“YANGI O’ZBEKISTON” UNIVERSITETI HUZURIDAGI ILG‘OR TADQIQOTLAR INSTITUTI HUZURIDAGI ILMIY DARAJALAR BERUVCHI DSC.03/07.07.2025.FM/T.192.01 RAQAMLI ILMIY KENGASH

“YANGI O‘ZBEKISTON” UNIVERSITETI ASTRONOMIYA INSTITUTI

DAVLATALIYEV AKBARJON AKMALJON O‘G‘LI

SKALYAR MAYDON MAVJUDLIGIDA KOMPAKT OBYEKTLAR ATROFIDA RELYATIVISTIK ASTROFIZIK JARAYONLAR

01.03.01 – Astronomiya
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FIZIKA-MATEMATIKA FANLARI BO‘YICHA FALSAFA DOKTORI (PhD) DISSERTATSIYASI AVTOREFARATI

TOSHKENT-2025

Fizika-matematika fanlari bo'yicha falsafa doktori (PhD) dissertatsiyasi mavzusi O'zbekiston Respublikasi Oliy ta'lim, fan va innovatsiyalar vazirligi huzuridagi Oliy attestatsiya komissiyasida B2025.2.PhD/FM1325 raqami bilan ro'yxatga olingan.

Dissertatsiya "Yangi O'zbekiston" universiteti va O'zbekiston Respublikasi Fanlar akdemiyasi Astronomiya institutida bajarilgan. Dissertatsiya avtoreferati uch tilda (o'zbek, rus, ingliz (rezyume)). Ilmiy Kengashning internet sahifasida (ias.newuu.uz) va "Ziyonet" axborot-ta'lim portalida (www.ziyonet.uz) joylashtirilgan.

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Dissertatsiya "Yangi O'zbekiston" universiteti huzuridagi Ilg'or tadqiqotlar instituti huzuridagi ilmiy darajalar beruvchi DSc.03/07.07.2025.FM/T.192.01 raqamli Ilmiy Kengashning 2025-yil "____" _______ soat _____ dagi majlisida bo'lib o'tadi. (Manzil: 100007, Toshkent shahri, Mirzo Ulug'bek tumani, Movarounnaxr ko'chasi, 1-uy, "Yangi O'zbekiston" universiteti huzuridagi Ilg'or tadqiqotlar instituti, tel.: +99871202-41-11, e-mail: info@newuu.uz.)

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KIRISH (PhD dissertatsiya annotatsiyasi)

Dissertatsiyaning dolzarbligi va talab yuqoriligi. Astronomik kuzatuvlarning aniqligi oshgani sari masalan, gravitatsion to‘lqinlarining aniqlanishi (LIGO/Virgo) va X-nurlari manbalaridan yuqori chastotali kvaziperiodik tebranishlar(QPOs)ning kuzatilishi natijasida klassik umumiy nisbiylik nazariyasidan tashqariga chiqadigan nazariy asoslarga ehtiyoj ortib bormoqda. Ushbu ish skalyar maydonlar va tashqi magnit maydonlar fazoviy-vaqt dinamikasiga qanday ta’sir qilishi bo‘yicha muhim tushunchalarni beradi hamda kelajakdagi kuzatuvlar uchun sinab ko‘rish mumkin bo‘lgan bashoratlarni taklif qiladi. Egzotik kompakt obyektlar (masalan, JNW naked(yalang‘och) singulyarliklari, Ellis wormhole(qurt teshigi))ni o‘rganish ham, Eynshteyn nazariyasining sinab ko‘rish va muqobil tortishish modellari bo‘yicha olib borilayotgan sa’y-harakatlarga mos keladi. Dissertatsiya zamonaviy nisbiy astrofizikaning muhim savollariga, xususan kompakt obyektlar (qora tuynuklar, naked singulyarliklar va wormhollar) bilan skalyar maydonlar o‘rtasidagi o‘zaro ta’sirga e’tibor qaratadi.

Dissertatsiyaning skalyar-tenzor o‘zaro bog‘lanishlariga va ularning astrofizik ahamiyatiga qaratilganligi uni muhim ahamiyatga ega. Ayniqsa qorong‘i materiya va qorong‘i energiya tadqiqotlaridagi so‘nggi yutuqlarni hisobga olgan holda modifikatsiyalangan tortishish nazariyasini qurishda muhim hissoblanadi. Skalyar maydonlar, f(R) tortishish nazariyasi va string nazariyasi kabi zamonaviy modellar asosida fundamental rol o‘ynaydi, bu esa ishni nazariy va kuzatuvli kosmologiya uchun dolzarbligini oshiradi. Bundan tashqari, kvazi-rezonans rejimlar va QPO lar tahlili LISA, Athena kabi yangi avlod teleskoplari hamda NICER kabi X-nur observatoriyalaridan olinadigan ma’lumotlarni talqin qilishga hissa qo‘shadi. IV-bobda ko‘rib chiqilgan magnitlangan qora tuynuklardagi buzilishlarning noodatiy kechkish davr "dumlari" esa, NANOGrav tomonidan qayd etilgan pulsar tayming massiv (PTA) signallaridagi nomuvofiqliklarni tushuntirishda yordam berishi mumkin.

Zarralar dinamikasi, buzilishlar nazariyasi va kuzatuv cheklovlari o‘rtasidagi ko‘prikni yaratish orqali, dissertatsiya ekstremal sharoitdagi zich obyektlarni o‘rganish vositalarini rivojlantiradi. Quasinormal rejimlar uchun WKB yaqinlashuvi va QPO lar uchun MCMC tahlili kabi metodologiyalar neytron yulduzlari yoki g‘ayrioddiy, gorizontsiz obyektlar kabi boshqa obyektlarga ham qo‘llanilishi mumkin. Ish shuningdek, umumiy nisbiylik, kvant maydon nazariyasi va ma’lumotlar fanini birlashtiruvchi tarmoqlararo yondashuvlarning ahamiyatini ta’kidlaydi. Bugungi fan umumiy nisbiylik va kvant mexanikasi o‘rtasidagi ziddiyatlarni hal etishga intilayotgan bir paytda, aynan shunday tadqiqotlar Standart Modeldan tashqaridagi yangi fizikani ochishga yo‘l ochadi. Dissertatsiyaning dolzarbligi, talabi va ahamiyati uning nisbiy astrofizikadagi ilg‘or savollar bilan bevosita shug‘ullanishida, mavjud va kelajakdagi kuzatuv dasturlariga tatbiq etilishida hamda tortishish va kosmologiya sohalaridagi nazariy paradigmalarga ta'sir ko'rsatish imkoniyatini beradi.

Shuningdek, ushbu dissertatsiya O'zbekiston va xalqaro ilmiy hamjamiyatning strategik ilmiy ustuvor yo'nalishlariga hamohangdir. Ushbu ilmiy ish quyidagi davlat me'yoriy hujjatlarida belgilangan vazifalarga mos keladi: O'zbekiston Respublikasi Prezidentining 2017-yil 7-fevraldagi PF-4947-sonli "O'zbekiston Respublikasini yanada rivojlantirish bo'yicha Harakatlar strategiyasi to'g'risida"gi farmoni hamda 2017-yil 18-fevraldagi PQ-2789-sonli "Fanlar akademiyasini yanada takomillashtirish, ilmiy tadqiqot faoliyatini tashkil etish, boshqarish va moliyalashtirish chora-tadbirlari to'g'risida"gi qarori.

Tadqiqotning respublika fan va texnologiyalari rivojlanishining ustuvor yo'nalishlariga mosligi. Dissertatsiya tadqiqoti O'zbekiston Respublikasidagi fan va texnologiyalarning ustuvor yo'nalishlariga muvofiq amalga oshirilgan, xususan: II. "Energetika, energiya va resurslarni tejash" yo'nalishi bo'yicha.

Muammoning o'rganilganlik darajasi.

Qora tuynuk fazo-vaqtlarida test zarrachalarining harakati umumiy nisbiylik nazariyasida (Subrahmanyan Chandrasekhar, Brandon Carter, James Bardeen, Valeriy Frolov, Andrey Zelnikov, Pankaj S. Joshi, Remo Ruffini) keng o'rganilgan bo'lsa-da, skalyar maydonlarni qo'shish, ayniqsa naked singulyarliklar (JNW fazosi) va wormhole (Ellis fazosi) kontekstida, yangi murakkabliklarni keltirib chiqaradi. Ushbu tadqiqot Misner, Wheeler va boshqalar tomonidan asos solingan ishlarga tayangan holda, bu yondashuvni skalyar o'zaro ta'sirlar, nurlanish reaksiyasi effektlari va magnit maydonlarni hisobga olgan holda kengaytiradi, bular esa ushbu fazolarda tizimli ravishda o'rganilmagan edi. Bu geodezik harakatning klassik tahlillaridan sezilarli darajada ilg'or yondashuv hisoblanadi.

