsiya bajarilgan oliy ta'lim muassasasining ilmiy tadqiqot rejalari bilan bog'liqligi, tadqiqotning maqsadi va vazifalari, obyekti, predmeti, predmetga oid qisqacha ma'lumotlar, qo'llanilgan usullar, ilmiy yangilik, amaliy natijalar, olingan natijalarning ishonchliligi, ilmiy va amaliy ahamiyati, natijalarning amaliyotga joriy etilishi, ularning aprobatsiyasi, chop etilgan ishlar, shuningdek, dissertatsiyaning tuzilishi va hajmi bayon etiladi.

Birinchi bob "Kvant-tuzatilgan qora tuynuk atrofida spinli zarrachalarning harakati" deb nomlanib, kvant-tuzatish ta'siridagi qora tuynuk yaqinida spinga ega sinov zarrachalarining dinamikasini o'rganishga bag'ishlangan. Ushbu bobda Mathisson-Papapetrou-Dixon tenglamalari, Tulzyjevning spin-supplementary sharti qo'llaniladi hamda harakat ekvatorial tekislikda o'rganiladi. Eng yaqin barqaror aylanish orbitasi (ISCO), effektiv potentsial va effektiv kuch aniqlanadi hamda ular qora tuynukning α parametri va zarracha spini s ga qanday bog'liq ekani tahlil qilinadi. Shuningdek, kinematik to'rt-tezlik va dinamik to'rt-impuls doimo parallel bo'lmaganligi sababli, spinga ega zarrachalarning harakati uchun yorug'likdan tezlashish chegarasi ham inobatga olinadi. Bundan tashqari, α parametri zarracha spining maksimal ruxsat etilgan qiymatiga qanday ta'sir ko'rsatishi ko'rsatiladi. Qora tuynuk gorizonti yaqinida spinga ega zarrachalarning to'qnashuvi tahlil qilinib, to'qnashuv sodir bo'lishi uchun zarur bo'lgan kritik burchak impulsi aniqlanadi.

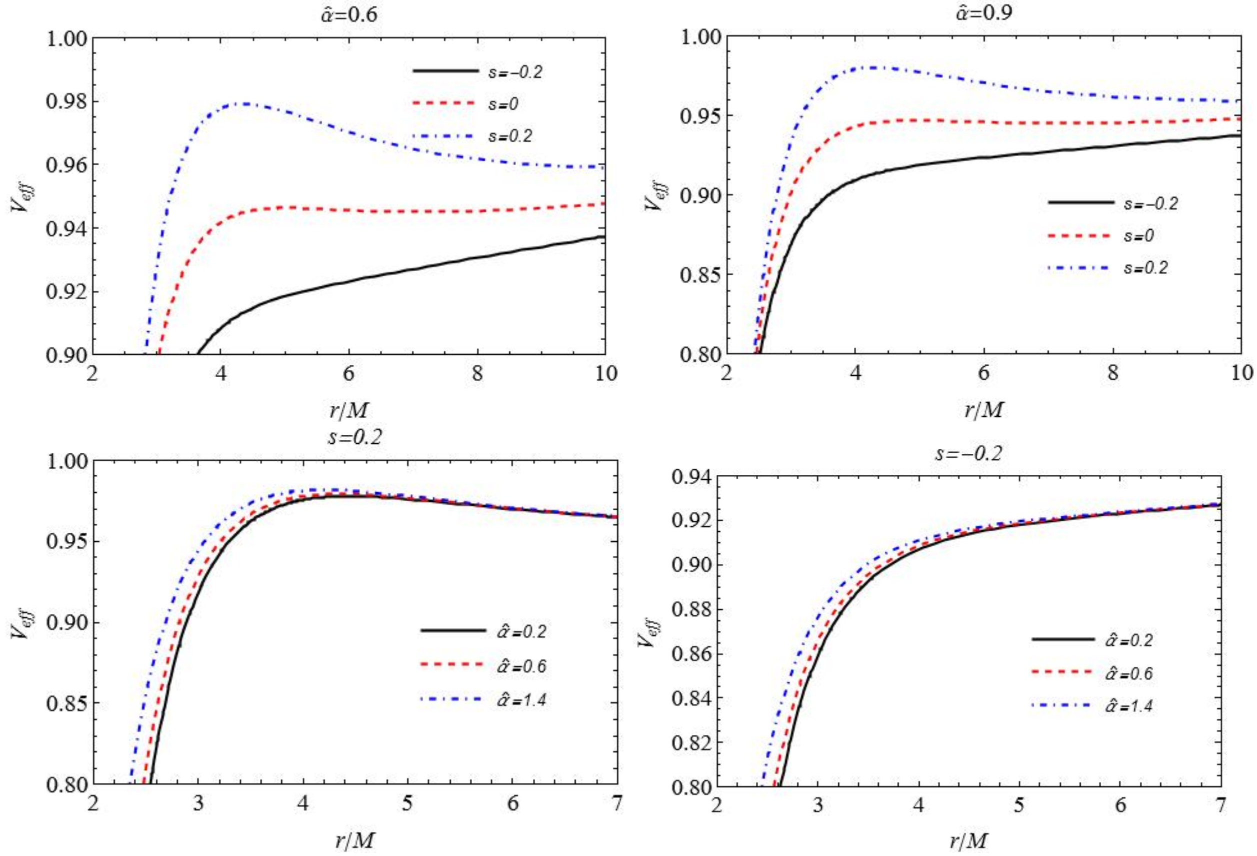
Biz kvant-tuzatilgan qora tuynuk yaqinidagi spinli zarrachalar dinamikasini sferik koordinatalarda quyidagi yechim orqali o'rganamiz:

$$ds^2 =- f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2. \tag{1}$$

Yuqoridagi ifodada $f(r)$ laps funksiyasi bo'lib, quyidagicha aniqlanadi:

$$f(r) = 1 - \frac{2M}{r} + \frac{\alpha M^2}{r^4}, \tag{2}$$

bu yerda M - Arnowitt-Deser-Misner massasi, $\alpha = 16\sqrt{3}\pi\gamma^3 l_P^2$ kvant tuzatishlari bilan bog'liq parametrdir. Bu yerda γ - Immirzi parametri.

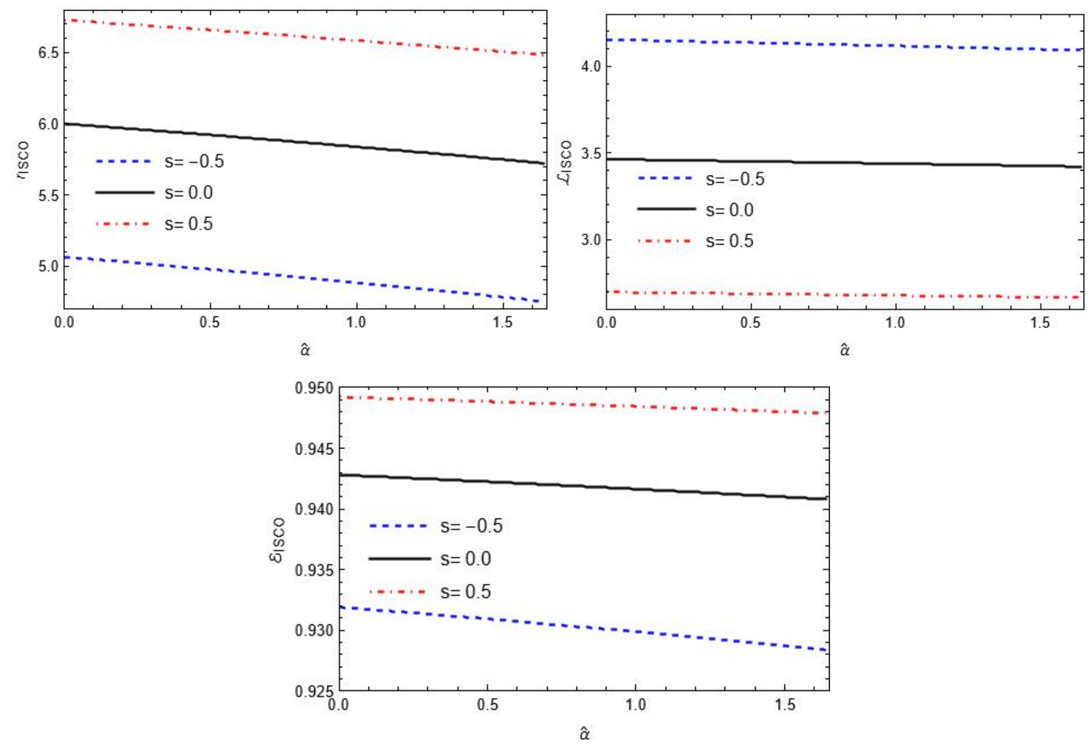


1-Rasm: Turli parametrlar uchun effektiv potentsial V_{eff} ning radial bog'liqligi. Yuqori qatorda $\hat{\alpha} = 0.6$ va 0.9 uchun $s = -0.2, 0, 0.2$ qiymatlaridagi V_{eff} ning o'zgarishi ko'rsatilgan. Pastki qatorda esa $s = -0.2$ va 0.2 belgilangan bo'lib, $\hat{\alpha} = 0.2, 0.6, 1.4$ qiymatlariga nisbatan o'zgarishi tasvirlangan.

Massiv, spinga ega zarrachalar harakatini tavsiflovchi differensial tenglamalar to'plami MPD tenglamalari deb ataladi va quyidagi ko'rinishga ega:

$$\frac{Dp^{\alpha}}{d\lambda} = -\frac{1}{2} R^{\alpha}{}_{\beta\delta\sigma} u^{\beta} S^{\delta\sigma}, \; \frac{DS^{\alpha\beta}}{d\lambda} = p^{\alpha}u^{\beta} - p^{\beta}u^{\alpha}, \tag{3}$$

bu yerda $D/d\lambda \equiv u^{\alpha}\nabla_{\alpha}$ - zarracha trayektoriyasi bo'ylab kovariant hosila, $u^{\mu} = dx^{\mu}/d\lambda$ - to'rt-tezlik, p^{α} - kanonik to'rt-impuls, $R^{\alpha}{}_{\beta\delta\sigma}$ - Riemann



2-rasm: Turli spin qiymatlari uchun $\hat{\alpha}$ parametri o'zgarishiga nisbatan ISCO radiusi, impuls momenti va energiyaning o'zgarishi.

tenzori, λ - affin parametr, $S^{\alpha\beta}$ esa antisimmetrik spin tenzori: $S^{\alpha\beta} =- S^{\beta\alpha}$. Agar $S^{\mu\nu}$ komponentlari nol bo'lsa, tenglama quyidagicha soddalashadi:

$$\frac{Dp^{\mu}}{d\tau} = 0. \tag{4}$$

MPD tenglamalarini yechishda zarrachalarning massa markazini tanlash muhim ahamiyatga ega. Shu sababli (3) tenglamalar tizimini yechish uchun markazni aniqlash sharti qo'llaniladi.

Statik va sferik simmetrik fazoda energiya E va to'liq burchak impulsi J (bu yerda $J = L + S$) saqlanadi, bu yerda S - spin, L - orbital burchak impulsi. Ularning ifodasi quyidagicha:

$$-E \ = p_t - \frac{1}{2} g_{tt,r} S^{tr} \tag{5}$$

$$J \ = p_{\phi} - \frac{1}{2} g_{\phi\phi,r} S^{\varphi r}, \tag{5}$$

Endi radial kanonik impulsni quyidagicha ifodalash mumkin:

$$(p^r)^2 =- g^{rr}\left[g^{tt} p_t^2 + g^{\phi\phi} p_{\phi}^2 + m^2\right]. \tag{6}$$

Shundan kelib chiqib, spinning zarracha energiyasi E uchun kvadratik tenglama olinadi:

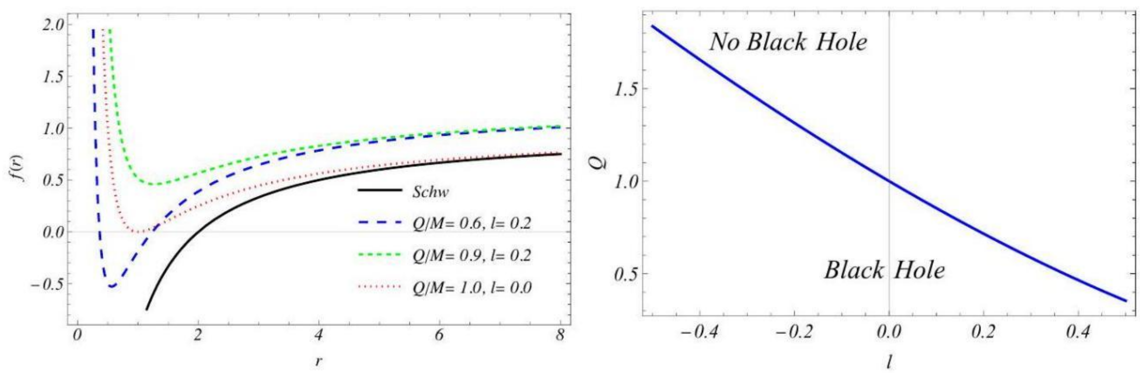
$$\rho(u^r)^2 = \alpha\mathcal{E}^2 + \delta\mathcal{E} + \gamma. \tag{7}$$

Bu tenglamani quyidagicha yozamiz:

$$(u^r)^2 = \frac{\alpha}{\rho}(\mathcal{E} - V_+)(\mathcal{E} - V_-). \tag{8}$$

Aylanish harakati uchun $p^r = 0$ sharti qo'yilib, effektiv potentsial quyidagicha aniqlanadi:

$$V_\pm = \frac{-\delta \pm \sqrt{\delta^2 - 4\alpha\gamma}}{2\alpha}. \tag{9}$$



3-rasm. Chapdagi grafik $f(r)$ funksiyasining o‘zgarishini ko‘rsatadi. O‘ngdagi diagramma esa Kalb–Ramond gravitatsiyasidagi RN-singari qora tuynuk uchun hodisa gorizonti radiusining qanday o‘zgarishini hamda qora tuynuk mavjud bo‘lishi yoki mavjud bo‘lmasligi mumkin bo‘lgan sohani aks ettiradi.