Astrofizikadagi skalyar maydonlar bo'yicha avvalgi tadqiqotlar asosan kosmologik ilovalar yoki qora tuynuk perturbatsiyalariga (Claudia de Rham, Thomas P. Sotiriou, Vitor Cardoso, Kostas D. Kokkotas, Toby Crisford, Hideo Kodama) qaratilgan bo'lib, ularning naked singulyarliklar va wormholelardagi roliga oid juda kam bilimlar mavjud. Ushbu dissertatsiya aynan shu bo'shliqlarni to'ldirib, aniq analitik yechimlar (masalan, JNW fazosidagi ISCO radiuslari), zarra trayektoriyalarining sonli modellashtirishlari va perturbatsiya tahlillari (masalan, magnitlangan qora tuynuklardagi kvazinormal modlar)ni taqdim etadi. Skalyar-modifikatsiyalangan fazoviy-vaqtlarda kvazi-periodik tebranishlar (QPOs)ni o'rganish ayniqsa yangilik bo'lib, nazariy bashoratlarni X-nurlari manbalaridan kuzatilgan eksperimental ma'lumotlar bilan bog'lashga harakat qiladi — bu esa kam o'rganilgan. Fazoviy-vaqt parametrlarini aniqlashda MCMC (Markov Chain Monte Carlo) metodlarining qo'llanilishi esa zamonaviy, ma'lumotlarga asoslangan model sinovini ko'rsatadi. O'xshash tadqiqotlar bilan solishtirilganda, ushbu ish skalyar maydonlar, elektromagnit o'zaro ta'sirlar va nurlanish reaksiyasini birlashtirgan holda zich obyektlarning turli turlarini yagona nazariy doirada o'rganishi bilan ajralib turadi. Biroq, ba'zi cheklovlar ham mavjud: masalan, aks ta'sirning kuchsiz deb hisoblanishi (zarraning fazoviy-vaqt geometriyasiga ta’siri e’tiborga olinmagan) va statik, sferik simmetrik yechimlargina ko‘rib chiqilgan. Aylanuvchi zich obyektlar (masalan, Kerr-tabiatli naked singulyarliklar) va nolinear skalyar o‘zaro ta’sirlar hali ham ochiq muammoligicha qolmoqda. Kelajakdagi tadqiqotlar ushbu ishni sonli nisbiylik texnikalari yoki yuqori o‘lchamli nazariyalar (masalan, braneworld modellari)ni qo‘shgan holda davom ettirishi mumkin, bu esa astrofizikada yanada realistik holatlarda skalyar maydon dinamikasini chuqurroq o‘rganishga olib keladi. Ushbu dissertatsiya muammoning har tomonlama, ammo to‘liq bo‘lmagan tahlilini taqdim etib, analitik aniqlikni innovatsion ilovalar bilan uyg‘unlashtiradi hamda kelajakdagi tadqiqotlar uchun aniq yo‘nalishlarni belgilab beradi.

Dissertatsiya tadqiqotining dissertatsiya bajarilgan oliy ta’lim muassasasining ilmiy-tadqiqot ishlari rejalari bilan bog‘liqligi. Dissertatsiya Innovatsion rivojlanish vazirligi tomonidan moliyalashtirilgan F-FA-2021-510 "Modifikatsiyalangan gravitatsiyada neytron yulduzlar yadroviy materiyasini tadqiq qilish" ilmiy loyihasi doirasida bajarilgan(2021-2026).

Tadqiqotning maqsadi: skalyar maydon o‘zaro ta’sirlarini o‘rganish uchun yangi nazariy model ishlab chiqish va zich obyektlar yaqinidagi zarralar dinamikasi bo‘yicha yangi natijalarni aniqlashdir.

Tadqiqotning vazifalari:

− JNW va Ellis fazoviy-vaqtlarida skalyar maydonlar va zarra dinamikasi o‘rtasidagi o‘zaro ta’sirni hisobga oluvchi nazariy model ishlab chiqish;

− Skalyar maydon ta’sirida massiv zarralar harakati uchun harakat tenglamalarini chiqarish;

− Skalyar o‘zaro ta’sir parametrlarining ISCO (ichki barqaror aylanish orbitasi)ga ta’sirini tahlil qilish;

− Yalang‘och singulyarliklar yaqinida zarra trayektoriyalariga nurlanish reaksiyasining ta’sirini o‘rganish;

− QPO (kvazi-periodik tebranishlar) ilovalari uchun asosiy chastotalarni (orbital, radial, vertikal) hisoblash;

− X-nurli qo‘sh yulduzlar (binary) ma’lumotlari asosida fazoviy-vaqt parametrlarini cheklash uchun MCMC tahlilini o‘tkazish;

− Magnitlangan qurilma (wormhole) va qora tuynuk fazoviy-vaqtlarida zaryadlangan zarralar dinamikasini o‘rganish.

− Skalyar modifikatsiyalangan geometriyalarda perturbatsiya spektrini (kvazinormal modlar) o‘rganish.

Tadqiqot obyekti — skalyar maydon bilan o‘zaro ta’sirlanuvchi Janis–Nyuman– Vinikur (JNW) yalang‘och singulyarlik fazoviy-vaqti; tashqi skalyar maydonlar bilan o‘zaro ta’sirlanuvchi Ellis qurilma (wormhole) geometriyasi; tashqi magnit maydonga joylashgan Shvartsshildga o‘xshash muntazam qora tuynuklar; va ushbu modifikatsiyalangan fazoviy-vaqt geometriyalaridagi test zarralari (neytral va zaryadlangan).

Tadqiqot predmeti — ekzotik fazoviy-vaqtlarida skalyar maydon ta’siridagi massiv zarralar dinamikasi; skalyar o‘zaro ta’sirlar natijasida orbital mexanika va asosiy chastotalarning o‘zgarishi; skalyar maydon bilan modifikatsiyalangan zich obyektlarning astrofizik ahamiyati (QPOlar, ISCOlar va boshqalar); skalyar o‘zaro ta’sirli zich obyektlarning buzilish spektrlari va barqarorlik xossalari.

Tadqiqot usullari — skalyar maydon bilan bog‘langan fazoviy-vaqtlar (JNW, Ellis)da zarralar uchun aniq harakat tenglamalari va effektiv potensiallarni chiqarish uchun analitik modellashtirish qo‘llanildi. Geodezik tenglamalarni yechish va nurlanish reaksiyasi ta’siridagi zarra trayektoriyalarini tahlil qilish uchun sonli simulyatsiyalar (Runge-Kutta integrallash usuli)dan foydalanildi. Kvazinormal modlar va perturbatsiya spektrlarini hisoblashda yarim-analitik WKB yaqinlashuvi hamda vaqt sohasidagi integrallash usullari qo‘llanildi. X-nurli qo‘sh yulduzlardan olingan kuzatuv ma’lumotlariga nazariy QPO chastotalarini moslashtirish uchun Bayes MCMC (Markov zanjiri Monte Karlo) tahlili olib borildi.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

Analitik asoslar birlamchi fizik prinsiplar asosida qat’iy tarzda chiqarilgan bo‘lib, umumiy nisbiylik nazariyasidagi tan olingan natijalar bilan solishtirilib tekshirilgan, bu esa barcha hisob-kitoblarda matematik izchillikni ta’minlaydi.

Sonli simulyatsiyalar bir nechta mustaqil usullardan (Runge-Kutta integrallash, WKB yaqinlashuvi) foydalangan holda amalga oshirildi va ular o‘zaro mos natijalarni berdi, bu esa zarra dinamikasi va perturbatsiya tahlillarining barqarorligini tasdiqlaydi.

MCMC orqali fazoviy-vaqt parametrlarini baholash konvergensiya testlari va mavjud astrofizik cheklovlar bilan solishtirish orqali tekshirildi, bu esa skalyar maydon o‘zaro ta’siri qiymatlarining statistik ishonchliligini ko‘rsatadi.

Barcha nazariy bashoratlar X-nurli qo‘sh yulduzlar va ma’lum qora tuynuk fenomenologiyasi bo‘yicha kuzatuv ma’lumotlariga mos keladi, shu bilan birga kelajakda tekshirilishi mumkin bo‘lgan yangi og‘ishlarni ham aniq ko‘rsatib beradi.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

Skalyar-modifikatsiyalangan fazoviy-vaqtlardagi zarra dinamikasi uchun chiqarilgan analitik ifodalar ekzotik zich obyektlar atrofidagi akkretatsion disk tuzilmalari va nurlanish profillarini aniq modellashtirish imkonini beradi.

Hisoblangan QPO chastotalari va ISCO radiuslari qora tuynuklarni JNW yalang‘och singulyarliklari hamda Ellis qurilmalaridan X-nurli qo‘sh yulduzlar ma’lumotlari asosida farqlash uchun kuzatiladigan belgilarni taqdim etadi.

MCMC usuli orqali aniqlangan skalyar maydon parametrlari ($n \sim 0.75$, $g_s \sim 0.2$) kelajakdagi tortish kuchi to‘lqinlari va ko‘p-manbali astrofizika kampaniyalari uchun sinovdan o‘tkazilishi mumkin bo‘lgan cheklovlarni beradi.

Aniqlangan perturbatsiya spektrlari (masalan, confluent Heun yechimlari, tebranuvchi dumlar) skalyar-tenzor tortishish effektlarini yangi avlod observatoriyalarida aniqlash uchun mezon (benchmark) bo‘lib xizmat qiladi.

Tadqiqot natijalarining ishonchliligi quyidagilardan iborat:

Analitik model birlamchi fizik prinsiplar asosida qat’iy tarzda chiqarilgan bo‘lib, umumiy nisbiylik nazariyasidagi mavjud natijalar bilan solishtirib tekshirilgan, bu esa barcha hisob-kitoblarda matematik izchillikni kafolatlaydi.

Sonli simulyatsiyalarda bir nechta mustaqil usullar (Runge-Kutta integrallash, WKB yaqinlashuvi) qo‘llanilgan bo‘lib, ular bir-birini tasdiqlovchi barqaror natijalar bergan, bu esa zarra dinamikasi va perturbatsiya tahlilining ishonchliligini tasdiqlaydi.