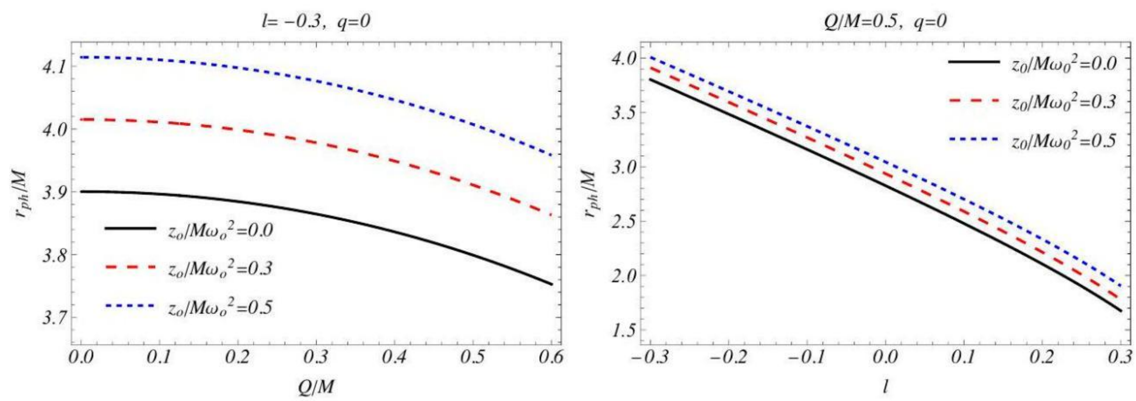
Musbat energiyaga ega spinning zarrachalar uchun effektiv potentsial quyidagicha aniqlanadi: $V_{eff} = E_+$. Fig. 1 da $\mathcal{L} = 3.5$ uchun $\hat{\alpha}$ va s ning turli qiymatlarida effektiv potentsialning radial o'zgarishi tasvirlangan. Yuqori qatorda $\hat{\alpha} = 0.6$ va $\hat{\alpha} = 0.9$ belgilangan holda s ning turli qiymatlariga nisbatan V_{eff} ning o'zgarishi ko'rsatilgan bo'lib, $\hat{\alpha}$ ortishi effektiv potentsialning ortishiga olib keladi. Pastki qatorda esa s belgilangan bo'lib, $\hat{\alpha}$ ning o'zgarishi natijasida V_{eff} ning turlicha xatti-harakati kuzatiladi. r/M ortishi bilan V_{eff} asimptotik qiymatga yaqinlashadi, bu esa katta radial masofalarda barqaror xatti-harakatni anglatadi. Grafikdan ko'rinib turibdiki, s ortishi effektiv potentsialning sezilarli ravishda ortishiga olib keladi. Pastki grafiklar turli $\hat{\alpha}$ qiymatlarida katta radial masofalarda V_{eff} deyarli bir xil bo'lishini ko'rsatadi.

Endi (1) fazoda spinning zarrachalarning barqaror aylanish orbitasini aniqlaymiz, chunki aynan shu orbitaga doimiy e'tibor qaratiladi. Ma'lumki, aylanish orbitasi uchun ikki shart bajarilishi zarur: (i) $dr/d\tau = 0$ yoki $\mathcal{E} = V_{\mathrm{eff}}(r)$ (radius o'zgarmas bo'lgan holat) (ii) $d^2r/d\tau^2 = 0$ yoki $V'_{\mathrm{eff}}(r) = 0$ (tezlanish nolga teng).

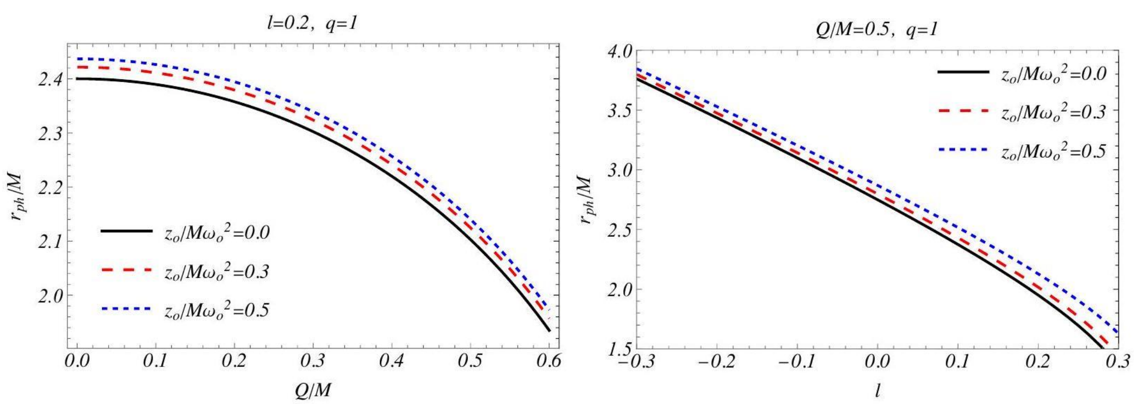
Biroq bu shartlar orbitaning barqarorligini kafolatlamaydi. Barqarorlik uchun effektiv potentsialning ikkinchi tartibli radial hosilasi musbat bo'lishi kerak, ya'ni $d^2V_{eff}/dr^2 \geq 0$. Tenglik esa ISCO nuqtasini bildiradi.

2-rasmda $\hat{\alpha}$ parametri o'zgarishiga nisbatan $r_{\mathcal{ISCO}}, \mathcal{L}_{\mathcal{ISCO}}$ va $\mathcal{E}_{\mathcal{ISCO}}$ ning turli spin s qiymatlarida qanday o'zgarishi tasvirlangan. Grafikdan ko'rinishicha, $r_{\mathcal{ISCOO}}$ va $\mathcal{E}_{\mathcal{ISCO}}\hat{\alpha}$ ga nisbatan chiziqli o'zgaradi, spin s ortishi esa ushbu funksiyalar qiymatining ortishiga olib keladi. $\mathcal{L}_{\mathcal{ISOO}}$ ning $\hat{\alpha}$ ga bog'liqligi esa deyarli o'zgarmasligicha qoladi.

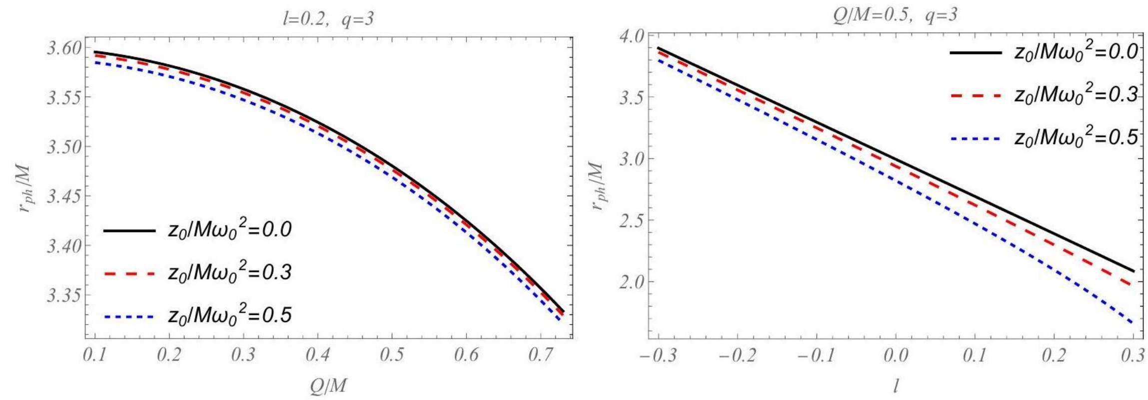
Ushbu bobda spinning zarrachalar harakati MPD tenglamalari yordamida fundamental tarzda o'rganildi. Spinning zarrachalarning effektiv potentsiali V_{eff} hisoblab chiqildi va uning radial bog'liqligi $\hat{\alpha}$ va s ning turli qiymatlari uchun



4-rasm**.** Chap panel qora tuynuk zaryadi $\boldsymbol{Q}$ ga bog'liq ravishda foton sferasining o'zgarishini ko'rsatadi. O'ng panel esa $\omega_0^2/\omega^2 =$ const holida $\boldsymbol{l}$ parametriga nisbatan foton sferasining o'zgarishini tasvirlaydi.



5-rasm**.** Bu rasmda $q = 1$ bo'lgan bir jinsli bo'lmagan plazma holatida foton sfera radiusining Q/M va l parametrlariga bog'liq holda qanday o'zgarishi ko'rsatilgan.



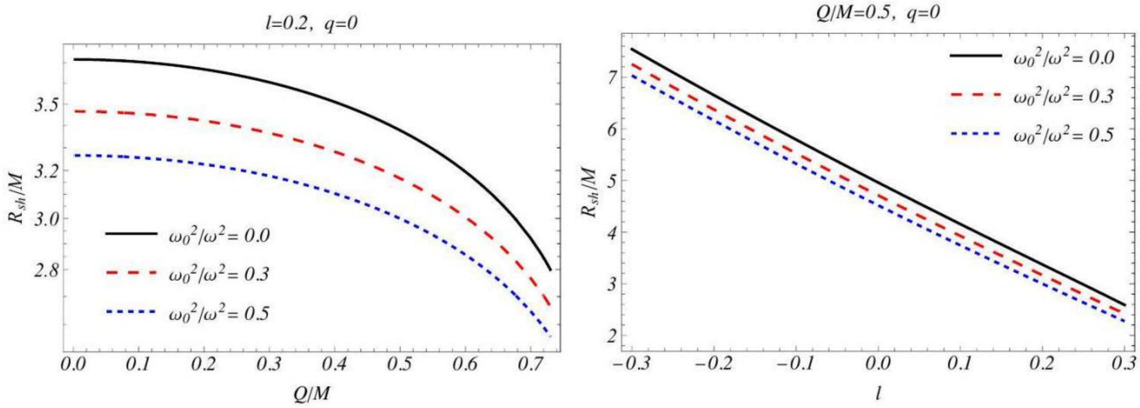
6-rasm. Bu rasmda q=3 bo'lgan bir jinsli bo'lmagan plazma holatida foton sfera radiusining Q/M va l parametrlariga bog'liq holda qanday o'zgarishi ko'rsatilgan.

grafik ko'rinishda tahlil qilindi. Natijada, har ikki parametrning ortishi effektiv potentsialning ortishiga olib kelishi aniqlandi.

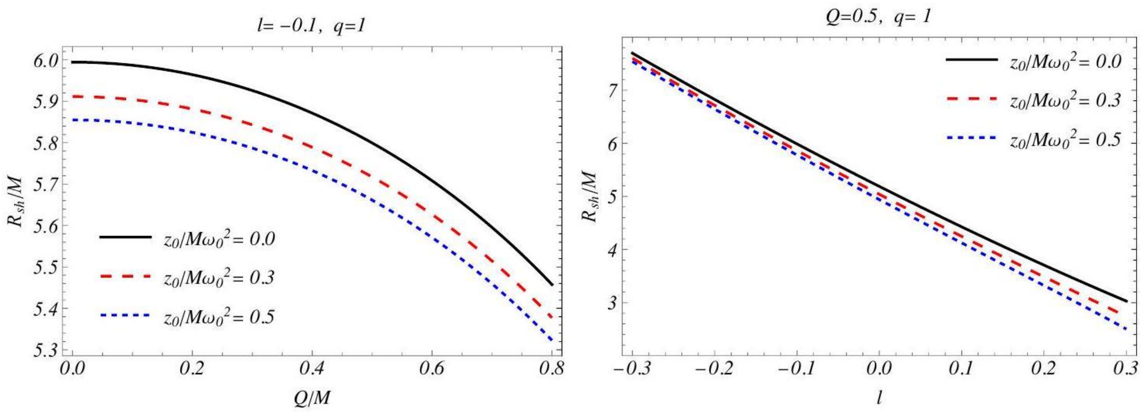
Bundan tashqari, $\hat{\alpha}$ va spin s ning ISCO radiusiga ta'siri tekshirildi. $\hat{\alpha}$ parametri mavjud bo'lganda ISCO radiusi kamayadi. Shu parametrlarning burchak impulsi va energiyaga ta'siri kuzatildi. $\hat{\alpha}$ ning ortishi impuls momenti va energiyani ham kamaytirishi grafikdan ma'lum.

Bundan tashqari, spin s uchun cheklovlar (ya'ni, superluminal bound condition) aniqlangan bo'lib, ushbu chegaradan oshilganda zarra trayektoriyasi fazosimon xarakterga ega bo'ladi va fizik ma'noga ega bo'lmay qoladi. Shuningdek, kritik spin qiymati $s_{\max}$ ning $\hat{\alpha}$ parametri bilan bog'liqligi tasvirlangan va tahlil qilingan. Shuningdek, kvant-tuzatilgan qora tuynuklar atrofidagi effektiv kuch r masofaning funksiyasi sifatida, $\hat{\alpha}$ va s parametrlarining turli qiymatlari uchun tahlil qilindi. Natijalar shuni ko'rsatadiki, effektiv kuch $\hat{\alpha}$ va s parametrlariga nisbatan o'xshash tendensiyani namoyon etadi: bu parametrlardan biri kichik bo'lganda kuch minimal bo'ladi, ammo ularning qiymati ortishi bilan kuch ham oshadi. Spin parametri s doimiy saqlanib, qora tuynuk parametri $\hat{\alpha}$ o'zgartirilganda, radial masofa r ortishi bilan effektiv kuch profillari bir-biriga yaqinlashadi. Biroq, s ning turli qiymatlari uchun radial masofa r ortgani sari effektiv kuchning farqli profillari kuzatiladi.

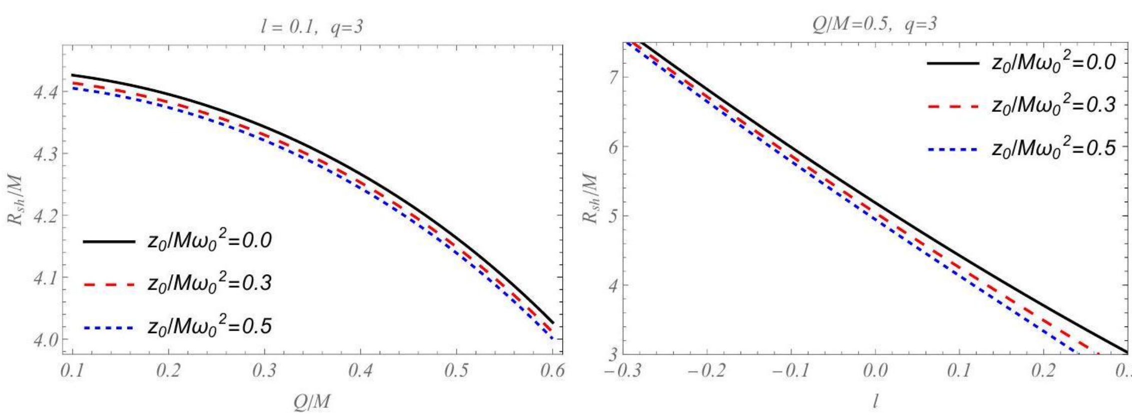
Ikkinchi bobda, "Plazma mavjudligidagi Reissner-Nordström (RN) qora tuynukning optik xususiyatlari" deb nomlangan bo'lib, unda Kalb-Ramond (KR) gravitatsiyasida RN-like qora tuynuk (BH) uchun kuchsiz gravitatsion linzalanish va soyasining radiusi o'rganiladi. Tadqiqot uchta turli sharoitda amalga oshiriladi: bir jinsli plazma, singulyar izotermal sfera (SIS) va nonsingulyar izotermal sfera (NSIS). Agar gravitatsion obyekt plazma bilan o'ralgan bo'lsa, linzalanish burchagi elektromagnit to'lqin chastotasi, plazmaning dis-



7-rasm. Bir jinsli plazmada qora tuynuk soyasi radiusining R_{sh}/M qiymati Q/M va l parametrlariga bog'liq holda qanday o'zgarishi tasvirlangan.