MCMC yordamida aniqlangan parametrlar konvergensiya testlari va mavjud astrofizik cheklovlar bilan solishtirish orqali tekshirildi, bu esa skalyar maydon o‘zaro ta’sir qiymatlarining statistik ishonchliligini ko‘rsatadi.

Barcha nazariy bashoratlar X-nurli qo‘sh yulduzlar va qora tuynuklar fenomenologiyasi bo‘yicha kuzatuv ma’lumotlari bilan mos keladi hamda kelajakda eksperimental tekshiruvlar uchun test qilinishi mumkin bo‘lgan og‘ishlarni aniq ko‘rsatib beradi.

Tadqiqotning ilmiy yangiligi quyidagilardan iborat:

Tadqiqot ekzotik zich obyektlardagi skalyar maydon effektlarini tahlil qilish uchun keng qamrovli nazariy asos yaratadi hamda klassik qora tuynuk paradigmalaridan tashqaridagi tortishish va fundamental maydonlar o‘rtasidagi o‘zaro ta’sirga yangi tushunchalarni taqdim etadi.

Hisoblangan QPO chastotalari va ISCO o‘zgarishlari qora tuynuklar, yalang‘och singulyarliklar va qurilmalarni farqlash uchun muhim kuzatuv mezonlari bo‘lib, hozirgi X-nur teleskoplari ma’lumotlarini tahlil qilishda bevosita qo‘llanishi mumkin.

Ishlab chiqilgan MCMC-QPO tahlil texnikasi va perturbatsiya usullari astrofizik ma’lumotlar asosida modifikatsiyalangan tortishish nazariyalarini cheklash uchun kuchli vositalarni taqdim etadi va tortishish nazariyasini aniqlik bilan sinovdan o‘tkazish imkoniyatlarini sezilarli darajada kengaytiradi.

Skalyar-tenzor o‘zaro ta’sirlarning o‘ziga xos spektral belgilarini aniqlash orqali bu natijalar yangi avlod tortish kuchi to‘lqinlari detektorlarini loyihalash talablari va ko‘p-manbali astronomiya kampaniyalari uchun muhim ma’lumot beradi.

Tadqiqotning amaliy natijalari quyidagilardan iborat:

Modifikatsiyalangan tortishish nazariyalarida qora tuynuklar atrofidagi zarralar dinamikasini o‘rganish natijalari quyidagi yo‘nalishlarda qo‘llanilgan:

Nazariy tadqiqot natijalari va usullari “A. Davlataliev, B. Narzilloev, I. Hussain, A. Abdujabbarov, B. Ahmedov, “Probing the Starobinsky-Bel-Robinson gravity by photon motion around the Kerr-type black hole in non-uniform plasma,” // *Phys. Dark Univ.* 42 (2023) 101340, https://doi.org/10.1016/j.dark.2023.101340” nomli ilmiy maqolada e’lon qilingan va A. Davlatalievning falsafa doktori (PhD) dissertatsiyasida taqdim etilgan.

Mazkur ilmiy natijalar Fudan universiteti tomonidan qo‘llab-quvvatlangan dasturlar doirasida qo‘llanilgan (Prof. Cosimo Bambi tomonidan taqdim etilgan rasmiy xat asosida).

Tadqiqot natijalarining aprobatsiyasi. Dissertatsiya natijalari 3 ta xalqaro va 1 ta mahalliy konferensiyalarda muhokama qilingan.

Tadqiqot natijalarining e'lon qilinganligi.

Tadqiqot natijalariga asosan 11 ta ilmiy nashr chop etilgan bo'lib, ularning barchasi xorijiy jurnallarda e'lon qilingan maqolalardir.

Dissertatsiya hajmi va tuzilishi

Dissertatsiya kirish qismi, to'rt bob, xulosa va adabiyotlar ro'yxatidan iborat bo'lib, jami 121 betni tashkil etadi.

DISSERTATSIYANING ASOSIY MAZMUNI

Dissertatsiyaning kirish qismida mavzuning dolzarbligi va zarurligi, tadqiqotning Respublika fan va texnologiyalarini rivojlantirishning ustuvor yo'nalishlariga muvofiqligi, muammo darajasi va uning tadqiqot olib borilgan oliy ta'lim muassasasining ilmiy rejalari bilan aloqasi, tadqiqotning maqsadi, vazifalari, obyekti, predmeti, tadqiqot usullari, ilmiy yangiligi, amaliy natijasi, natijalarning ishonchliligi, ilmiy va amaliy ahamiyati, amaliyotga joriy etilishi, tasdiqlanishi, chop etilishi hamda dissertatsiyaning tuzilishi va hajmi haqida qisqacha ma'lumot berilgan.

Dissertatsiyaning birinchi bobi — "JNW fazosida skalyar maydon ta'sirida massiv zarralar harakati" deb nomlanib, unda Janis–Nyuman–Vinikur (JNW) yalang'och singulyarligi atrofidagi zarra dinamikasi o'rganiladi. JNW yalang'och singulyarligi — bu massaga ega bo'lmagan skalyar maydon ishtirokidagi Eynshteyn tenglamalarining aniq yechimidir. Bu bobda Eynshteyn–skalyar maydon–massiv zarra sistemasi qisqacha tavsiflanadi va butun tizim uchun harakat tenglamasi chiqariladi. Shuningdek, JNW fazoviy-vaqtida skalyar maydon ishtirokida massiv zarralarning aylana trayektoriyasi tahlil qilinadi. Bundan tashqari, nurlanish reaksiyasi (radiation reaction) a'zosi hisobga olingan holda zarralarning aylana harakati va ularning trayektoriyalari o'rganilgan.

Eynshteyn-skalyar maydon tenglamalarining eng oddiy yechimlaridan biri — bu Janis–Nyuman–Vinikur yalang'och singulyarligidir. Unga mos fazoviy-vaqt metrikasi quyidagicha ifodalanadi:

$$\begin{aligned} ds^2 &= -\, f^n dt^2 + f^{-n} dr^2 + f^{1-n} r^2 (d\theta^2 + \sin^2\theta d\phi^2)\ , \\ \varphi(r) &= \frac{\sqrt{1-n^2}}{2} \ln f\ , \qquad f = 1 - \frac{2M}{nr}\ , \end{aligned} \tag{1}$$

bu yerda M — tortishish obyektining massasi, n esa skalyar maydonning parametri hisoblanadi. Fazoviy-vaqtning singulyarligi $r_* = 2M/n$ nuqtada joylashgan.

Massiv zarra uchun harakat tenglamasi quyidagicha aniq ifodalanadi:

$$\ddot{t} + \frac{2M\dot{t}\dot{r}}{fr^2}(1-B) = 0 \ , \tag{2}$$

$$\ddot{r} + \frac{Mf^n}{fr^2}\left(f^n\dot{t}^2 - 2B\right) - \frac{M\dot{r}^2}{fr^2}(1+2B) \tag{3}$$

$$+ \left(\frac{M(n+1)}{n} - r\right)\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right) = 0 \ ,$$

$$\ddot{\theta} - \frac{1}{2}\sin 2\theta\dot{\phi}^2 - \frac{2M\dot{\theta}\dot{r}}{fr^2}\left(1 + \frac{1}{n} - \frac{r}{M} + B\right) = 0 \ , \tag{4}$$

$$\ddot{\phi} + 2\cot\theta\dot{\theta}\dot{\phi} - \frac{2M\dot{r}\phi}{fr^2}\left(1 + \frac{1}{n} - \frac{r}{M} + B\right) = 0 \ , \tag{5}$$

bu yerda B

$$B = \frac{2g_s\sqrt{1-n^2}}{n(g_s\sqrt{1-n^2}\ln f + 2)} \ . \tag{6}$$

1-rasmda biz skalyar parametr nn ning qiymatiga qarab, turli xil o‘zaro ta’sir parametrlari g_s uchun massiv zarraning ISCO (ichki barqaror aylanish orbitasi) pozitsiyasining bog‘liqligini ko‘rsatamiz.

2-rasmda esa skalyar maydon ishtirokida, shu jumladan nurlanish reaksiyasi a’zosi hisobga olingan holda, massiv zarraning trayektoriyasi tasvirlangan. Sonli hisob-kitoblar yordamida shuni ko‘rsatdikki, nurlanish reaksiyasining hissasi unchalik katta emas. Shu sababli, aylana harakatdagi nurlanish reaksiyasining ta’sirini yaqqol ko‘rish uchun biz susayish vaqtini τ_0 =0 s ga, τ_0=0.5 s ga teng deb oldik. Bu esa susayish vaqti 1 sekunddan ancha kichik ekanligini anglatadi, va zarra 2rasmda ko‘rsatilganidan ko‘ra ko‘proq marta JNW fazoviy-vaqti bilan ifodalangan zich obyekt atrofida aylanishga ulgursa-da, nurlanish reaksiyasi kuchi ta’sirida oxiroqibat tizimni tark etadi. 3-rasmda esa massiv zarraning ikki xil holatdagi trayektoriyasi ko‘rsatilgan:

(i) skalyar maydonning ta’siri hisobga olingan, biroq nurlanish reaksiyasi hisobga olinmagan holat;

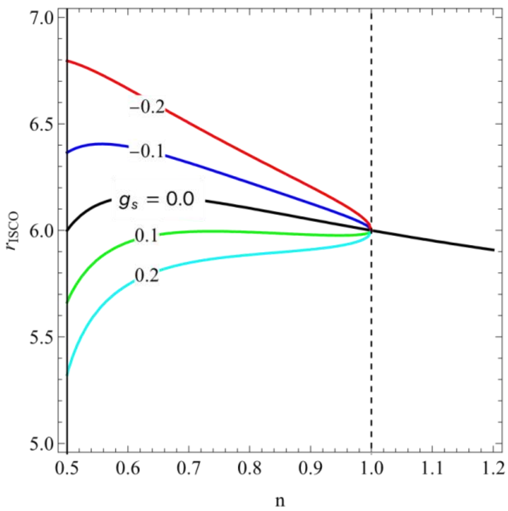
(ii) skalyar maydonning ta’siri hisobga olinmagan, lekin nurlanish reaksiyasi a’zosi kiritilgan holat.