8-rasm. $\omega_p^2 = z_0/r^1$ bo'lgan plazma modelida soyani ifodalovchi radius R_{sh}/M va l parametrlariga bog'liq o'zgarishi tasvirlangan.



9-rasm. $\omega_p^2 = z_0/r^3$ bo'lgan plazma modelida soyani ifodalovchi radius R_{sh}/M va l parametrlariga bog'liq o'zgarishi tasvirlangan.

persiyasi, plazmaning notekisligi va tortishish kuchi bilan belgilanadi. Hatto bir jinsli plazmada ham fotonning gravitatsion og'ishi vakuumdagidan farq qiladi va ikkinchi effekt tufayli foton chastotasiga bog'liq bo'ladi. Biz har ikki ta'sirni hisobga olib, KR gravitatsiyasi mavjud bo'lgan holda kuchli bir jinsli bo'lmagan plazma uchun linzalanish burchagini hisoblaymiz. Event Horizon Telescope (EHT) loyihasining Sgr A* bo'yicha kuzatuv ma'lumotlaridan foydalanib, RN-ga o'xshash qora tuynukning Q va l parametrlarining qaysi qiymatlari kuzatuvlarga mos kelishini aniqladik. Shuningdek, Lorens buzilishi parametri l oshgan sari qora tuynuk soyasining radiusi kamayishi ko'rsatildi. Kuzatuvlar bilan bog'lash uchun, linzalanish natijasida hosil bo'lgan tasvirlarning kattalashuvi va joylashuvi, shuningdek, turli galaktikalar yaqinidagi manbalar uchun kuchsiz og'ish burchagi va kattalashtirish tahlil qilindi.

Boyer-Lindquist koordinatalarida RN-tipidagi BH metrikasi quyidagicha beriladi:

$$ds^2 =- f(r)dt^2 + f(r)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right), \tag{10}$$

bu yerda

$$f(r) = \frac{1}{1-l} - \frac{2M}{r} + \frac{Q^2}{(1-l)^2 r^2}, \tag{11}$$

l esa o'lchamsiz Lorens-buzilishi parametridir. (20) dan foydalanib, BH gorizontining radiusi quyidagicha topiladi:

$$r_h = (1-l)\left(M \pm \sqrt{M^2 - \frac{Q^2}{(1-l)^3}}\right). \tag{12}$$

(12) ga ko'ra, Kalb-Ramond maydoni mavjud bo'lgan holatda qora tuynukning gorizonti Shvarschildnikidan torroq bo'lishi mumkin. Ayni paytda $l < 1$ sharti har doim bajarilishi kerak. Bundan tashqari, $l \to 0$ chegarasida metrika Schwarzchild metrkasiga qaytadi hodisalar gorizonti 3-rasmda $f(r)$ funksiyasining radial o'zgarishi, shuningdek, hodisalar gorizonti radiusining qora tuynukning umumiy zaryadi va l parametri bilan bog'liqligi ko'rsatilgan. Q va l ning qiymatlariga qarab, bitta, ikkita yoki umuman gorizont bo'lmasligi mumkin. Shuningdek, RN-ga o'xshash qora tuynuklar mavjud bo'ladigan Q/M va l parametrlarining diapazoni aniqlanishi mumkin (quyi panelga qarang). RN-ga o'xshash fazo-vaqt yordamida qora tuynuklar mavjudligi tasdiqlandi va ularni ajratuvchi chegaraning Q/M va l parametrlari bilan bog'liqligi aniqlab olindi (quyi panelga qarang). Bundan tashqari, l parametrining qiymatini aniqlash oson bo'lib, qora tuynuk zaryadi $Q/M > 0.3$ bo'lganda l parametri $0.5 > l >- 0.5$ oralig'ida o'zgarishi mumkin.

Foton plazma muhitida tarqalayotganda, uning Gamiltoniani quyidagicha beriladi

$$H(x^\alpha, p_\alpha) = \frac{1}{2}\tilde{g}^{\alpha\beta} p_\alpha p_\beta \tag{13}$$

Bu yerda x^α fazo-vaqt koordinatalarini, $\tilde{g}_{\alpha\beta}$ esa quyidagicha ifodalanadigan effektiv metrikani bildiradi:

$$\tilde{g}^{\alpha\beta} = g^{\alpha\beta} - (n^2 - 1)u^\alpha u^\beta \tag{14}$$

Bu yerda n - plazmaning sinish ko'rsatkichi, p_α - fotonning to'rt impuls komponentlari, u^β esa fotonning to'rt tezligidir. Plazmaning sinish ko'rsatkichi n quyidagicha beriladi:

$$n^2 = 1 - \frac{\omega_p^2(r)}{\omega(r)^2} \tag{15}$$

bu yerda $\omega_p^2(r) = 4\pi e^2 N(r)/m_e(e$ va $m_e -$ elektron zaryadi va massasi, N - elektron zichligi), va foton chastotasi tortishish kuchi tufayli quyidagicha qizil siljishga uchraydi:

$$\omega(r) = \frac{\omega_0}{\sqrt{f(r)}} \tag{16}$$

Uzoq masofada $\omega_0 =$ const bo'lib, $\omega_0 = \omega(\infty) = -p_t$ fotonning fazoviy cheksizlikdagi energiyasini bildiradi. Yorug'lik plazmadan faqat uning chastotasi plazma chastotasidan yuqori bo'lganda o'tadi. Shuning uchun (15) faqat plazma chastotasi yorug'lik chastotasidan kichik bo'lganda qo'llaniladi. Agar bu nisbat birdan katta bo'lsa, foton plazma orqali tarqala olmaydi. $\omega_0 \approx \omega$ bo'lsa, defleksiya burchagi vakuumga nisbatan ancha katta bo'ladi ($\omega_p = 0$ holatdagi defleksiya bilan solishtirilganda).
Plazma bir jinsli bo'lgan maxsus holatda $\omega_p^2(r) =$ const deb olinadi. photon sfera tenglamasini sonli yechgandan so'ng natijalar 4-rasmda keltirilgan. Qora tuynukning zaryadi va l parametri oshishi bilan foton orbitasi radiusi kamayishi aniq ko'rinadi. Bundan tashqari, plazma muhiti foton sfera radiusini oshiradi.

Bizning hozirgi tadqiqotimiz bir jinsli bo'lmagan plazma muhitidagi foton sferasini tahlil qilishga bag'ishlangan bo'lib, bunda plazma chastotasi quydagicha qonun bilan o'zgaradi:

$$\omega_p^2(r) = \frac{z_0}{r^q} \tag{17}$$

bu yerda z_0 - erkin parametr. Kuchli xususiyatlarni o'rganish uchun ikki misol ko'rib chiqiladi: (17) ni photon sfera ifodasiga qo'yib, foton sfera radiusining sonli qiymatini olamiz va grafiklarda tasvirlaymiz. Natijalarga ko'ra, Q/M va l parametrlarining oshishi foton sfera radiusining kamayishiga olib keladi. Aksincha, $\omega_p^2(r) = z_0/r$ bo'lsa $(q = 1)$, plazma muhiti foton sferasini kengaytiradi, $q = 3$ holatida esa biroz kamaytiradi. Bundan tashqari, bir jinsli va bir jinsli bo'lmagan plazma ta'siri orasidagi farq juda kichik bo'lib, qora tuynuk soyalarini kuzatish orqali ularni farqlash qiyin bo'lishi mumkinligini anglatadi. Bu bo'limda RN-ga o'xshash fazo-vaqt metrikasining soyasi radiusi bir jinsli va bir jinsli bo'lmagan plazma muhitida o'rganiladi. Geometriyaga asoslangan yondashuv asosida qora tuynuk soyasining burchak radiusi α_{sh} quyidagicha beriladi:

$$\sin^2 \alpha_{sh} = \frac{\gamma^2(r_p)}{\gamma^2(r_0)} \tag{18}$$

bu yerda r_0 kuzatuvchi joylashuvi, r_p esa foton sferasining radiusidir. Kuzatuvchi qora tuynukdan juda uzoqda joylashganda, soyani taxminiy radiusi quyidagicha bo’ladi:

$$R_{sh} \approx r_0 \sin\alpha_{sh} = \sqrt{r_p^2\left[\frac{1}{f(r_p)} - \frac{\omega_p^2(r_p)}{\omega_0^2}\right]}. \qquad (19)$$

Ushbu tahlil har ikkala plazma modeli uchun ham $\ell(r) \to r$ limitiga asoslanadi. Xususan, $r_p = 3M$ bo'lganda, vakuum holati $\omega(r) \equiv 0$ uchun Shvartsshild qora tuynugiga xos soyalanish radiusi $R_{\mathrm{sh}} = 3\sqrt{3}M$ qiymati tiklanadi.

5-rasmda bir jinsli plazma mavjud bo'lgan holatda qora tuynuk soyasining radiusi Q/M va l parametrlarining turli qiymatlari uchun keltirilgan. Natijalar shuni ko'rsatadiki, Q/M va l ortishi bilan soyaning radiusi ancha keskin kamayadi. Bundan tashqari (19) dagi $\omega_p^2(r) = z_0/r^q$ ko'rinishidagi bir jinsli bo’lmagan plazma ($q = 0,1,3$) ham o'rganildi. Plazmaning mavjudligi sababli soya o'lchami izchil ravishda qisqarib boradi.

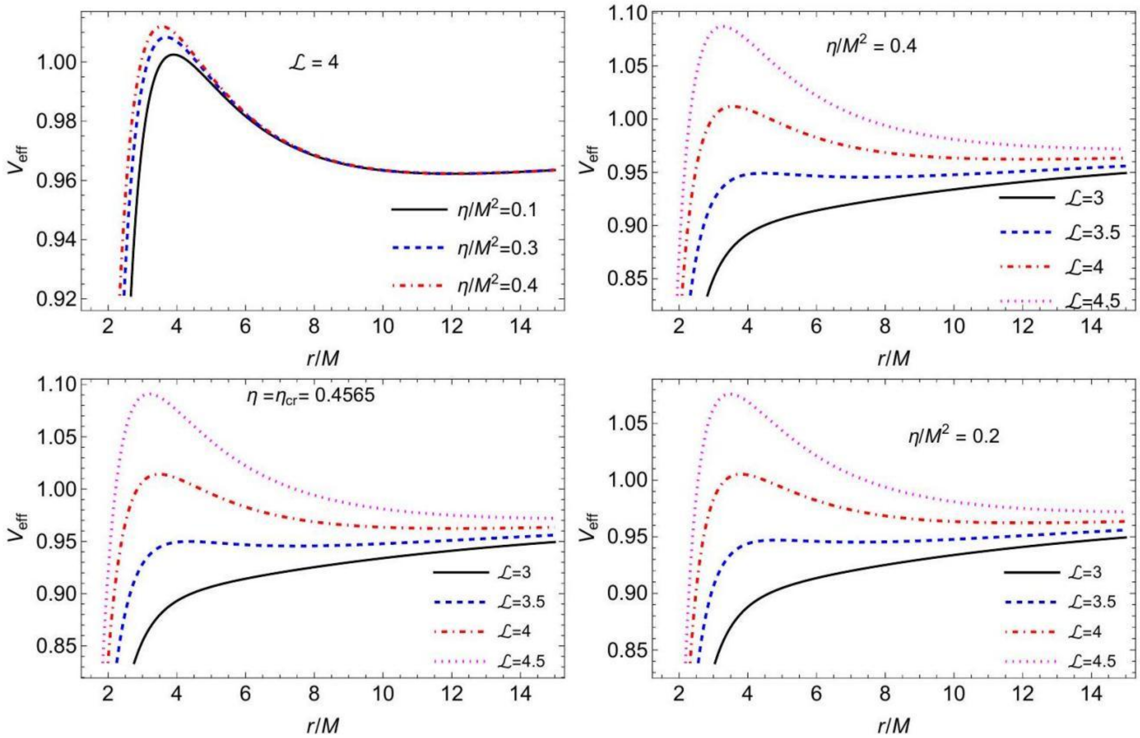
Shuningdek, Sgr A* qora tuynugiga doir kuzatuviy cheklovlar ham qo'llandi. Olingan natijalar Event Horizon Telescope tomonidan olingan ma'lumotlar bilan to'liq mos keladi va nazariy bashoratlarning kuzatuvlar bilan yuqori darajadagi uyg'unligini tasdiqlaydi.

Uchinchi bobda, "Asimptotik xavfsiz gravitatsiyada qora tuynuklarning astrofizik xususiyatlari" deb nomlangan bo'lib, biz asimptotik xavfsiz gravitatsiya doirasidagi aylanmaydigan regular qora tuynuk atrofida neytral sinov zarralarining harakatini o'rganamiz hamda qora tuynuk parametrlarining zarracha dinamikasiga ta'sirini tahlil qilamiz. Ushbu qora tuynuk yechimi standart Schwarzchild yechimidan farq qiluvchi qo'shimcha parametr η bilan tavsiflanadi. Biz ekvatorial aylana orbitaning energiyasi va impuls momentining qora tuynuk parametri orqali analitik ifodalarini hosil qilamiz. Ushbu aylana orbitaning barqarorligi esa samarali potensial yordamida tekshiriladi. Bundan tashqari, ichki eng barqaror ekvatorial aylana orbitaning qora tuynuk parametri funksiyasi sifatida grafik tahlili keltiriladi hamda aylana orbitaga ta'sir etuvchi effektiv kuch o'rganiladi. Shuningdek, radial va latitudinal garmonik tebranish chastotalari model parametrlarining funksiyasi sifatida chiqariladi va ekvatorial tekislikdagi barqaror aylana orbitaga yaqin sinov zarralarining kvazi-periodik tebranishlarining asosiy xususiyatlari muhokama qilinadi. Periastron pretsessiyasi hodisasi ham ko'rib chiqiladi.

Qora tuynuk yaqinida zarrachalar to'qnashuvi o'rganilib, bunday to'qnashuvlar hodisa gorizonti yaqinida yuqori energiya hosil qilishi ko'rsatiladi. Natijalarimiz shuni ko'rsatadiki, qora tuynuk parametri asimptotik xavfsiz gravitatsiya doirasidagi muntazam qora tuynuk atrofidagi sinov zarralarining harakatiga sezilarli darajada ta'sir ko'rsatadi. Shuningdek, biz qora tuynuk soyasini o'rgandik va Sgr A* hamda M87* uchun Event Horizon Telescope (EHT) tomonidan taqdim etilgan kuzatuv ma'lumotlaridan foydalanib, η parametrining kuzatuvlarga mos keluvchi oraliqlarini aniqladik.