Bu natijalar shuni ko‘rsatadiki, skalyar maydon haqiqatan ham tortuvchi kuch hosil qiladi, holbuki nurlanish reaksiyasi massiv zarra harakatida itaruvchi (repulsiv) kuch sifatida namoyon bo‘ladi.

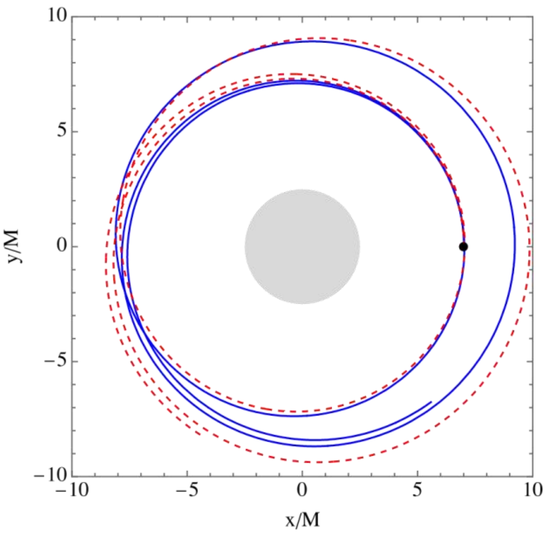
Bizning natijalarimiz shuni ko‘rsatadiki, JNW fazosidagi yalang‘och singulyarlik atrofida harakatlanayotgan massiv zarra trayektoriyasi skalyar maydon tomonidan yaratilgan tortuvchi kuch hamda nurlanish reaksiyasi tomonidan ta’sir qilinuvchi itaruvchi kuchga bog‘liq. Bu holat sonli simulyatsiyalarimiz orqali aniq namoyish etilgan.

Dissertatsiyaning ikkinchi bobi — “JNW fazosida skalyar maydon ta’sirida massiv zarra harakati” deb nomlanadi. Ushbu bobda zarra dinamikasi natijalari JNW zich obyektlari atrofidagi kvaziperiodik tebranishlarni (QPO) oʻrganishda qoʻllanildi. Biz JNW fazoviy-vaqtida barqaror aylana orbitaga yaqin joylashgan massiv zarraning tebranma harakatini koʻrib chiqdik. Astrofizik jihatdan, massiv zarraning tebranma harakatiga asoslangan egizak choʻqqili QPO hodisasi tahlil qilindi va bu natijalardan foydalanib markaziy obyektning massasi, skalyar parametr nn va bogʻlanish doimiysi gs kabi ikki asosiy parametrga cheklovlar olindi.

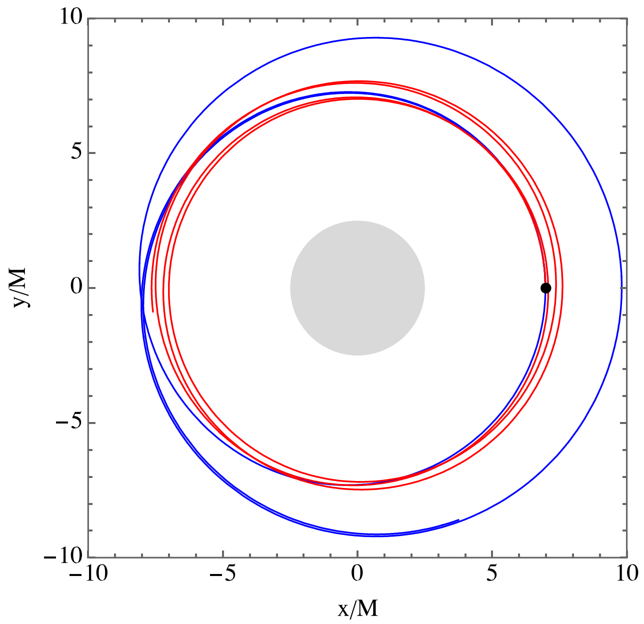
Gravitatsion zich obyektlar atrofida test zarralarning tebranma harakati nisbiylik astrofizikasining muhim jihatlaridan biridir. Bu harakat odatda ikki turga boʻlinadi: (i) radial va (ii) vertikal tebranishlar, va bu tebranishlar fazoviy-vaqtning tuzilishi va obyekt dinamikasini aniqlashda muhim ahamiyatga ega.



1-rasm: Massiv zarraning ISCO pozitsiyasining parametr n ga bogʻliqligi turli bogʻlanish parametrlari gsgs qiymatlari uchun.



2-rasm: JNW yalangʻoch singulyarligi yaqinidagi zarra trayektoriyalari: nurlanish reaksiyasi a’zosisiz (τ_0=0, uzluksiz koʻk chiziq) va u bilan (τ_0=1.0, uzilgan qizil chiziq) holatlar uchun, parametrlar g_s=0.03 va n=0.8 tanlanganda.



3-rasm: JNW yalang‘och singulyarligi yaqinidagi zarra trayektoriyalari uchun tanlangan parametrlar holatida: ko‘k uzluksiz chiziq g_s=0.03, τ_0=0 holatni; qizil uzluksiz chiziq esa g_s=0, τ_0=0.5 holatni ifodalaydi.

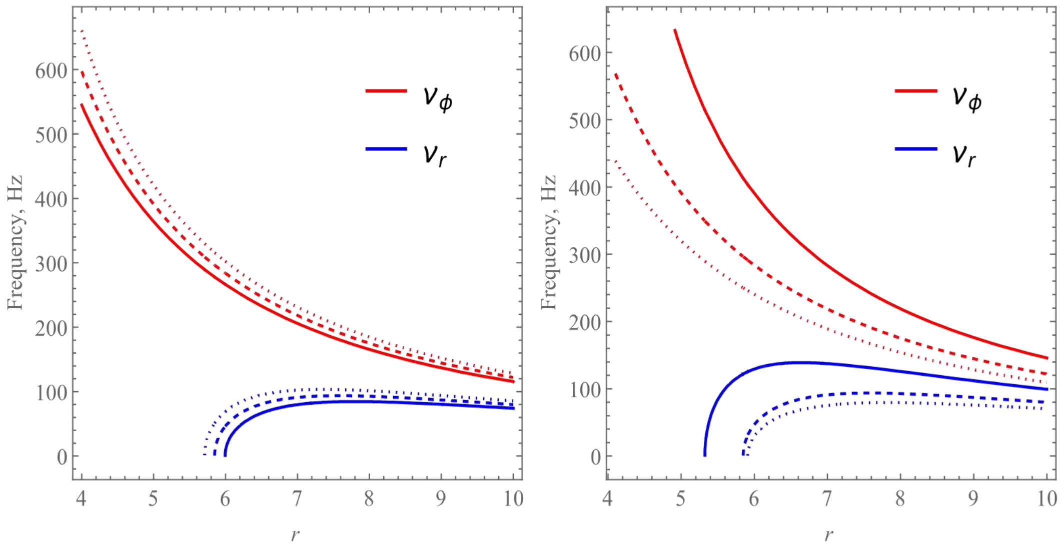
(i) Radial tebranishlar — zarra markaziy obyektga nisbatan oldinga-orqaga (yaqinlashish-va-uzoqlashish) yo‘nalishdagi harakatini bildiradi. Bunday tebranishlar barqaror aylana orbitalar atrofida sodir bo‘ladi. Bu tebranishlarning barqarorligi va chastotasi fazoviy-vaqt geometriyasi va zich obyektning xossalariga bog‘liq. Masalan, Shvartsshild qora tuynugi holatida radial tebranish chastotasi qora tuynuk massasiga va zarraning undan bo‘lgan radial masofasiga bog‘liq bo‘ladi. Qora tuynukka yaqinlashgan sari tortishish kuchi ortadi va bu tebranish chastotasining oshishiga olib keladi.

(ii) Vertikal tebranishlar — orbitaning tekisligiga nisbatan perpendikulyar yo‘nalishda sodir bo‘ladigan harakatdir. Bu harakatlar ba’zida latitudinal yoki epitsiklik tebranishlar deb ham yuritiladi. Ular orbitaning vertikal barqarorligi haqida ma’lumot beradi va markaziy obyektning burchak impulsi hamda aylanishiga bog‘liq. Vertikal epitsiklik chastota — bu zarra markaziy obyektning ekvatorial tekisligi atrofida vertikal ravishda tebranadigan chastotadir.

Bu ikki turdagi chastotalar orbital (yoki Kepler) chastotasi bilan bog‘langan bo‘lib, ular birgalikda asosiy chastotalardeb yuritiladi. Shvartsshild qora tuynugi uchun bu chastotalar orasida $\Omega_\theta=\Omega_K$ bog‘lanishi mavjud bo‘lib, bu yerda Ω_K — Shvartsshild fazosidagi Kepler chastotasidir.