Asimptotik xavfsiz gravitatsiya doirasidagi regular qora tuynuk quyidagi metrika bilan tasvirlanadi:

$$ds^2 =- f(r)dt^2 + f^{-1}(r)dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{20}$$



10-rasm: Asimptotik xavfsiz gravitatsiya fonidagi regular qora tuynuk atrofidagi test zarralari uchun samarali potensialning xatti-harakatlari.

bu yerda $\delta = 1$ vaqtsimon zarralar uchun va $\delta = 0$ null zarralar uchun. Nuqta τ vaqtiga nisbatan hosilani bildiradi. Bizning e'tiborimiz faqat vaqtsimon zarralarga qaratiladi.

Bu yerda samarali potensial $V_{eff}(r,\theta)$ quyidagicha beriladi:

$$V_{eff}(r,\theta) = \left(1 + \frac{\mathcal{L}^2\csc^2\theta}{r^2}\right)\left(1 - \frac{r^2}{3\eta}\ln\left(\frac{6M\eta}{r^3} + 1\right)\right). \tag{21}$$

$\eta = 0$ bo'lganda, samarali potensial (32) Schwarzchild holatiga o'tadi. Effektiv potensial $V_{eff}(r,\theta)$ zarralar dinamikasini tushunishda juda muhimdir. U zarralarning harakatini aniq harakat tenglamalarini yechmasdan tasvirlash imkonini beradi. Barqaror va barqaror bo'lmagan aylana orbitlar mos ravishda V_{eff} ning minimal va maksimal qiymatlariga mos keladi. Ekvatorial orbitaning effekiv potensial xatti-harakatlari 10-rasmda ko'rsatilgan. Birinchi qator va birinchi ustunda BH parametri η ning turli qiymatlari uchun effektiv potensialning xatti-harakati berilgan, $\eta < \eta_{cr}$ va $\eta = \eta_{cr}$ uchun esa impuls moment L ning turli qiymatlari ko'rsatilgan. $\eta < \eta_{cr}$ bo'lsa, effektiv potensial impuls momentni oshirsak ham ko'p o'smaydi. Qiziq tomoni shundaki, BH parametri η va zarraning impuls momenti L samarali potensialga o'xshash ta'sir ko'rsatadi. BH parametri η yoki zarraning impuls momenti L oshganda effektiv potensial ham ortadi.

Biz asimptotik xavfsiz gravitatsiya doirasidagi aylanmaydigan regular BH atrofidagi ekvatorial aylana orbitaning energiya va impuls momentining analitik ifodalarini hosil qildik. Qayd etish joizki, BH parametri η kichik bo'lsa, zarralar yuqori energiyaga ega; katta η qiymatlarida esa energiya kamayadi. Bundan tashqari, energiya radial masofa r ortishi bilan monotonik ravishda oshadi. BH parametri η ning turli qiymatlari uchun impuls momentning r ga bog'liqligi ko'rsatildi. BH parametri η kichik bo'lsa, zarralar katta impuls momentga ega, katta η qiymatida esa impuls moment kichik bo'ladi. Impuls moment radial masofa r ortishi bilan monotonik ravishda oshadi.

Biz asimptotik xavfsiz gravitatsiya fonidagi muntazam BH atrofida test zarralarining epitsiklik tebranishlari va asosiy chastotalarini o'rgandik. BH parametri η test zarralari asosiy chastotalarining BH gorizontiga yaqin siljishiga ta'sir qiladi. Schwarzchild

BH atrofidagi zarralar yuqori chastotalarga ega, asimptotik xavfsiz gravitatsiya fonidagi muntazam BH atrofidagi zarralarga nisbatan. Shuningdek, biz periastron chastotasini va BH parametrining unga ta'sirini o'rgandik. Periastron chastotasi η oshishi bilan kamayadi va radial masofa r ortishi bilan ham kamayadi.
Bundan tashqari, biz asimptotik xavfsiz gravitatsiya fonidagi muntazam qora tuynukning foton sferasining radiusi va soyasini o'rgandik. Tahlilimiz shuni ko'rsatadiki, η parametri oshishi bilan foton sferasining radiusi va soyasi radiusi kamayadi. EHT hamkorligi ma'lumotlaridan foydalanib, Sgr A* va M87* supermassiv qora tuynuklari uchun η parametri chegarasini aniqladik. Natijada, muntazam BH parametri ηSgrA* va M87* uchun quyidagi oraliqqa cheklangan:
$0 < \eta/M^2 < 0.4565.$

Xulosa

Quyidagi xulosalar "Modifikatsiyalangan gravitatsiya nazariyalari doirasida qora tuynuklar atrofidagi optik jarayonlar va zarralar dinamikasi" mavzusida olib borilgan tadqiqot asosida PhD dissertatsiyasi uchun taqdim etildi:

- Kvant-tuzatilgan qora tuynuklar atrofidagi aylanadigan zarralar harakat tenglamalari effektiv potentsial, ISCO radiusi va superluminal chegaralarning ham spin, ham kvant tuzatish parametrlari bilan sezilarli bog'liqligini ko'rsatadi.
- Kvant-tuzatilgan qora tuynuklar yaqinidagi yuqori energiyali zarra to'qnashuvlar ma'lum kritik burchak momenti qiymatlarida sodir bo'lishi aniqlangan, ba'zi spin konfiguratsiyalari esa markaziy massaning maksimal energiyasiga olib keladi.
- Kalb-Ramond gravitatsiyasidagi Reissner-Nordström turidagi qora tuynuklarning optik xususiyatlari Lorentz buzuvchi parametrlar va plazma taqsimotlari tomonidan sezilarli darajada ta'sirlanishi aniqlangan, shuningdek, bu omillar foton sohasiga va soyalar radiusiga qanday ta'sir qilishini ko'rsatildi.
- Qora tuynuk soyasi o'lchamlarining nazariy prognozlari Sgr A* va M87* Event Horizon Telescope kuzatuvlari bilan solishtirilganda, modifikatsiyalangan gravitatsiya nazariyalaridagi parametrlar uchun cheklovlarni o'rnatishga imkon beradi.
- Asimptotik xavfsiz gravitatsiya doirasidagi regular qora tuynuklarning epitsiklik tebranishlar va davriy precessiya effektlari kabi o'ziga xos astrofizik va orbital xususiyatlari mavjudligi aniqlangan, shuningdek, potentsial kuzatuv belgilarini ko'rsatadi.

SCIENTIFIC COUNCIL No. DSc.03/07.07.2025 FM/T.192.01 ON AWARD OF SCIENTIFIC DEGREE AT THE INSTITUTE FOR ADVANCED STUDIES AT “NEW UZBEKISTAN” UNIVERSITY

TASHKENT STATE TECHNICAL UNIVERSITY AFTER ISLAM KARIMOV
NEW UZBEKISTAN UNIVERSITY

01.03.01 - Astronomy
01.04.02 - Theoretical Physics

ALIMOVA ASALKHON

OPTICAL PROPERTIES AND DYNAMICS OF PARTICLES AROUND BLACK HOLES IN MODIFIED GRAVITIES

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) IN PHYSICS AND MATHEMATICS

Tashkent-2025

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INTRODUCTION (presentation abstract)

Topicality and demand of the theme of the dissertation.

The study of black holes and modified gravity theories is at the forefront of modern theoretical and observational astrophysics. With groundbreaking discoveries such as the first images of black hole shadows by the Event Horizon Telescope (EHT) and the detection of gravitational waves by LIGO-Virgo, there is a growing need to explore alternative theories of gravity that extend or modify General Relativity (GR). These theories aim to address unresolved issues in GR, such as singularities, dark matter, and dark energy, while providing testable predictions for future observations.

This dissertation investigates the dynamics of particles and light around black holes in modified gravity frameworks, including quantum-corrected, asymptotically safe, and Kalb-Ramond gravity. Such research is highly relevant as it bridges theoretical predictions with observational data, offering insights into the nature of compact objects and their environments. The analysis of spinning particles and gravitational lensing in plasma has direct implications for interpreting astrophysical phenomena, such as accretion disk dynamics, quasi-periodic oscillations (QPOs), and black hole shadows.

Furthermore, the demand for this work is underscored by ongoing and future observational missions, such as next-generation EHT upgrades, LISA, and X-ray telescopes, which will probe strong-field gravity with unprecedented precision. By constraining parameters of modified gravity models through black hole observations, this research contributes to the broader effort to uncover deviations from GR and explore new physics. Thus, the dissertation aligns with cutting-edge astrophysical inquiries and supports the advancement of fundamental gravitational physics.

Moreover, the dissertation aligns with the strategic scientific priorities in Uzbekistan and the broader international community. This dissertation work corresponds to the tasks of the following state regulatory documents: Decree of the President of the Republic of Uzbekistan No. UP-4947 "On the Strategy of Actions for the Further Development of the Republic of Uzbekistan" dated February 07, 2017, Decree of the President of the Republic of Uzbekistan No. PP-2789 "On

Measures for Further Improvement of Academy of Sciences, Organization, Management, and Financing of Research Activities from 18.02.2017.

Relevance of the research to the priority areas of science and technology development of the Republic of Uzbekistan. Dissertation research has been carried out in accordance with the priority areas of science and technology in the Republic of Uzbekistan: II. "Power, energy and resource-saving".

Degree of study of the problem:

The study of black holes and modified gravity theories has been a central focus in theoretical and observational astrophysics for decades. Classical black hole solutions in General Relativity (GR), such as the Schwarzschild and Kerr metrics, have been extensively analyzed in terms of particle dynamics, gravitational lensing, (Atamurotov, Ahmedov, Toshmatov, Bambi). However, the limitations of GR-such as singularities, the lack of a quantum description, and discrepancies in cosmological observations-have motivated the exploration of alternative gravity models. Modified theories, including quantum-corrected gravity, asymptotically safe gravity, and metric-Palatini extensions, have been proposed to address these issues, with varying degrees of investigation. While some aspects of these theories, such as their basic black hole solutions and stability conditions, have been studied (Atamurotov, Rayimbaev), the detailed behavior of particles and light in these spacetimes remains an active area of research.

The motion of spinning particles around black holes, governed by the Mathisson-Papapetrou-Dixon equations, has been examined in GR and some modified gravity scenarios (Mustafa, Rayimbaev, Herdeiro, Sotiriou). However, the effects of quantum corrections and additional fields (e.g., Proca fields or Kalb-Ramond backgrounds) on spinning particle dynamics are less explored. Similarly, gravitational lensing in plasma environments has been studied in standard GR (Bambi, Stuchlík, Tsupko, Bisnovatyi-Kogan, Mustafa, Rayimbaev), but its implications in modified gravity, particularly in the presence of Lorentz-violating terms or nonminimal couplings, require further investigation.

Despite significant progress, many aspects of these modified gravity theories remain underdeveloped. Observational constraints from black hole shadows, gravitational waves, and X-ray spectroscopy (Bambi, Frolov, Konoplya, Radu, Benavides-Gallego) provide new opportunities to test these models, but systematic studies linking theoretical predictions to measurable phenomena are still needed. This dissertation contributes to filling these gaps by analyzing particle dynamics, optical properties, and thermodynamic behavior in specific modified gravity frameworks, thereby advancing our understanding of black holes beyond classical GR.

Connection of the topic of the dissertation with the scientific research of the higher educational research institutions, where the dissertation was carried out. The dissertation was done in the framework of the scientific projects funded by the Ministry of Innovative Development: F-FA-2021-510 "Investigations of nuclear matter of neutron stars in modified gravity" (2021-2026).

The aim of the research is to develop a model to probe black hole astrophysics via modified gravity using particles orbits, black hole shadows, and thermodynamics.

The tasks of the research:

- to analyze the dynamics of spinning particles using the Mathisson-PapapetrouDixon (MPD) equations;
- to determine the innermost stable circular orbit (ISCO), effective potential, and effective force, and study their dependence on the black hole's quantumcorrected parameter and the particle's spin;
- to examine the superluminal bound condition for spinning particles;
- to investigate weak gravitational lensing and shadow formation in KalbRamond (KR) gravity;
- to analyze the effects of uniform and inhomogeneous plasma on photon trajectories and black hole shadows;
- to compare theoretical predictions with observational data from the Event Horizon Telescope (EHT) for Sgr A*;
- to examine the motion of neutral test particles around regular black holes;
- to derive expressions for energy, angular momentum, and stability of circular orbits;
- to investigate epicyclic oscillations, quasi-periodic oscillations (QPOs), and periastron precession;
- to compare black hole shadows with EHT data for M87* and Sgr A* to constrain model parameters;

The objects of the research are he physical phenomena and theoretical models describing the motion, optical behavior, thermodynamical characteristics, and astrophysical properties of particles and fields in the vicinity of black holes within various modified gravity frameworks.

The subjects of the research are the mathematical models, analytical methods, and computational techniques used to study spinning particle dynamics, black hole shadows, gravitational lensing in plasma environments and observational constraints on black hole properties in the context of modified gravity theories.

The methods of the research are analytical and numerical methods grounded in general relativity and modified gravity theories, including the Mathisson-Papapetrou-Dix equations for spinning particle dynamics and geodesic analysis for photon trajectories for black hole systems. The study integrates effective potential analysis, stability criteria for circular orbits, and superluminal bound conditions, along with optical geometry techniques for shadow radius estimation and gravitational lensing in plasma media. Numerical simulations and graphical analysis are used to explore parameter dependencies and compare theoretical predictions with observational data from sources such as the Event Horizon Telescope.

The scientific novelty of the research is the following:

- A comprehensive analysis of spinning particle motion around quantum-corrected black holes was conducted, revealing the influence of quantum gravity parameters on effective potential, ISCO characteristics, and superluminal bounds.
- New results were obtained on the optical properties of Reissner-Nordströmlike black holes in Kalb-Ramond gravity, including shadow radius and gravitational lensing effects in both homogeneous and inhomogeneous plasma environments.

Practical results are as follows:

- Developed theoretical models and computational tools for predicting observable signatures of modified gravity effects in black hole environments, applicable to interpreting Event Horizon Telescope data.
- Established parameter constraints for quantum-corrected and Lorentz-violating black holes based on black hole shadow observations, enabling their use as astrophysical tests of gravity theories.
- Provided a methodological framework for analyzing particle dynamics and lensing in modified gravities, which can be adapted for further astrophysical modeling and educational purposes.

Reliability of the research: results are provided by the use of well-established theoretical frameworks, including general relativity, modified gravity theories, and the Mathisson-Papapetrou-Dixon formalism for spinning particle dynamics. The mathematical models and analytical derivations are based on rigorous physical principles, while numerical simulations were carried out using verified computational methods. The obtained results are consistent with known limiting cases, such as Schwarzschild and Kerr solutions, and have been cross-checked against observational constraints from the Event Horizon Telescope and other astrophysical data, confirming their physical plausibility and applicability.

The scientific and practical significance of the research results are the following:

- Expands the theoretical understanding of particle dynamics, optical effects, and thermodynamical properties of black holes in modified gravity frameworks, contributing to the development of alternative gravitational models.
- Provides analytical and numerical tools for predicting observational signatures-such as black hole shadows, gravitational lensing patterns, and orbital dynamics-that can be directly compared with current and future astronomical data.
- Offers parameter constraints for quantum-corrected, Lorentz-violating, and vector-field-modified black holes, enabling their use as practical tests of gravity theories in astrophysical observations.
- Establishes a methodological basis applicable in further astrophysical research and higher education for modeling extreme gravitational environments, thereby supporting both scientific investigations and training of specialists in relativistic astrophysics.

Application of the research results.

The results of the study of the dynamics of particles around black holes in modified gravity theories have been applied as follows: the theoretical research results and methods, published in the scientific paper 'A. Alimova, Z. Turakhonov, F. Atamurotov, A. Abdujabbarov, Shadow and weak gravitational lensing of RN-like BH in plasma, Physics of the Dark Universe, Vol. 47, 101749 (2025), https://doi.org/10.1016/j.dark.2024.101749" and presented in the Doctorate (PhD) thesis of Ms. Asalkhon Alimova have been used in the framework of the programs supported by Fudan University (Letter from Prof. Cosimo Bambi).

Approbation of the research results:

The dissertation results have been discussed in 3 international conferences.

Publication of the research results:

The principal results of the dissertation have been published in scientific journals and conference proceedings. A total of 11 scientific publications related to the dissertation topic have been produced, of which 6 articles appeared in international peer-reviewed journals recognized by the Supreme Attestation Commission of the Republic of Uzbekistan as appropriate outlets for the publication of PhD research results.

THE MAIN CONTENT OF DISSERTATION

The introduction of the dissertation indicates the relevance and necessity of the topic, the correspondence of the research to the priority directions of development of science and technology of the republic, the degree of knowledge of the problem, its connection with the research plans of the higher educational institution in which the dissertation was carried out, and the purpose, objectives, object of research, brief information about the subject, methods, scientific novelty, practical result, reliability, scientific and practical significance of the results, introduction of the results into practice, approval of the results, publication of the results, as well as the structure and scope of the dissertation.

The first chapter, entitled "Spinning particle motion around a black hole with quantum-corrected parameter" is devoted to study the motion of spinning particles around a quantum-corrected black hole. We investigate the dynamics of spinning test particles by using the Mathisson-Papapetrou-Dixon equations, the Tulczyjew spin-supplementary condition, and restricting the motion to the equatorial plane. We determine the innermost stable circular orbit (ISCO), effective potential, and effective force and examine how these depend on the black hole's α parameter and the particle's s spin. However, we also take into account a superluminal bound on the motion of the spinning particle since its kinematical four-velocity and dynamical four-momentum are not always parallel. We also show how the parameter α affects the maximum value of the spin parameter s. We determine the critical angular momentum of the particle for which a collision is possible by investigating collisions of spinning particles close to the horizon of a black hole. We aim to examine the dynamics of spinning particles near the quantumcorrected black hole, described in spherical coordinates by the solution

$$ds^2 =- f(r)dt^2 + f(r)^{-1}dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2. \tag{1}$$

In the above equation $f(r)$ representing the lapse function, which is further defined as:

$$f(r) = 1 - \frac{2M}{r} + \frac{\alpha M^2}{r^4}, \tag{2}$$

where M represents the Arnowitt-Deser-Misner mass, and $\alpha = 16\sqrt{3}\pi\gamma^3 l_P^2$ is a parameter influenced by quantum corrections. In this expression, γ denotes the Immirzi parameter.

The set of differential equations for describing the motion of massive spinning particles is called the MPD equations and has the following form:

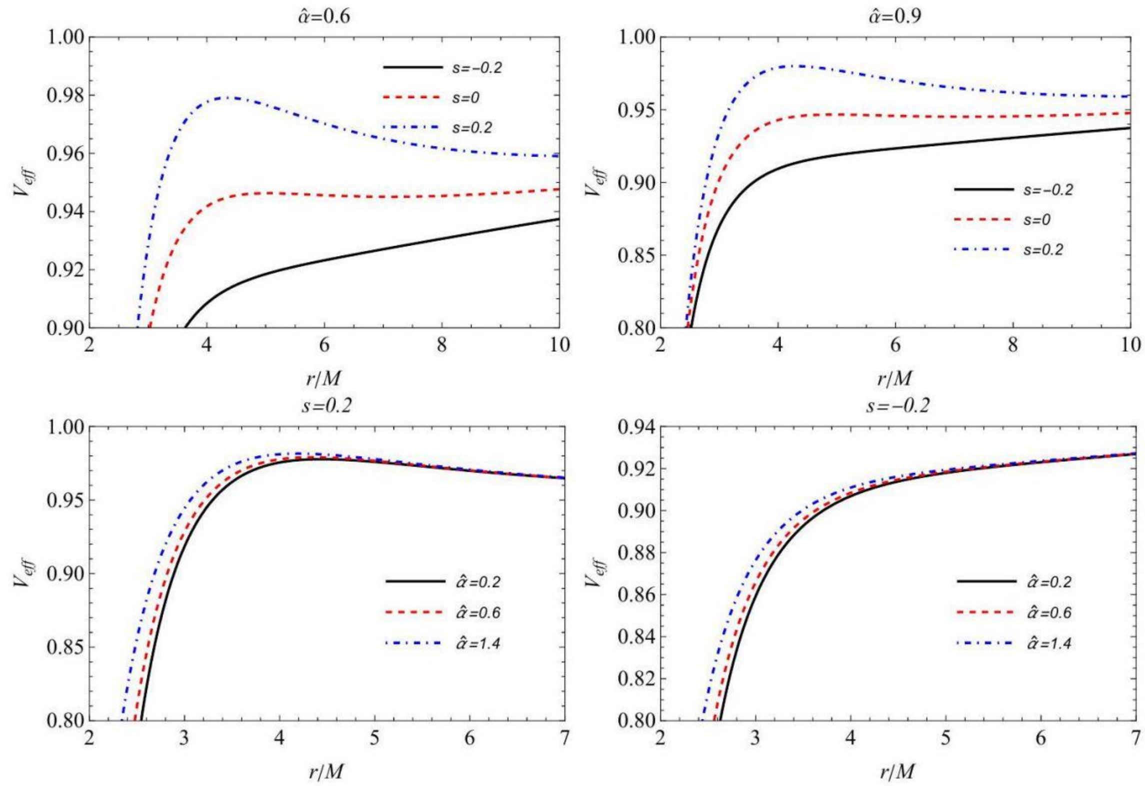

Figure 1: The radial dependence of the effective potential, V_{eff}, for varying parameter sets. The upper row illustrates the behavior of V_{eff} for fixed values of the parameter $\hat{\alpha}=0.6$ and 0.9 , with different spin values $s=-0.2,0,0.2$. The lower row highlights fixed spin values $s=-0.2$ and 0.2 , while varying $\hat{\alpha}=0.2,0.6,1.4$.

$$
\frac{Dp^{\alpha}}{d\lambda}=-\frac{1}{2}R^{\alpha}_{\beta\delta\sigma}u^{\beta}S^{\delta\sigma},\ \frac{DS^{\alpha\beta}}{d\lambda}=p^{\alpha}u^{\beta}-p^{\beta}u^{\alpha}, \tag{3}
$$

where $D/d\lambda\equiv u^{\alpha}\nabla_{\alpha}$ is the projection of the covariant derivative along the trajectory of the particle as $u^{\mu}=dx^{\mu}/d\lambda$ is the test particle's 4 -velocity, p^{α} is the canonical 4 -momentum, $R^{\alpha}{}_{\beta\delta\sigma}$ is the Riemann curvature tensor, λ is an affine parameter and $S^{\alpha\beta}$ is the antisymmetric spin tensor: $S^{\alpha\beta}=-S^{\beta\alpha}$. Alternatively,

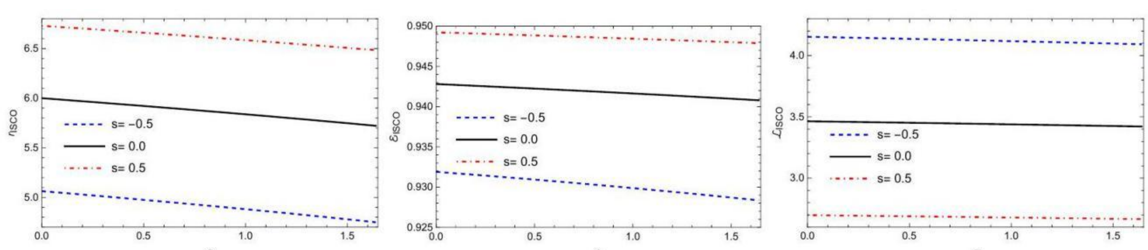

Figure 2: The variation of the radius, specific angular momentum, and energy at the ISCO with respect to the parameter $\hat{\alpha}$ of the BH for various particle spin values.

when the components of $S^{\mu\nu}$ are null, the differential equation simplifies as:

$$\frac{Dp^{\mu}}{d\tau} = 0. \tag{4}$$

The center of mass of the spinning test particle is an essential component of the MPD equations. In this sense, to solve the system in Eq. (3), one needs to fix its center of mass.

In static and spherically symmetric spacetimes, there exist two conserved quantities: energy E and total angular momentum J (where $J = L + S$), with S representing spin and L representing orbital angular momentum. Further, both quantities are defined as:

$$-E = p_t - \frac{1}{2} g_{tt,r} S^{tr}$$

$$J = p_{\phi} - \frac{1}{2} g_{\phi\phi,r} S^{\varphi r} \tag{5}$$

Now, one can get the expression for the radial canonical momentum as:

$$(p^r)^2 =- g^{rr}\left[g^{tt} p_t^2 + g^{\phi\phi} p_{\phi}^2 + m^2\right]. \tag{6}$$

One can easily derive the quadratic equation for the energy E of the spinning particle as:

$$\rho (u^r)^2 = \alpha \mathcal{E}^2 + \delta \mathcal{E} + \gamma. \tag{7}$$

We rewrite Eq. (7) as:

$$(u^r)^2 = \frac{\alpha}{\rho} (\mathcal{E} - V_+)(\mathcal{E} - V_-). \tag{8}$$

To achieve the circular motion of the spinning particles, we impose the condition $u^r = 0$, from which we can define the effective potential in the following manner:

$$V_{\pm} = \frac{-\delta \pm \sqrt{\delta^2 - 4\alpha\gamma}}{2\alpha}. \tag{9}$$

The effective potential can be defined under the following assumption that the spinning particles possess positive energy: $V_{eff} = E_+$. Fig. 1 demonstrates the radial dependence of the effective potential at fixed $\mathcal{L} = 3.5$ for different values of the parameter $\hat{\alpha}$ and s. The upper row of the plot shows the dependence of the effective potential on the radial motion of the spinning particle for different values of s, with the parameter fixed at $\hat{\alpha} = 0.6$ and $\hat{\alpha} = 0.9$. An increase in the parameter $\hat{\alpha}$ results in an increase in the effective potential. In contrast, the bottom row of the plot depicts the influence of the various values of the parameter $\hat{\alpha}$ on the effective potential at a fixed value $s = 0.2$ and $s =- 0.2$. As r/M increases, $V_{\rm eff}$ approaches an asymptotic value, indicating stable behavior at large radial distances. We can see from the graph that, if s increase $V_{\rm eff}$ is also increases significantly. The bottom graphs show that for different values of the parameter $\hat{\alpha}, V_{eff}$ is nearly similar at large radial distances.

We now examine the dynamics of a spinning particle in spacetime (1) by identifying the stable circular orbits, which are consistently the focal point of attention. It is widely recognized that two conditions must be satisfied for circular orbits: (i) $dr/d\tau = 0$ or $\mathcal{E} = V_{\rm eff}(r)$ (the motion with a constant radius R) and (ii) $d^2r/d\tau^2 = 0$ or $V'_{eff}(r) = 0$ (the motion with zero acceleration). However, these constraints alone do not ensure the stability of circular orbits. To guarantee stability, the second radial derivative of the effective potential must be positive, specifically $d^2V_{eff}/dr^2 \geq 0$. Moreover, the equality of the last condition signifies the ISCO.

The influence of the physical quantities $r_{\mathcal{ISCOO}}, \mathcal{L}_{\mathcal{ISCO}}$, and $\mathcal{E}_{\mathcal{ISCO}}$ on the parameter α for different spin s values is illustrated in Fig. 2. The plot illustrates a linear relationship between $r_{\mathcal{ISCO}}$ and $\mathcal{E}_{\mathcal{ISCO}}$ with respect to $\hat{\alpha}$, and a rise in spin s results in higher function values. The dependence of the particle's angular momentum $\mathcal{L}_{\mathcal{ISCO}}$ on $\hat{\alpha}$ remains largely unaffected.

In this chapter we have examined the fundamental principles of spinning particle motion using the MPD equations. The effective potential V_{eff} of the spinning particle has been computed and its radial dependency has been graphed for various parameter values of $\hat{\alpha}$ and s. Consequently, both parameters increase the effective potential.

Furthermore, the influence of parameters $\hat{\alpha}$ and spin s on the ISCO has been examined. The presence of the parameter $\hat{\alpha}$ reduces the radius of the ISCO. The influence of the same parameters on the angular momentum and energy at ISCO has been observed. The presence and variation of the parameter $\hat{\alpha}$ increase the specific angular momentum while diminishing the energy at the ISCO up to the joining point.

Moreover, constraints on spin s (the superluminal bound condition) have been identified, beyond which the particle's trajectory becomes space-like and nonphysical. The relationship between the critical values of spin $s_{\max}$ and $\hat{\alpha}$ has been illustrated and analyzed. We have also analyzed the effective force surrounding the quantum-corrected BHs as a function of r , for different values of the parameters $\hat{\alpha}$ and s. The effective force acting on the particles exhibits a similar trend for both the parameter $\hat{\alpha}$ and the spin parameter s. Specifically, the effective force is minimal when $\hat{\alpha}$ or s is small but increases as these parameters grow. When the parameter s is maintained constant and the BH parameter $\hat{\alpha}$ is varied, the radial profiles of the effective force converge as the radial distance r increases. However, for different values of s, distinct radial profiles of the effective force can be observed as the radial distance r increases.

In the second chapter, entitled "Optical properties of Reissner-Nordström (RN)-like BH in the presence of plasma" we investigate weak gravitational lensing and shadow radius in Kalb-Ramond (KR) gravity for a Reissner-Nordström-like black hole (BH). The examination is carried out in three distinct frameworks: uniform plasma, singular isothermal sphere(SIS), and non-singular isothermal sphere (NSIS). When a gravitating object is surrounded by plasma, the lensing angle is determined by the frequency of the electromagnetic wave, plasma dispersion, plasma inhomogeneity, and gravity. Even in uniform plasma, the gravitational photon deflection angle differs from that in a vacuum and depends on photon frequency due to the second effect. We consider both effects and calculate the lensing angle for a significantly non-uniform plasma in the existence of KR gravity. Utilizing observational data from the Event Horizon Telescope (EHT) project for SgrA* , we have obtained that which range of RN-like black hole's parameters Q and l correspond observations as well as when the Lorentz-violating parameter l rise, radius of black hole shadow decrease. To connect a relationship with observations data, we examine the magnification and positioning of images produced by lensing, as well as the weak deflection angle and magnification for sources located near various galaxies.