4-rasmda asosiy chastotalarning radial bog‘liqligi maxsus parametr sozlamalari asosida tasvirlangan. Chap panelda skalyar bog‘lanish parametri gsgs ning turli qiymatlari uchun, n parametri o‘zgarmas keltirilgan qiymatida grafiklar ko‘rsatilgan. O‘ng panelda esa g_s parametri qo‘zgarmas keltirilgan qiymatida n ning turli qiymatlari uchun grafiklar berilgan. Grafiklardan ko‘rinadiki, skalyar maydon bilan o‘zaro ta’sir sababli asosiy chastotalar ortadi, biroq nn parametri oshishi bilan ularning qiymatlari kamayadi.

Orbital, radial va vertikal tebranishlarning birgalikdagi ta’siri test zarralar uchun murakkab trayektoriyalarni hosil qiladi. Bu tebranishlar X-nurli qo‘sh yulduzlarda kuzatiladigan kvaziperiodik tebranishlar (QPO) ni tushunishda muhim rol o‘ynaydi. QPO hodisalari qora tuynuk yoki neytron yulduz atrofidagi akkretatsion diskdagi moddaning tebranuvchi harakatiga bog‘liq deb hisoblanadi. Tebranish harakati akkretatsion disklarning barqarorligi va tuzilmasiga ta’sir qiladi. Ushbu tebranishlarni o‘rganish nurlanish spektrlari va disklarning o‘zgaruvchanligini modellashtirishga yordam beradi. Kuzatilayotgan chastotalar va tebranish modlarini tahlil qilish qora tuynuklar va neytron yulduzlarning massasi, aylanishi va atrofdagi fazoviy-vaqt geometriyasi haqida muhim ma’lumot beradi.

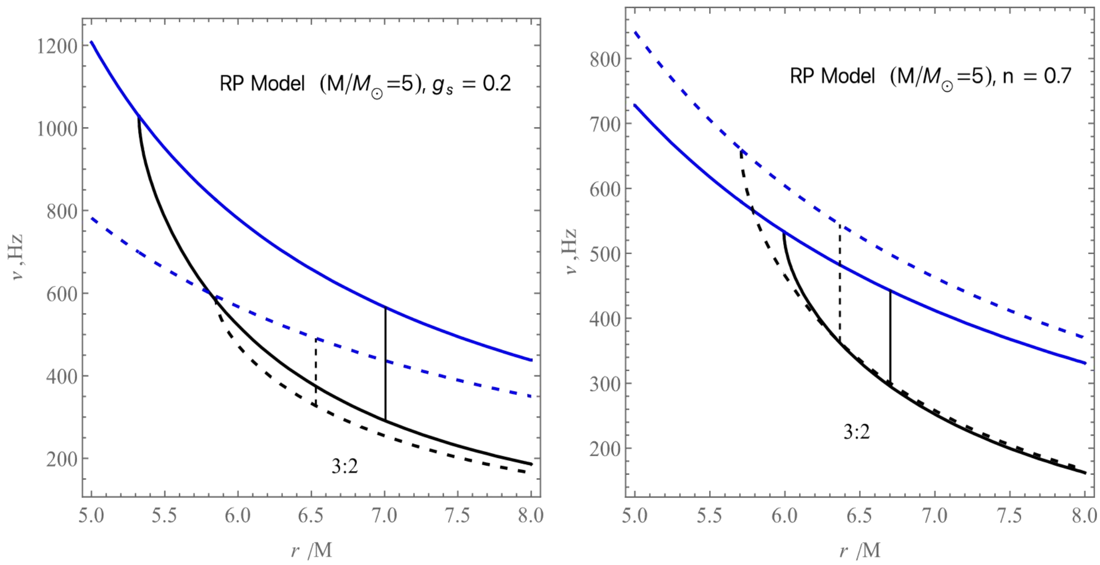


4-rasm: (Chap panel) – Massiv zarraning asosiy chastotalarining radial bog‘liqligi turli bog‘lanish parametrlari qiymatlari uchun: g_s=0.1— uzluksiz chiziq, g_s=0.2— uzilgan chiziq, va g_s=0.3 — nuqtali chiziq holatida; bu yerda n=0.7qat’iy olingan. **(**O‘ng panel) – Massiv zarraning asosiy chastotalarining radial bog‘liqligi turli nn parametr qiymatlari uchun: n=0.5 — uzluksiz chiziq, n=0.7 — uzilgan chiziq, va n=0.9— nuqtali chiziq holatida; bu yerda g_s=0.2. Har ikkala holatda ham markaziy obyekt massasi M=10M⊙ deb olingan.

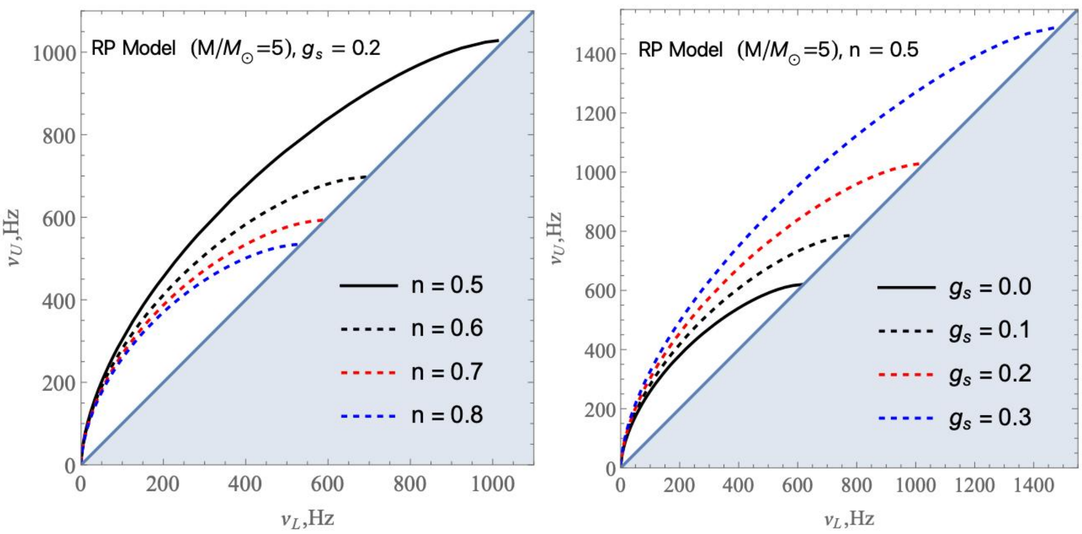
Relativistik pretsessiya (RP) modeli kvaziperiodik tebranishlar (QPO) hodisasini tushuntirish uchun nazariy asos bo‘lib xizmat qiladi. Ushbu modelga ko‘ra, QPOlar qora tuynuklar va qurilmalar atrofida harakatlanayotgan zaryadlangan zarralarning radial va burchak bo‘yicha kvazigarmonik tebranishlari bilan bog‘liq. RP model doirasida egizak cho‘qqili QPOlar quyidagicha talqin qilinadi: yuqori chastota zarra orbital chastotasiga to‘g‘ri keladi — bu $\nu_U=\nu_\varphi$ bilan belgilanadi; pastki chastota esa orbital va radial chastotalar farqi sifatida ifodalanadi — $\nu_L=\nu_\varphi-\nu_r$. Boshqacha aytganda, egizak cho‘qqili QPOlarda yuqori chastota zarra aylanish chastotasini, pastki chastota esa radial tebranish chastotasini ayirish orqali olinadi.

5-rasm RP modeli doirasida egizak QPOlarning yuqori (ko‘k chiziq) va pastki (qora chiziq) chastotalarining radial bog‘liqligini ko‘rsatadi. Chap panelda uzluksiz chiziq n=0.5 holatini, uzilgan chiziq esa n=0.7 holatini tasvirlaydi, bu yerda g_s=0.2 qat’iy saqlangan. O‘ng panelda esa uzluksiz chiziq g_s=0.1, uzilgan chiziq esa g_s =0.3 qiymatlariga mos keladi, bu yerda n=0.7 qat’iy olingan. Grafiklardan ko‘rinib turibdiki, 3:2 chastotalar nisbati sodir bo‘ladigan radial pozitsiya n va g_s parametrlari oshgan sari yalang‘och singulyarlik tomon biroz siljiydi.

6-rasmda esa turli n va g_s qiymatlari uchun egizak cho‘qqili QPOlarning yuqori va pastki chastotalari orasidagi bog‘liqlik ko‘rib chiqilgan. Tahlil shuni ko‘rsatadiki, nn ortgani sari yuqori hamda pastki chastotalar kamayadi. Aksincha, gsgs ortishi bu chastotalarning oshishiga olib keladi. Grafiklardan shuni ham ko‘rish mumkin: 6-rasmda soya bilan belgilangan hudud ichida chastotalar kuzatib bo‘lmaydi, sababi bu oraliqlar ISCO pozitsiyasining ichki sohasiga to‘g‘ri keladi.



5-rasm: RP modelida 3:2 nisbatda kuzatiladigan yuqori va pastki chastotalarning radial bog‘liqligi.