In Boyer-Lindquist coordinates, the RN-like BH metric is given by

$$ds^2 =- f(r)dt^2 + f(r)^{-1}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{10}$$

with

$$f(r) = \frac{1}{1-l} - \frac{2M}{r} + \frac{Q^2}{(1-l)^2 r^2} \tag{11}$$

where l is a dimensionless Lorentz-violating parameter. By using Eq.(20) we can

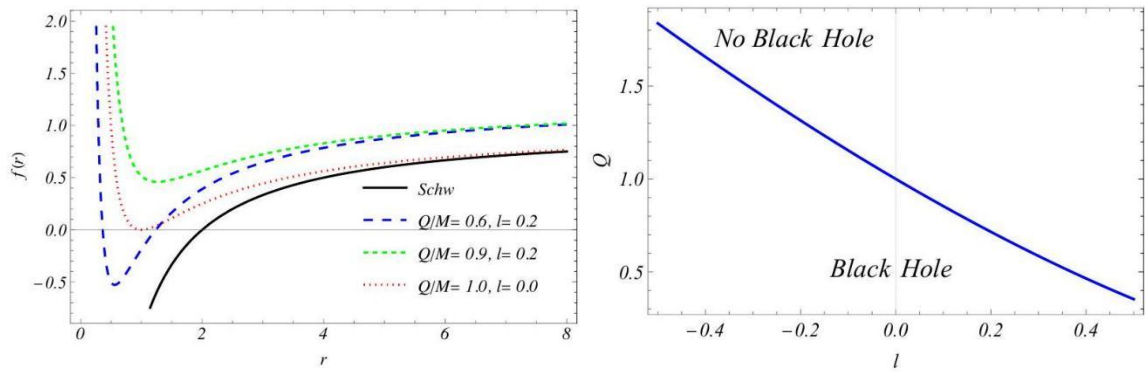

Figure 3: This graph shows a variation of f(r) (top panel) and event horizon radius due to the RN-like BH in the KR gravity and region (bottom panel), where a black hole can exist or not.

find the radius of the BH horizon as

$$r_h = (1-l)\left(M \pm \sqrt{M^2 - \frac{Q^2}{(1-l)^3}}\right). \tag{12}$$

According to (12), BH's horizon could be narrower than Schwarzschild's in the presence of a Kalb-Ramond field. It is obvious that the $l < 1$ requirement must be met at all times. Furthermore, when $l \to 0$, this measure simplifies to the Schwarzschild metric.

The top panel of Fig. 3 illustrates the radial dependence of $f(r)$ and the relationship between the event horizon radius and both the total charge of the black hole and the l parameter. Given the values of Q and l, it is possible to see either one, two, or no event horizon. Additionally, the range of RN-like black holes for parameters Q/M and l may be determined, as shown in Fig. 3. The bottom panel. By utilizing a RN-like spacetime, we have established the presence of black holes and derived the relationship between the boundary separating them and the charge-to-mass ratio Q/M and l parameters. Refer to Fig. 3. (bottom panel) Obtaining the value of the l parameter is also straightforward. The parameter l can range in value from $0.5 > l >- 0.5$ as we obtain the black hole charge $Q/M > 0.3$.

As a photon propagates through a plasma medium, the Hamiltonian is given as

$$H(x^\alpha, p_\alpha) = \frac{1}{2}\tilde{g}^{\alpha\beta}p_\alpha p_\beta. \tag{13}$$

In this context, x^α denotes spacetime coordinates, and $\tilde{g}_{\alpha\beta}$ stands for the effective metric, which can be expressed in a particular mathematical format as

$$\tilde{g}^{\alpha\beta} = g^{\alpha\beta} - (n^2-1)u^\alpha u^\beta, \tag{14}$$

The variables n, p_α, and u^β represent the refractive index of the plasma, the fourmomentum of the photon, and the four-velocity of the photon, respectively. The plasma refractive index, denoted as n, is

$$n^2 = 1 - \frac{\omega_p(r)^2}{\omega(r)^2} \tag{15}$$

where $\omega_p^2(r) = 4\pi e^2 N(r)/m_e$ (e and m_e are the electron charge and mass, respectively, and N is the electron number density) is the electron frequency in the plasma, and the gravitational red-shift formula is used to calculate the photon frequency $\omega(r)$ as measured by a static observer

$$\omega(r) = \frac{\omega_0}{\sqrt{f(r)}} \tag{16}$$

At infinity, $\omega_0 =$ const is the frequency ($f(\infty) = 1$) and $\omega_0 = \omega(\infty) =- p_t$, which denote the energy of the photon at spatial infinity. Light can travel through plasma only if its frequency exceeds the plasma frequency. Therefore, Equation (15) is applicable only when the ratio of the plasma frequency to the light frequency is less than 1. If this ratio is greater than 1, the photon cannot propagate through the plasma medium. If the frequency ω_0 is approximately equal to ω, then the deflection angle is significantly greater than in the vacuum situation where $\omega_p = 0$.

For a homogeneous plasma, in the exceptional case where the plasma $\omega_p^2(r) =$ const frequency is constant throughout the medium. After solving photon sphere equation numerically, the result presented in Fig. 4. It is clear that the radius of the photon surrounding the black hole decreases as its charge and l parameter rise. Moreover, the plasma environment increases the photon sphere radius.

Our current research involves the analysis of photon spheres in an inhomogeneous plasma, where the plasma frequency needs to follow a straightforward power-law relationship.

$$\omega_p^2(r) = \frac{z_0}{r^q} \tag{17}$$

where z_0 is the free parameter. We limit our analysis to the following examples in order to examine the main characteristics of the power law model: $q = 3$ and z_0 as a constant that is connected to the stellar surface based on Goldreich-Julian (GJ) density, and $q = 1$ and z_0 as a constant that precisely reproduces the negativemass divergent lens. After putting Eq. (17) into photon sphere equation, we get numerical expression for the radius of the photon sphere in the inhomogeneous plasma medium, as plotted in Fig. 5 and Fig. 6. It is noticeable that increasing

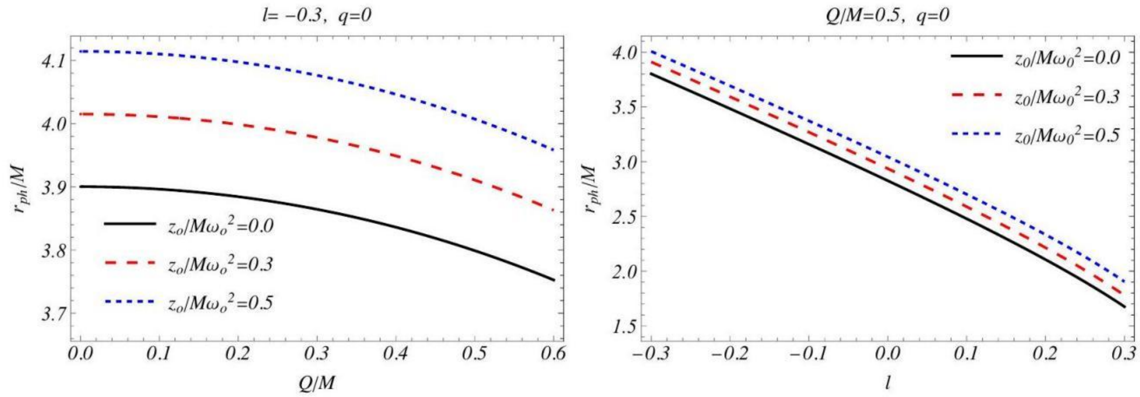

Figure 4: Plots of photon sphere as a function of black hole charge Q (top panel) and l parameter (bottom panel) in the case of $\omega_0^2/\omega^2 = \text{const}$.

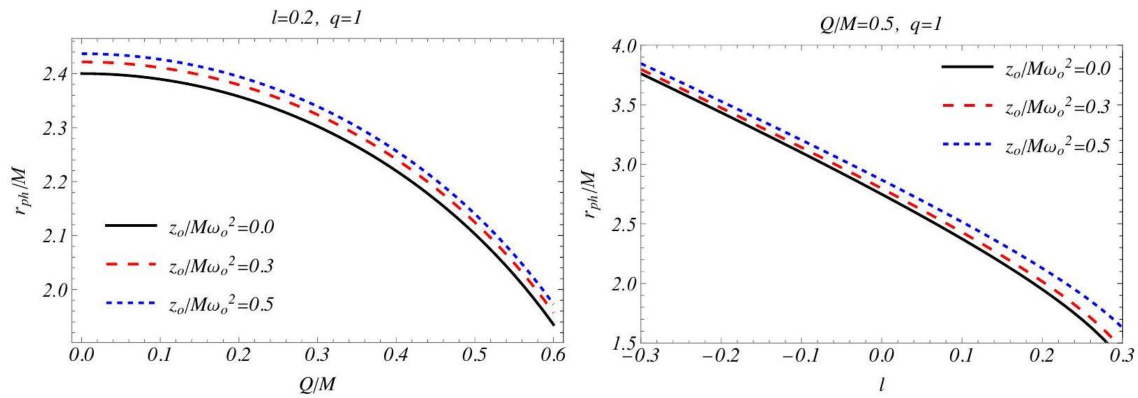

Figure 5: This figures illustrate radius of photon sphere with respect to Q/M and l parameters as a fixed $q = 1$ in-homogeneous case .

values of both Q/M and l parameters lead to a decrease in the radius of photon sphere. On the other hand, if $\omega_p^2(r) = z_0/r$ describes the distribution of the photon sphere, then the photon sphere radius rises due to the plasma medium $(q = 1)$, and decreases slightly when there is inhomogeneous plasma $(q = 3)$ surrounding the black hole. Furthermore, it is difficult to distinguish the difference in the effect on the photon radius between non uniform and uniform plasma mediums. It implies that testing and differentiating between homogeneous and inhomogeneous plasma using the shadows of black holes will be difficult.

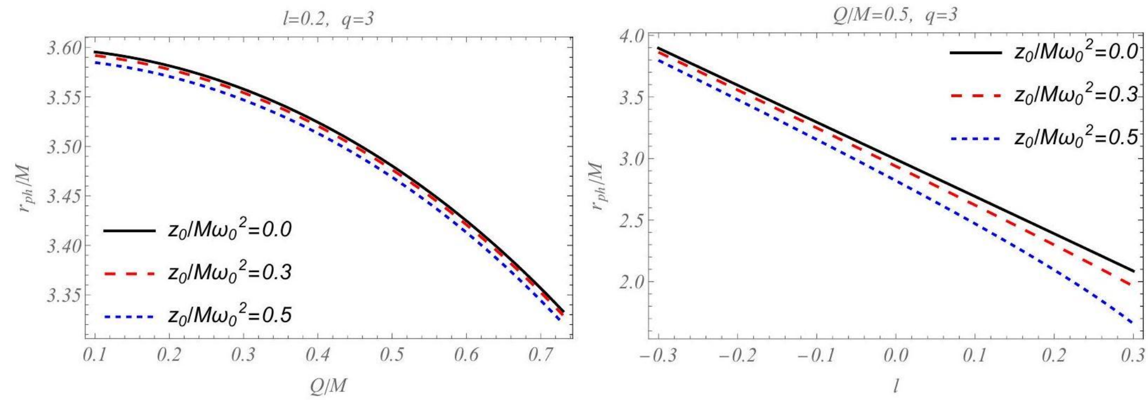

Figure 6: These figures give information about radius of photon sphere as a function of Q/M (top panel) and l (bottom panel) parameters with a $q = 3$ inhomogeous plasma medium.

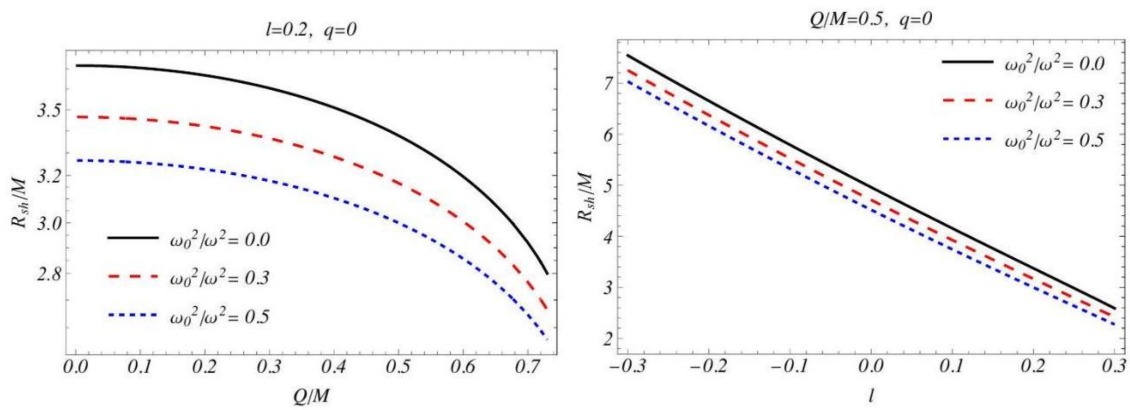

Figure 7: This graph illustrates variation of the radius of shadow R_{sh}/M concerning Q/M and l (bottom panel)for the case $\omega_p^2 = z_0/r$.

The radius of the RN-like space-time metric's shadow in the presence of both a uniform and a non-uniform plasma medium is examined in this section. An geometry-based technique yields the angular radius α_{sh} of the black hole shadow.

$$\sin^2 \alpha_{sh} = \frac{\gamma^2(r_p)}{\gamma^2(r_0)}, \tag{18}$$

The observer's position and the photon sphere's radius are denoted by the variables r_0 and r_p, respectively. By positioning the observer at a significant distance from the black hole, we can get an approximation formula for the radius of the black hole's shadow

$$R_{sh} \approx r_0 \sin \alpha_{sh} = \sqrt{r_p^2 \left[\frac{1}{f(r_p)} - \frac{\omega_p^2(r_p)}{\omega_0^2}\right]}. \tag{19}$$

This is predicated on the fact that, for both plasma models, $l(r) \to r$. When $r_p = 3M$, we recover the radius of the Schwarzshild black hole shadow, $R_{sh} = 3\sqrt{3}M$, for vacuum $\omega(r) \equiv 0$.

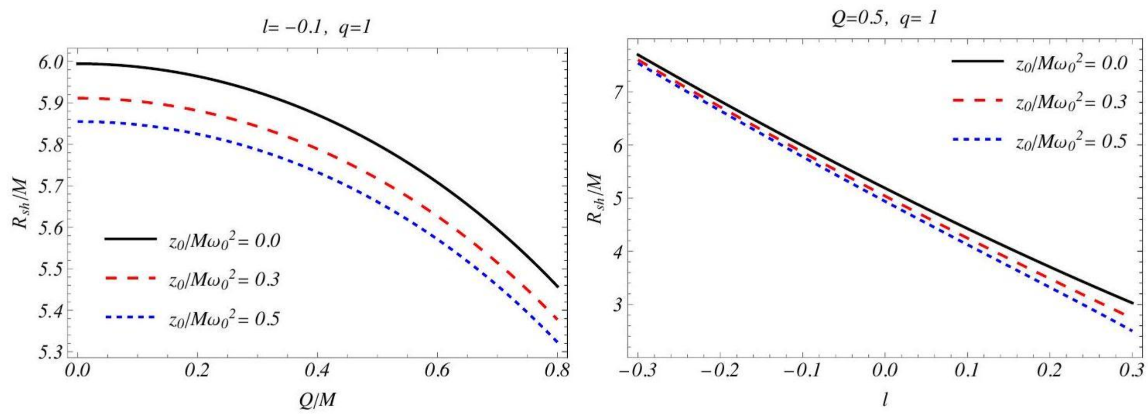

Figure 8: This graph illustrates variation of the radius of shadow R_{sh}/M concerning Q/M and l (bottom panel)for the case $\omega_p^2 = z_0/r$.

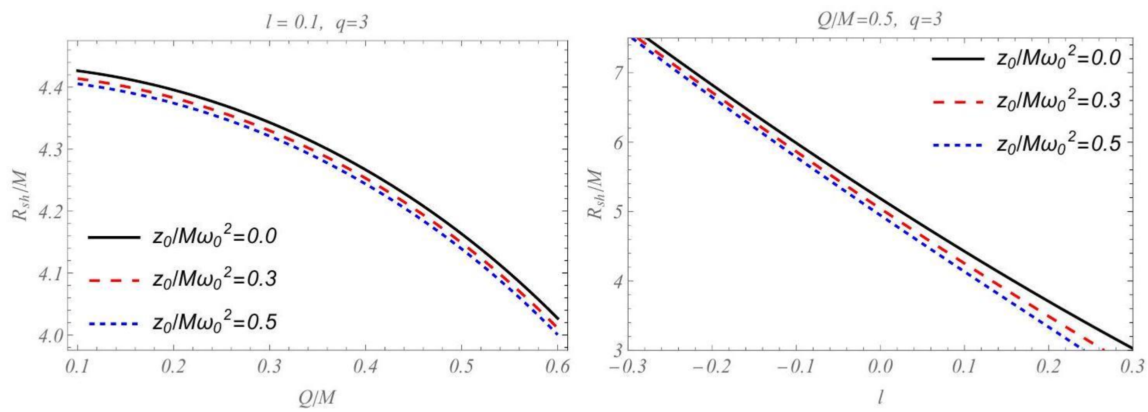

Figure 9: This graph illustrates variation of the radius of shadow R_{sh}/M concerning Q/M and l (bottom panel)for the case $\omega_p^2 = z_0/r^3$.

radius decreases much more steeply with Q/M and l. Furthermore, we have investigated inhomogeneous plasma with $\omega_p^2(r) = z_0/r^q(\mathrm{q} = 0{,}1{,}3)$ in Eq. (19). From Figs. 7-9, It is evident that the black hole's shadow is decreasing due to the existence of plasma in the radius. Besides, we have constraints on the shadow of Sgr A* black hole. Our results exactly coincide with observational data that were token from the EHT.

In the third chapter, entitled "Astrophysical properties of black holes in asymptotically safe gravity" we investigate the motion of neutral test particles around a non-rotating regular black hole within the framework of asymptotically safe gravity, analyzing the impact of the black hole's parameters on particle motion. This black hole solution is characterized by an additional parameter, η, which differentiates it from the standard Schwarzschild black hole solution. We obtain analytical formulations for the energy and angular momentum of equatorial circular orbits as functions of the black hole parameter. The stability of these circular orbits is examined by the effective potential. Additionally, we present a graphical study of the innermost stable equatorial circular orbits as functions of the black hole parameter and investigate the effective force exerted on circular orbits.

We also derive the frequencies of radial and latitudinal harmonic oscillations as functions of the model parameters and discuss the key features of quasi-periodic oscillations of test particles near stable circular orbits in the black hole's equatorial plane. The phenomenon of periastron precession is also considered.

Additionally, we explore particle collisions near the black hole and demonstrate that such collisions can produce high energy near the event horizon. Our findings indicate that the black hole parameter significantly influences the motion of test particles around a regular black hole in the context of asymptotically safe gravity. We examine the shadow of black hole and using observational data from the Event Horizon Telescope (EHT) collaboration for Sgr A* and M87*, we have determined the range of η parameter that corresponds to the observations.

Regular BHs in the framework of the asymptotically safe gravity is described by following metric

$$ds^2 =- f(r)dt^2 + f^{-1}(r)dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) \tag{20}$$

where the unknown function $f(r)$ reads

$$f(r) = 1 - \frac{r^2}{3\eta}\ln\left(\frac{6M\eta}{r^3} + 1\right) \tag{21}$$

where M is the Gravitational mass. For vanishing $\eta = 0$, the metric (20) leads to the Schwarzshild metric. The position of the horizon is calculated by solving the equation $f(r) = 0$, which can gives interesting results. For any specified value of M, critical values exist given by

$$\eta_{cr} = \frac{2}{3}(3 + 2y)y^2M^2 \tag{22}$$

and

$$r_h^{ext} =- 2My, y = W_0(x) = W_0\left(-\frac{3}{2e^{\frac{3}{2}}}\right) \tag{23}$$

where $W_0(x)$ defines the Lambert function and the numerical values are provided as:

$$\eta_{cr} = 0.4565M^2, r_{\text{hoz}}^{ext} = 1.2516M \tag{24}$$

The Hamiltonian equations of motion are expressed as:

$$\frac{dx^\alpha}{d\zeta} \equiv mU^\alpha = \frac{\partial H}{\partial P_\alpha}, \frac{dP_\alpha}{d\zeta} =- \frac{\partial H}{\partial x^\alpha}, \tag{25}$$

where $\zeta = \tau/m$ represents the affine parameter. Owing to the symmetries of the black hole geometry, two constant of motion are present: a specific energy E and specific angular momentum L, which both are defined as follows

$$\frac{P_t}{m} =- \left(1 - \frac{r^2}{3\eta}\ln\left(\frac{6M\eta}{r^3} + 1\right)\right)\frac{dt}{d\tau} =- \mathcal{E} \tag{26}$$

$$\frac{P_\phi}{m} = r^2\sin^2\theta\frac{d\phi}{d\tau} = \mathcal{L} \tag{27}$$

where $\mathcal{E} = E/m$ and $\mathcal{L} = L/m$ denote a particular energy and a specific angular momentum, respectively. The temporal U^t, azimuthal U^ϕ and radial U^r components of the four-velocity U^α adhere to the following equations as:
-like particles and $\delta = 0$ for null particles. The dot represents the derivative for the appropriate time τ. Our effort will concentrate exclusively

$$\dot{t} = \frac{\mathcal{E}}{1 - \frac{r^2}{3}\eta \ln\left(\frac{6M\eta}{r^3} + 1\right)} \tag{28}$$

$$\dot{\phi} = \frac{\mathcal{L}}{r^2 \sin^2\theta} \tag{29}$$

$$\dot{r}^2 + \left(\delta + \frac{L^2}{r^2\sin^2\theta}\right) f(r) = \mathcal{E}^2 \tag{30}$$

where $\delta = 1$ for time
on time-like particles. The Hamiltonian for a conventional black hole (20) under the framework of asymptotically safe gravity is expressed as

$$H = \frac{1}{2}\left(1 - \frac{r^2}{3\eta} ln\left(\frac{6M\eta}{r^3} + 1\right)\right) P_r^2 + \frac{1}{2r^2} P_\theta^2 + \frac{1}{2}\frac{m^2}{\left(1 - \frac{r^2}{3\eta} ln\left(\frac{6M\eta}{r^3} + 1\right)\right)}\left[V_{eff}(r,\theta) - \mathcal{E}^2\right] \tag{31}$$

where the effective potential $V_{eff}(r,\theta)$ can be written as

$$V_{eff}(r,\theta) = \left(1 + \frac{\mathcal{L}^2 \csc^2\theta}{r^2}\right)\left(1 - \frac{r^2}{3\eta}\ln\left(\frac{6M\eta}{r^3} + 1\right)\right). \tag{32}$$

For $\eta = 0$, the effective potential (32) leads to the Schwarzschild case. The effective potential $V_{eff}(r,\theta)$ is crucial for understanding the dynamics of the test particles.

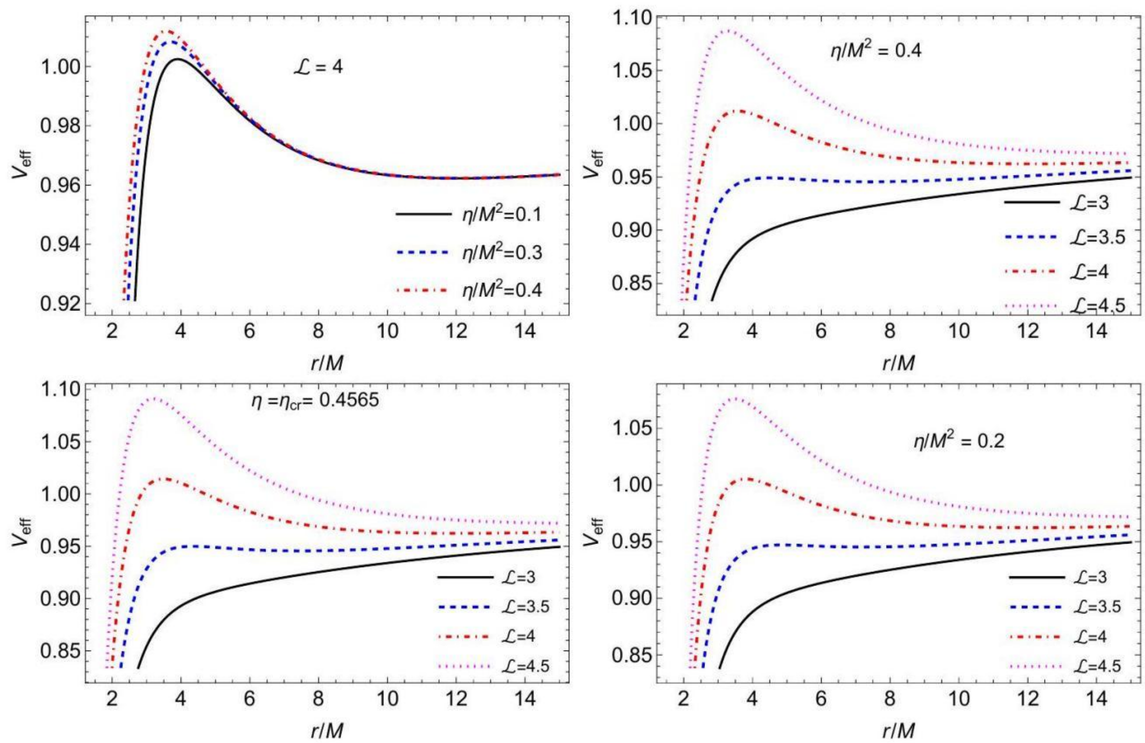

Figure 10: Behaviour of the effective potential for the test particles in the background of regular BH in asymptotically safe gravity.

It provides a means to describe the motion of particles independently of explicit equations of motion. The stable and unstable circular orbits correspond to the minimum and maximum values of V_{eff} , respectively. The behavior of the effective potential for the equatorial orbits is shown in Fig. 10. In the first row and first column, the behavior of effective potential is shown for different values of the BH parameter η, while the behavior of effective potential for different values of angular momentum is shown for $\eta < \eta_{cr}$ and $\eta = \eta_{cr}$. When $\eta < \eta_{cr}$, the effective potential does not increase much as we increase the values of angular momentum. It is interesting to note that both the BH parameter η and the particle's angular momentum behave similarly on effective potential. The effective potential increases with increasing either the BH parameter η or the angular momentum of the particle. When the particles move close to the event horizon of the BH , the circular orbits are stable, and stability decreases as the particles move away from the BH .

We have obtained the analytical expressions for energy and angular momentum of equatorial circular orbits around non-rotating regular BH within asymptotically safe gravity, as a function of BH parameters. It is noted that when the value of the BH parameter η is small, the particles have a high energy; however, the energy decreases for large values of η. Furthermore, the energy increases monotonically as the radial distance r increases. We have shown the behavior of the angular momentum as a function of r for varying values of the BH parameter η.

It is shown that when the value of the BH parameter η is small, the particles have a large angular momentum, while the particles have a small angular momentum for increasing values of the BH parameter η. However, the angular momentum increases monotonically as the radial distance r increases.

We have explored the epicyclic oscillations and fundamental frequencies of test particles moving around a regular BH within asymptotically safe gravity. The BH parameter η is shown to contribute to the shift of the fundamental frequencies of the test particles close to the BH horizon. The particles around Schwarzschild BH have higher frequencies as compared to those particles traveling around regular BH within asymptotically safe gravity. We also examine the frequency of the periastron and the effects of the BH parameter on it. We observed that the frequency of the periastron decreases as the parameter η increases and also decreases with increasing radial distance r.

Also, we have studied the radius of the photon sphere and the shadow of the regular black hole in the background of asymptotically safe gravity. Our analysis confirms that an increase in the parameter η leads to a decrease in both the radius of the photon sphere and the radius of the shadow. Using EHT collaboration data, we obtained a constraint on the parameter η for the supermassive black hole Sgr A* and M87*. Consequently, the regular BH parameter η has been constrained to $0 < \eta/M^2 < 0.4565$ for both SgrA* and M87*.

Conclusion

The following conclusions were presented based on research carried out on the topic of 'Optical Properties and Dynamics of Particles Around Black Holes in Modified Gravities' for the Doctor of Philosophy (PhD) dissertation:

1. It has been obtained that the equations of motion for spinning particles around quantum-corrected black holes reveal a significant dependence of the effective potential, ISCO radius, and superluminal bounds on both spin and quantum gravity parameters.
2. It has been obtained that the optical properties of Reissner-Nordström like black holes in Kalb-Ramond gravity are strongly affected by Lorentz violating parameters and plasma distributions, and it has been shown how these factors influence the photon sphere and shadow radius.
3. It has been demonstrated that theoretical predictions of black hole shadow sizes, when compared with Event Horizon Telescope observations of Sgr A* and M87*, allow for the establishment of constraints on parameters in modified gravity theories.
4. It has been shown that regular black holes in asymptotically safe gravity exhibit distinct astrophysical and orbital properties, including characteristic epicyclic oscillations and precession effects, with potential observational signatures.