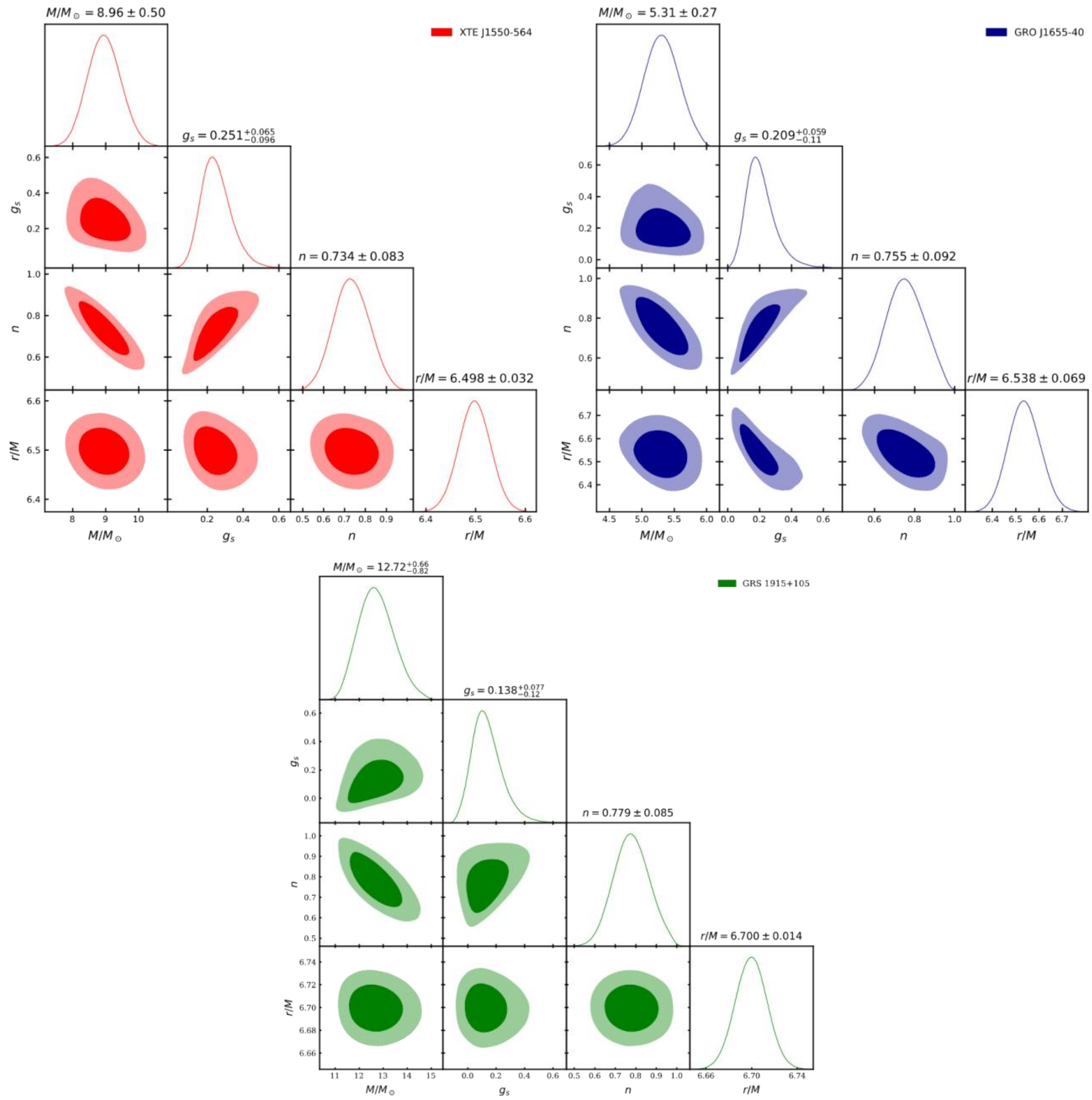
6-rasm: RP modelida massa bilan bog‘liq egizak cho‘qqili QPOlarning yuqori va pastki chastotalari orasidagi o‘zaro bog‘liqlik M = 5M⊙

7-rasmda ilgari tanlab olingan uchta turli manba uchun JNW fazoviy-vaqtiga o‘xshash parametrlar (M, g_s, n, r) uchun eng yaxshi mos keluvchi qiymatlar burchakli (corner) grafiklar orqali tasvirlangan. Bu natijalar MCMC tahlillari yordamida 1σ ishonchlilik darajasida olingan. Mos keluvchi eng yaxshi qiymatlar 1-jadvalda keltirilgan.

Olingan natijalar har bir mikrokvazar uchun JNW parametrlariga cheklovlar qo‘yadi. Shuningdek, QPO ma’lumotlari asosida aniqlangan eng yaxshi mos parametrlar qiymatlari ham keltirilib, bu RP modelining mazkur X-nurli qo‘sh yulduz tizimlarida QPO hodisasini tavsiflashdagi samaradorligini ko‘rsatadi. Ushbu tadqiqot kuchli tortishish maydonlaridagi zich obyektlar dinamikasi va ularning xossalarini chuqurroq tushunishga xizmat qiladi.

1-Jadval: Tanlangan X-nurli manbalar uchun kvaziperiodik tebranishlar (QPO) asosida aniqlangan JNW fazoviy-vaqtiga mos eng yaxshi parametr qiymatlari.

	XTE J1550-564	GRO J1655-40	GRS 1915+105
$M(M_\odot)$	8.96 ± 0.50	5.31 ± 0.27	$12.72^{+0.66}_{-0.82}$
g_s	$0.251^{+0.065}_{-0.096}$	$0.209^{+0.059}_{-0.11}$	$0.138^{+0.077}_{-0.12}$
n	0.734 ± 0.083	0.755 ± 0.092	$0.779^{\pm}0.085$
r/M	6.498 ± 0.032	6.538 ± 0.069	6.700 ± 0.014



7-rasm. JNW massasiga, shuningdek**,** gsgs va n parametrlariga qo'yilgan cheklovlar: XTE J1550-564 (yuqori chap), GRO J1655-40 (yuqori o'ng) va GRS 1915+105 (pastki panel) mikrokvazarlari uchun MCMC tahlili yordamida aniqlangan.

Dissertatsiyaning uchinchi bobi — "Ellis fazoviy-vaqtining yangi xususiyatlarini o'rganish: skalyar maydon dinamikasiga oid qarashlar" deb nomlanadi. Ushbu bobda biz Ellis fazoviy-vaqtini tashqi skalyar maydon ishtirokida chuqurcha (wormhole) atrofida zarra harakatini ko'rib chiqish orqali sinovdan o'tkazdik. Tashqi skalyar maydon mavjudligida fon fazoviy-vaqtiga va test zarraning dinamik harakatiga tegishli asosiy tenglamalar keltirildi. Shuningdek, zarraning harakati va unga nurlanish reaksiyasi ta'siri ham o'rganildi.

Ellis chuqurchasi (wormhole) quyidagi fazoviy-vaqt chiziqli element (line element) bilan ifodalanadi:

$$ds^2 = -dt^2 + dr^2 + (r^2 + r_0^2)(d\theta^2 + \sin^2\theta d\phi^2)\ , \tag{7}$$

tegishli skalyar maydon bilan birgalikda

$$\Phi = \frac{\pi}{2} - \tan^{-1}\left(\frac{r}{r_0}\right)\ , \tag{8}$$

bu yerda r_0 — chuqurchaning (wormhole) bo‘yin (tor yo‘lak) radiusi hisoblanadi. Radial koordinata r, r0 dan boshlab cheksizlikkacha o‘zgaradi, ya’ni $r_0<r<\infty$. Fazo-vaqtning skalyar invariantlari, masalan, Ritchi skalyari va Kretschmann skalyari quyidagicha ifodalash mumkin:

$$R=-\frac{2r_0^2}{(r^2+r_0^2)^2}\ , \qquad K=\frac{12r_0^4}{(r+r_0^2)^4}\ , \tag{9}$$

bu invariantlar $r>r_0$ bo‘lgan fazoviy-vaqtning istalgan nuqtasida regulyar **(**singulyarsiz**)** hisoblanadi. 8-rasmda skalyar maydon, shuningdek, **Ritchi** va **Kretschmann** invariantlarining radial bog‘liqligi tasvirlangan.

Bundan tashqari, Ellis fazoviy-vaqtida massiv zarraning asosiy chastotalari**,** ya’ni orbital va epitsiklik chastotalari ham muhokama qilindi. 9-rasmda orbital, radial va vertikal chastotalarning radial bog‘liqligi grafik tarzda ko‘rsatilgan.

Tashqi magnit maydon mavjud bo‘lgan holatda zaryadlangan zarra uchun Lagranj funksiyasi quyidagicha ifodalanadi:

$$L=\frac{1}{2}m_* g_{\mu\nu}u^\mu u^\nu+qA_\mu u^\mu\ , \tag{10}$$

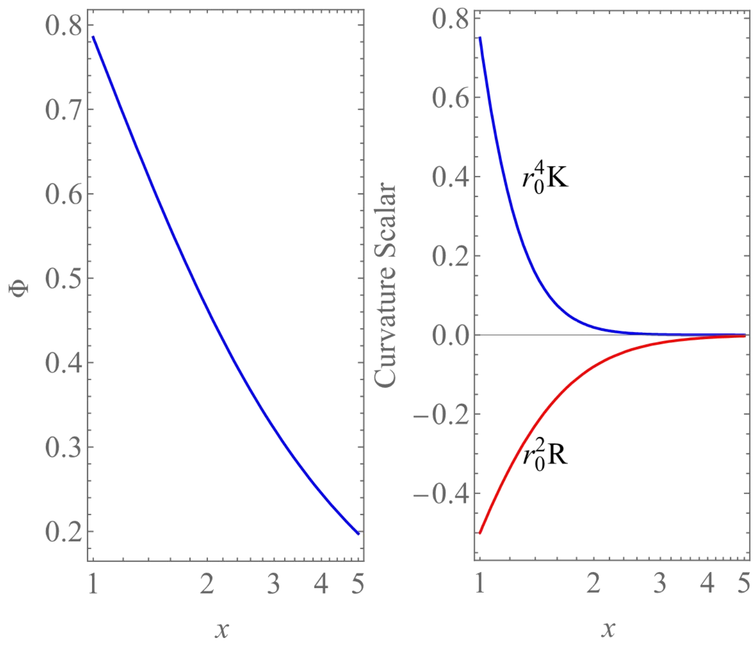
bu yerda q — test zarraning zaryadi. Lagrang tenglamasidan kelib chiqqan harakat tenglamasi quyidagicha hosil qilinadi:

$$\frac{Du^\mu}{d\tau}=\frac{q}{m}F^\mu{}_\nu u^\nu+(g^{\mu\nu}+u^\mu u^\nu)\partial_\nu \ln\frac{m_*}{m}\ , \tag{11}$$

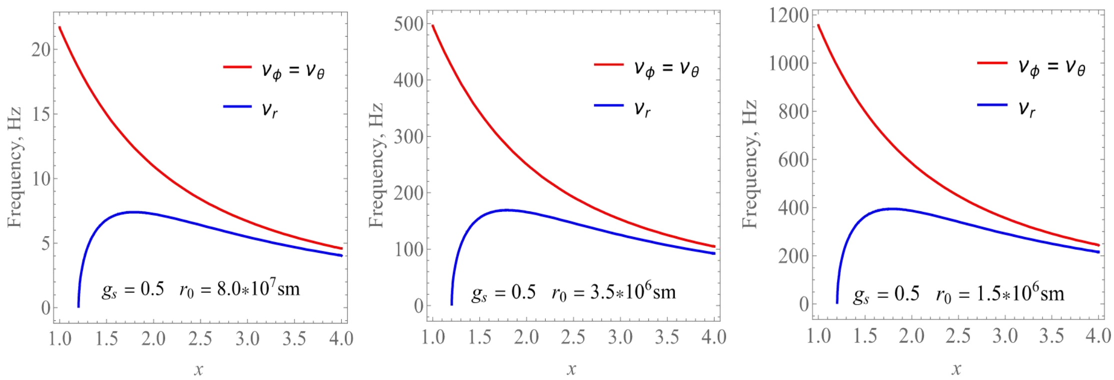
hamda oddiy algebraik o‘zgartirishlardan so‘ng, radial harakat tenglamasi quyidagi ko‘rinishni oladi:

$$\dot{r}^2=\frac{1}{(1+\mathrm{g}_s\Phi)^2}\left[\mathcal{E}^2-\left(r^2+r_0^2\right)\left(\frac{\mathcal{L}}{r^2+r_0^2}-\omega\right)^2\right]-1\ , \tag{12}$$

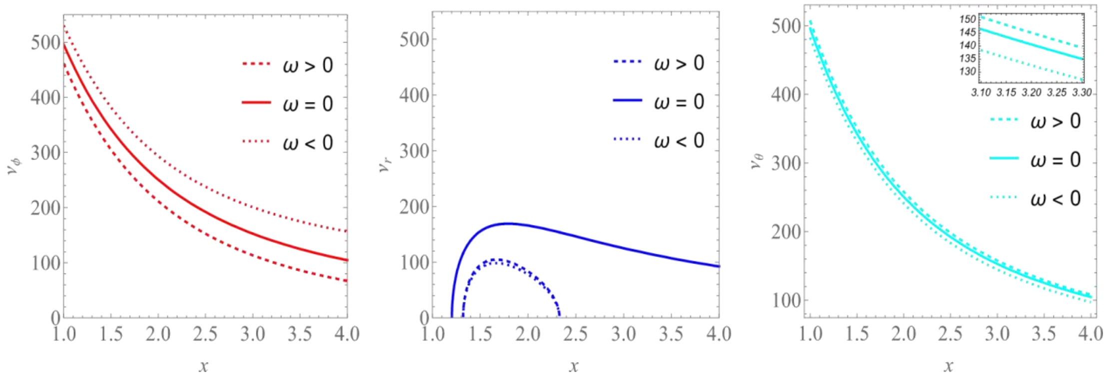
bu yerda $\omega=qB/2m$— magnit parametri hisoblanadi. Zaryadlangan test zarrasi uchun chegaraviy bog‘langan aylana orbitaning radiusi quyidagi shartlar asosida aniqlanadi:



8-rasm: Skalyar maydon Φ(x) ning va egri chiziqlik invariantlari o‘lchamsiz Ritchi skalyari hamda o‘lchamsiz Kretschmann skalyari ning radial bog‘liqligi.



9-rasm: Massiv zarraning asosiy chastotalarining radial bog‘liqligi.



10-rasm: Zaryadlangan zarraning Ellis chuqurchasi atrofidagi asosiy chastotalari uchun ω=±0.4va bog‘lanish parametri g_s=0.5 deb olingan.

$$\mathrm{g}_s x + \left(1 - x^2\right)\left(1 + \mathrm{g}_s \Phi\right) + 4\omega^2 x^3 (1 + x^2) = 0 \ . \tag{13}$$

Ellis chuqurchasi yaqinida tashqi magnit maydon ishtirokida zaryadlangan zarraning asosiy chastotalarini (orbital, radial va vertikal) aniqlash oson. Kepler chastotasi (orbital chastota) quyidagi ifodadan hosil qilinadi:

$$\Omega^2 + \omega\Omega\sqrt{1 - \Omega^2(x^2+1)} = \frac{\mathrm{g}_s\left[1 - \Omega^2(x^2+1)\right]}{x(x^2+1)(1+\mathrm{g}_s\Phi)} \ . \tag{14}$$

Xuddi shuningdek, Ellis chuqurchasi atrofida tashqi magnit maydon ishtirokida harakatlanuvchi zaryadlangan zarraning epitsiklik chastotalari ham aniqlanishi mumkin. Biroq, biz bu yerda batafsil hisob-kitoblarni keltirmaymiz. 10-rasmda zaryadlangan zarraning asosiy chastotalarining radial bog‘liqligi ko‘rsatilgan. Rasmga ko‘ra, tashqi magnit maydon ta’sirida asosiy chastotalar bir-biridan ajraladi (ya’ni, chastotalarning ajralishi sodir bo‘ladi). Shu bilan birga, natijalarimiz Kepler chastotalari magnit parametriga kuchli bog‘liqligini ko‘rsatadi.

Qora tuynuklardagi buzilish (perturbatsiya) hozirgi astrofizikadagi dolzarb mavzulardan biridir, ayniqsa, LIGO va Virgo hamkorliklari ikkilik tizimlardan gravitatsion to‘lqinlarni qayd eta boshlagandan so‘ng. Bunday perturbatsiyalar chuqurcha (wormhole) fazoviy-vaqtiga ham qo‘llanilishi mumkin. Bu yerda biz Ellis fazoviy-vaqtining perturbatsiyasini ko‘rib chiqamiz.

Yuqorida ta’kidlanganidek, Ellis fazoviy-vaqti — bu Eynshteyn-skalyar maydon tenglamalarining yechimidir. Shuning uchun skalyar maydon va metrik tensor quyidagicha ifodalanadi:

$$\Phi=\Phi_0+\delta\Phi, \quad g_{\mu\nu}=g_{\mu\nu}(0)+\delta g_{\mu\nu}$$

bu yerda Φ_0 va $g_{\mu\nu}(0)$ — mos ravishda skalyar maydon va fon fazoviy-vaqt metrikasi (7) va (8)-tenglamalarda berilgan, $\delta\Phi$ esa buzilgan (perturbatsiyalangan) skalyar maydonni bildiradi.

$$\delta\Phi = e^{-i\omega t} F(r) P_\ell(\cos\theta) \ , \tag{15}$$

hamda buzilgan metrik tenzor komponentalari quyidagicha aniqlanadi:

$$h_{\mu\nu} = e^{-i\omega t}\begin{pmatrix} H_0 & H_1 & 0 & 0 \\ H_1 & H_2 & 0 & 0 \\ 0 & 0 & K(r^2+r_0^2) & 0 \\ 0 & 0 & 0 & K(r^2+r_0^2)\sin^2\theta \end{pmatrix} P_\ell(\cos\theta) \ , \tag{16}$$

bu yerda $H_0(r)$, $H_1(r)$, $H_2(r)$, $K(r)$, $F(r)$ — hali aniqlanmagan radial funksiyalar, $P_\ell=P_\ell(\cos\theta)$ esa Legendre polinomi bo‘lib, u quyidagi differensial tenglamani qanoatlantiradi:

$$\frac{1}{\sin\theta}\frac{d}{d\theta}(\sin\theta\frac{d}{d\theta})P_\ell+\ell(\ell+1)P_\ell=0\ . \tag{17}$$