НАУЧНЫЙ СОВЕТ DSc.03/07.07.2025 FM/T.192.01 ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ИНСТИТУТЕ ПЕРСПЕКТИВНЫХ ИССЛЕДОВАНИЙ УНИВЕРСИТЕТА “НОВЫЙ УЗБЕКИСТАН”

ТАШКЕНТСКИЙ ГОСУДАРСТВЕННЫЙ ТЕХНИЧЕСКИЙ УНИВЕРСИТЕТ ИМЕНИ ИСЛАМА КАРИМОВА
УНИВЕРСИТЕТ “НОВЫЙ УЗБЕКИСТАН”

01.03.01 — Астрономия
01.04.02 — Теоретическая физика

АЛИМОВА АСАЛХОН

ОПТИЧЕСКИЕ ПРОЦЕССЫ И ДИНАМИКА ЧАСТИЦ В ОКРЕСТНОСТИ ЧЁРНЫХ ДЫР В РАМКАХ МОДИФИЦИРОВАННЫХ ТЕОРИЙ ГРАВИТАЦИИ

АВТОРЕФЕРАТ диссертации на соискание степени **доктора философии (PhD) по физико-математическим наукам**

Ташкент — 2025

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министерстве высшего образования, науки и инноваций Республики Узбекистан под номером B2025.4.PhD/FM_____.

Данная диссертационная работа выполнена в Ташкентском государственном техническом университете имени Ислама Каримова и в Университете Новый Узбекистан.

Автореферат диссертации на трех языках (узбекский, английский, русский (резюме)) размещен на веб-странице Научного совета (www.ias.newuu.uz) и Информационно-образовательном портале «Ziyonet» (www.ziyonet.uz).

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Зашита диссертации состоится «__»_________ **2025 года в ____ часов** на заседании Научного Совета **DSc.03/07.07.2025. FM/T.192.01** при Институте перспективных исследований университета "Новый Узбекистан" (Адрес: 100007, г. Ташкент, ул. Мовароуннахр 1, Институт перспективных исследований университета "Новый Узбекистан", Тел.: +99871 202-41-11)

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ВВЕДЕНИЕ (Аннотация к представлению)

Актуальность темы диссертации

Изучение чёрных дыр и модифицированных теорий гравитации является одним из наиболее актуальных направлений современной астрофизики. После получения первых изображений теней чёрных дыр проектом Event Horizon Telescope (EHT), а также обнаружения гравитационных волн детекторами LIGO–Virgo, необходимость исследования альтернативных гравитационных теорий, расширяющих или модифицирующих Общую теорию относительности (ОТО), значительно возросла. Эти теории направлены на решение нерешённых проблем ОТО, таких как сингулярности, тёмная материя и тёмная энергия, а также на формирование проверяемых прогнозов для будущих наблюдений.

В настоящей диссертации изучается движение частиц и света в окрестности чёрных дыр в рамках модифицированных гравитационных моделей, включающих квантовые поправки, асимптотически безопасную гравитацию. Такие исследования обладают высокой актуальностью, поскольку они связывают теоретические прогнозы с наблюдательными данными и помогают лучше понять физику плотных объектов и их окрестностей. Анализ движения спиновых частиц и гравитационного линзирования в плазме играет ключевую роль в интерпретации динамики аккреционных дисков, квазипериодических осцилляций (QPO) и теней чёрных дыр.

Необходимость выполнения подобных исследований усиливается в связи с планируемыми наблюдательными миссиями нового поколения, такими как обновлённый EHT, космический интерферометр LISA и будущие рентгеновские телескопы, которые будут изучать гравитацию сильных полей с беспрецедентной точностью. Ограничение параметров модифицированных гравитационных моделей по наблюдениям чёрных дыр позволяет выявлять отклонения от ОТО и исследовать новое физическое содержание. Таким образом, диссертация согласуется с современными направлениями астрофизики и способствует развитию фундаментальной гравитационной физики.

Кроме того, тема работы соответствует стратегическим научным приоритетам Республики Узбекистан и международного научного сообщества.

Соответствие исследования приоритетным направлениям развития науки и технологий Республики Узбекистан

Диссертационная работа выполнена в соответствии с приоритетными направлениями развития науки и технологий Республики Узбекистан:

“Энергия, ресурсосбережение и эффективность”.

Степень изученности проблемы

Исследования чёрных дыр и модифицированных теорий гравитации в течение десятилетий занимают центральное место как в теоретической, так и в наблюдательной астрофизике. Классические решения в рамках ОТО — такие как метрики Шварцшильда и Керра — глубоко изучены с точки зрения динамики частиц и гравитационного линзирования. Однако ограничения ОТО — существование сингулярностей, отсутствие квантовых эффектов и несоответствия с космологическими наблюдениями — стимулировали развитие альтернативных моделей гравитации. Были предложены такие направления, как квантово-корректированная гравитация и асимптотически безопасная гравитация; тем не менее, детальная динамика частиц и света в подобных пространствах-времени остаётся актуальной областью исследований.

Движение вращающихся частиц, описываемое уравнениями Матиссона–Папапетру–Диксона, изучено в рамках ОТО и отдельных модифицированных моделей; однако влияние квантовых поправок и дополнительных полей, таких как поля Прока или фоны Калба–Рамонда, исследовано недостаточно. Аналогично, гравитационное линзирование в плазме хорошо изучено в стандартной ОТО, но влияние модифицированной гравитации и нарушений симметрий (например, Лоренцевой симметрии) требует дальнейшего анализа.

Несмотря на существенный прогресс, многие аспекты модифицированных гравитационных моделей остаются недостаточно исследованными. Наблюдательные ограничения, полученные из теней чёрных дыр, гравитационных волн и рентгеновской спектроскопии, открывают возможности для проверки этих моделей, однако необходимы системные теоретические исследования, связывающие модели с наблюдаемыми явлениями. Настоящая диссертация вносит вклад в заполнение этих пробелов, анализируя динамику частиц и оптические эффекты в конкретных модифицированных моделях гравитации.

Связь диссертационной работы с научно-исследовательскими проектами

Работа выполнена в рамках научного проекта Министерства инновационного развития Республики Узбекистан: F-FA-2021-510 “Исследование ядерного вещества нейтронных звёзд в условиях модифицированной гравитации” (2021–2026).

Цель исследования

Целью исследования является изучение модифицированной гравитации на основе анализа астрофизики чёрных дыр с использованием орбит частиц и теней чёрных дыр.

Задачи исследования

• анализ динамики спиновых частиц с помощью уравнений Матиссона–Папапетру–Диксона (МПД);
• определение ISCO, эффективного потенциала и эффективной силы, а также изучение зависимости этих величин от квантового параметра чёрной дыры и спина частицы;
• исследование сверхсветового предельного условия (superluminal bound);
• изучение слабого гравитационного линзирования и формирования теней в гравитации Калба–Рамонда;
• анализ влияния однородной и неоднородной плазмы на траектории фотонов и тени чёрных дыр;
• сравнение теоретических прогнозов с наблюдениями EHT для Sgr A*;
• изучение движения нейтральных частиц вокруг регулярных чёрных дыр;
• вывод выражений для энергии, углового момента и устойчивости круглых орбит;
• исследование эпициклических частот, QPO и перицентральной прецессии;
• сопоставление теней чёрных дыр с данными EHT для Sgr A* и M87*;

Объект исследования: физические явления и теоретические модели, описывающие движение частиц и полей и оптические свойства в окрестности чёрных дыр в различных модифицированных гравитационных теориях.

Предмет исследования: математические модели, аналитические методы и вычислительные техники, используемые для анализа движения спиновых частиц, теней чёрных дыр и гравитационного линзирования в плазме и наблюдательных ограничений.

Методы исследования

Используются аналитические и численные методы в рамках ОТО и модифицированных теорий гравитации: формализм МПД, геодезический анализ для фотонов, методы эффективного потенциала, техники оптической геометрии, моделирование теней и линзирования в плазме, численные симуляции и сравнение с наблюдениями EHT.

Научная новизна

- Проведён всесторонний анализ движения спиновых частиц вокруг квантово-корректированных чёрных дыр; выявлено влияние квантовых параметров на эффективный потенциал, ISCO и сверхсветовые пределы.
- Получены новые результаты по оптическим свойствам RN-подобных чёрных дыр в гравитации Калба–Рамонда, включая радиус теней и эффекты линзирования в однородной и неоднородной плазме.

Практическая значимость

- Разработаны теоретические модели и вычислительные методы для прогноза наблюдаемых эффектов модифицированной гравитации в окрестности чёрных дыр.
- Установлены ограничения на параметры квантово-корректированных и нарушающих Лоренцеву симметрию чёрных дыр по данным EHT.
- Создана методологическая база для анализа динамики частиц, линзирования и термодинамики в модифицированной гравитации.

Достоверность результатов

Результаты основаны на строгих математических моделях, согласуются со стандартными решениями ОТО (Шварцшильда и Керра), проверены численными методами и сопоставлены с наблюдениями EHT.

Применение результатов исследования

Результаты исследования оптических процессов и динамики частиц в окрестности чёрных дыр в рамках модифицированных теорий гравитации были использованы следующим образом: теоретические результаты и разработанные методы опубликованы в научной статье *A. Alimova, Z. Turakhonov, F. Atamurotov, A. Abdujabbarov, "Shadow and weak gravitational lensing of RN-like BH in plasma", Physics of the Dark Universe, Vol. 47, 101749 (2025),* https://doi.org/10.1016/j.dark.2024.101749, а также представлены в диссертации на соискание степени PhD Алимовой Асалхон. Полученные результаты использованы в рамках программ, поддержанных Фуданьским университетом (что подтверждено письмом профессора Косимо Бамби).

Подтверждение результатов исследования

Результаты диссертационной работы были представлены и обсуждены на трёх международных конференциях.

Публикации по теме диссертации

По теме исследования опубликовано 11 научных статей.

Объем и структура диссертации

Основные результаты диссертации опубликованы в научных журналах и материалах конференций. По теме диссертации опубликовано всего 11 научных работ, из которых 6 статей вышли в международных рецензируемых научных журналах, рекомендованных Высшей аттестационной комиссией Республики Узбекистан для публикации основных научных результатов PhD-диссертаций

Заключение

Ниже приведены выводы, представленные на основе проведённого исследования по теме "Оптические процессы и динамика частиц в окрестности чёрных дыр в рамках модифицированных теорий гравитаци", выполненного для диссертации на соискание степени PhD:

1. Показано, что уравнения движения вращающихся частиц в окрестности квантово-корректированных чёрных дыр демонстрируют существенную зависимость эффективного потенциала, радиуса ISCO и сверхсветовых пределов как от спина частицы, так и от параметров квантовых поправок.
2. Оптические свойства чёрных дыр типа Рейсснера–Нордстрёма в гравитации Калба–Рамонда существенно зависят от параметров, нарушающих симметрию Лоренца, а также от распределения плазмы; в работе показано их влияние на параметры фотонной сферы и радиус тени.
3. Сравнение теоретически предсказанных размеров тени с наблюдениями Event Horizon Telescope для Sgr A* и M87* позволило установить ограничения на параметры модифицированных теорий гравитации.
4. Показано, что регулярные чёрные дыры в асимптотически безопасной гравитации обладают характерными астрофизическими и орбитальными особенностями, такими как эпициркулярные колебания и перицентральная прецессия, что открывает потенциал для наблюдательного тестирования таких моделей.

E’LON QILINGAN ISHLAR RO‘YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ

I bo‘lim (part I; I часть)

1. Mushtaq F., Tiecheng X., Ditta A., Atamurotov F., Abduvokhidov A., Alimova A. //*Weak gravitational lensing and fundamental frequencies of Einstein–Euler–Heisenberg black hole* //New Astronomy, 2024, Volume 108, Article ID 102185 (Q1 Web of Science, IF: 2.0).
2. Alimova A., Turakhonov Z., Atamurotov F., Abdujabbarov A. //*Shadow and weak gravitational lensing of RN-like BH in plasma* // Physics of the Dark Universe, 2025, Volume 47, Article ID 101749 (Q1 Web of Science, IF: 6.5).
3. Alimova A., Atamurotov F., Abdujabbarov A., Mustafa G., Channuie P.//*Impact of quantum-corrected parameter on spinning particle motion around a black hole* //The European Physical Journal C, 2025, Volume 85, Article No. 646 (Q1 Web of Science, IF: 4.8).
4. Mustafa G., Alimova A., Atamurotov F., Ibraheem A.A., Channuie P., Bahaddinova G.//*Testing regular black holes in the framework of asymptotically safe gravity using particle dynamics, QPOs, and shadow constraints* //The European Physical Journal C, 2025, Volume 85, Article No. 741 (Q1 Web of Science, IF: 4.8).
5. Alimova A., Ghorani E., Pulіće B., Atamuratov F., Abdujabbarov A. //*Thermodynamics of Einstein–Geometric Proca AdS Compact Objects* // The European Physical Journal C, 2025, Volume 85, Article No. 962 (Q1 Web of Science, IF: 4.8).
6. Ditta A., Ashraf A., Maurya K., Ali A., Alimova A., Atamurotov F. //*Oscillatory motion and QPO signatures around rotating short-hairy black holes: Bridging theory and observation* //Physics of the Dark Universe, 2025, Volume 50, Article ID 102071 (Q1 Web of Science, IF: 6.5).
7. Davlataliev A., Alimova A., Khamidov V.//*Gravitational lensing in Starobinsky–Bel–Robinson gravity in non-uniform plasma* //Вестник Национального университета Узбекистана**,** 2024, [3/1/1], pp. 471–473.
8. Alimova A., Abduqayumova G., Juraeva N.//*Spinning particles motion around the Reissner–Nordström-like black hole* //Вестник Национального университета Узбекистана, 2025, [3/1/1], pp. 403–406.

II bo‘lim (II часть; II part)

1. Ortiqboev D., Alimova A., Umaraliyev M., Photon motion in the KalbRamond gravity / "Fizika fanining rivojida iste'dodli yoshlarning o'rni" Respublika ilmiy-amaliy konferensiya (RIAK-XVII-2024) materiallari to'plami: O'zMU,5-6-aprel,2024-yil. Toshkent.80-82 betlar.
2. Orzuev Sh., Ibragimov I., Alimova A., Weak gravitational lensing in SIS plasma with Gauss-Bonnet theory// "Fizika fanining rivojida iste'dodli yoshlarning o'rni" Respublika ilmiy-amaliy konferensiya (RIAK-XVII-2024) materiallari to'plami: O'zMU, 5-6-aprel, 2024-yil.Toshkent.31-32 betlar
3. Alibekov H., Alimova A., Khamidov V., Gravitational lensing and redshift of photons in the spacetime of deformed RN BH// "Fizika fanining rivojida iste'dodli yoshlarning o'rni" Respublika ilmiy-amaliy konferensiya (RIAK-XVII-2024) materiallari to'plami: O'zMU, 5-6-aprel, 2024-yil. Toshkent.4748 betlar.