Biz endi radial funksiyalar uchun maydon tenglamalarining to‘lqinli yechimlarini ko‘rib chiqamiz. Keyingi tahlillarga o‘tishdan oldin, quyidagicha ta’riflanuvchi o‘lchamsiz chastota kiritiladi: $\omega_0=\omega_{r0}$.Bu yerda ω — fizik chastota, r_0 esa chuqurchaning bo‘yin radiusi. F funksiyasi uchun tenglama esa quyidagicha oson hosil qilinadi:

$$\left[\left(x^2+1\right)F'\right]'+\left[\omega_0^2x^2+\frac{4}{x^2+1}-4\eta\right]F=0\ , \tag{18}$$

bu yerda η=ℓ²+ℓ— separatsiya konstantasi bo‘lib, yuqoridagi tenglama uchun analitik yechimbirlashtirilgan Heun funksiyasi (ya’ni, HeunC(a,b,c,d,x) yordamida quyidagicha yozilishi mumkin:

$$F(r)=\left(x^2+1\right)\left[c_1F_{1\ell}(x)+c_2xF_{2\ell}(x)\right]\ , \tag{19}$$

bu yerda

$$F_{1\ell}(x)=\mathrm{HeunC}\left[\eta-\frac{3}{2},-\frac{\omega_0^2}{4},\frac{1}{2},3,0,-x^2\right]\ , \tag{20}$$

$$F_{2\ell}(x)=\mathrm{HeunC}\left[\eta-3,-\frac{\omega_0^2}{4},\frac{3}{2},3,0,-x^2\right]\ . \tag{21}$$

Biz aniqladikki, Ellis fazoviy-vaqtida skalyar perturbatsiyaning yechimi aniq analitik ifoda bilan – ya’ni birlashtirilgan Heun funksiyasi bilan tasvirlanadi. 11-rasmda F(x) funksiyasining radial bog‘liqligi keltirilgan. Grafikdan ko‘rinib turibdiki, radial profil funksiyasi F(x) tebranma harakat qilgan holda asta-sekin kamayib boradi.

Ushbu tadqiqot doirasida biz Ellis fazoviy-vaqtidagi skalyar va gravitatsion perturbatsiyalarni tahlil qildik. Faraz qilamizki, skalyar va gravitatsion to‘lqinlar bir xil chastotada tarqaladi va bu to‘lqinlar sferik garmonikalar asosida kengaytirilgan. Tahlil natijalariga ko‘ra, skalyar profil funksiyasi uchun tenglama butunlay tenzor profil funksiyalaridan mustaqil bo‘lsa-da, tenzor profil funksiyalari uchun tenglamalar Ellis fazoviy-vaqtida skalyar profil funksiyasiga kuchli bog‘liq ekanligi ko‘rsatildi.

Shuningdek, biz vaqtga bog‘liq bo‘lmagan yechimlar — ya’ni skalyar va gravitatsion buzilishlar uchun yechimlar kompleks argumentga ega Legendre va biriktirilgan Legendre funksiyalari orqali ifodalanishini aniqladik. Biroq to‘lqin zonasi doirasida statsionar yechimlar ko‘rib chiqilganda, skalyar buzilishlar uchun aniq analitik yechim birlashtirilgan Heun funksiyasi yordamida olinadi.

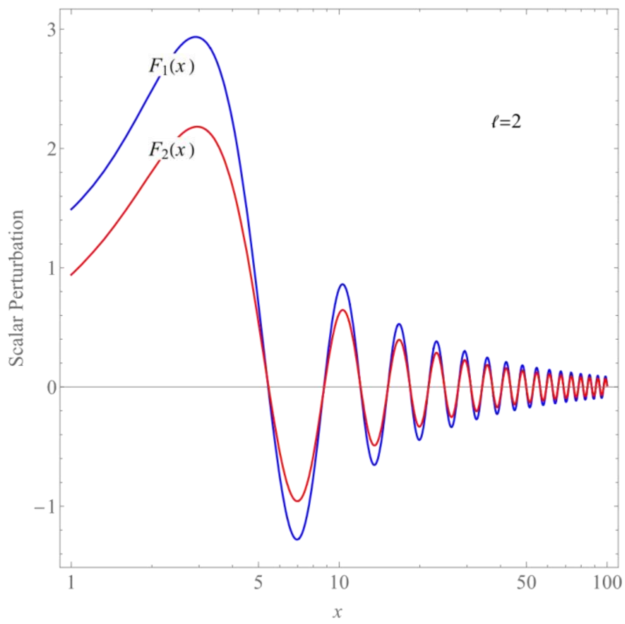
Qayd etish joizki, gravitatsion buzilishlarni boshqaruvchi tenglamalar ancha murakkab bo‘lishiga qaramay, ularni Regge–Wheeler–Zerilli tenglamasi shakliga keltirish orqali soddalashtirish mumkin. Nihoyat, biz bu tenglama uchun radial funksiyalar bo‘yicha sonli yechimlarni ham taqdim etdik.

$$ds^2 = -f\,dt^2 + f^{-1}\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right) , \tag{22}$$

bu yerda metrik funksiya quyidagicha ifodalanadi:

$$f = 1 - \frac{2Me^{-\mathrm{a}/r}}{r} . \tag{23}$$

Ushbu kontekstdagi M parametri qora tuynuk massasini, aa esa Simpson va Visser tomonidan kiritilgan og‘ish (deviatsiya) parametrini bildiradi. Shuni ta'kidlash joizki, yuqorida keltirilgan fazoviy-vaqt metrikasi umumiy nisbiylik nazariyasidagi standart Shvartsshild qora tuynugiga a→0 limitida mos keladi.



11-rasm: $\ell = 2$ holda skalyar g’alayonlanishning radial bog’liqligi.

Zaryadlangan, massiv skalyar maydon Ψ uchun elektromagnit maydon mavjudligida relativistik Klein–Gordon tenglamasi quyidagicha ifodalanadi:

$$g^{\alpha\beta}(\Delta_\alpha - iqA_\alpha)(\Delta_\beta - iqA_\beta)\Psi - \mu^2\Psi = 0 , \tag{24}$$

Bu yerda μ — skalyar maydon massasi, q — skalyar va elektromagnit maydonlar orasidagi kupling konstantasi, ∇α esa kovariant hosila, i — imaginariy (xayoliy) birlikni bildiradi.

(23)-tenglamada oʻzgaruvchilarni ajratish murakkab boʻlishiga qaramay, quyidagi fizik jihatdan asosli taxminlar orqali muammoni soddalashtirish mumkin:

- Vektor potentsial uchun Lorens kalibrlash sharti: $\nabla_\alpha A^\alpha = 0$;
- Kuchsiz oʻzaro ta'sir limitida, yuqori tartibli hadlarni, masalan, q^2B^2, e'tiborsiz qoldirish mumkin, ya'ni, $q^2B^2 \rightarrow 0$. Shunda (23)-tenglama quyidagi koʻrinishga keladi:

$$\frac{1}{\sqrt{-g}}\partial_\alpha(\sqrt{-g}g^{\alpha\beta}\partial_\beta\Psi) - 2iqA^\alpha\partial_\alpha\Psi - \mu^2\Psi = 0 \ , \tag{25}$$

Yechimni quyidagicha yozishimiz mumkin:

$$\Psi(t,r,\theta,\phi) = e^{-i\omega t}Y_{lm}(\theta,\phi)\frac{R(r)}{r}. \tag{26}$$

Biz tashqi asimptotik jihatdan bir jinsli magnit maydonga joylashtirilgan Shvartsshildga oʻxshash qora tuynuk uchun kvazinormal rejimlar (QNM) boʻyicha olib borilgan raqamli hisoblash natijalarini qisqacha taqdim etamiz.

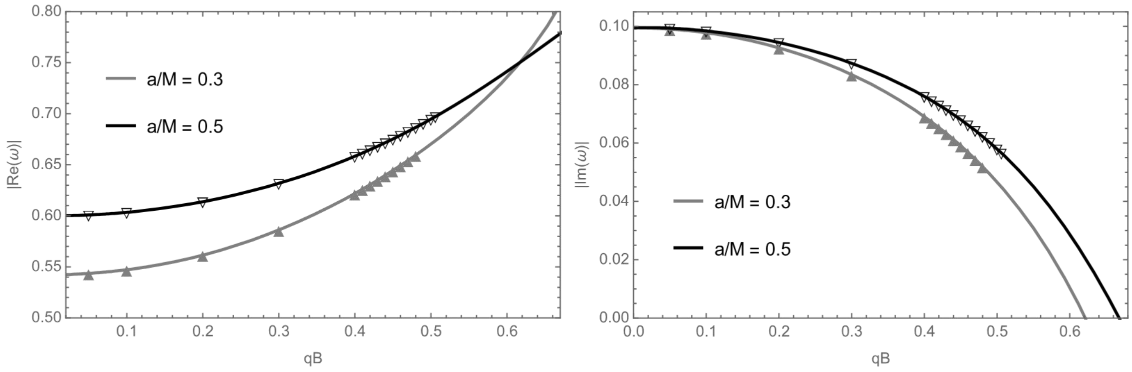
Chastota sohasidagi tahlil uchun biz yarim-analitik WKB usulidan foydalanamiz. Ushbu usulda yechim har ikki cheksizlikda WKB qatoriga kengaytiriladi va bu asimptotik kengaytmalar samarali potensialning maksimumi yaqinidagi Teylor qatori bilan moslashtiriladi (match qilinadi).
Yuqori tartibli WKB formulasi quyidagicha beriladi:The higher order

$$\begin{aligned}\omega^2 &= V_0 + A_2(K^2) + A_4(K^2) + A_6(K^2) + \ldots \\ &- iK\left(-2V_1 + A_3(K^2) + A_5(K^2) + A_7(K^2) + \ldots\right)\end{aligned} \tag{27}$$

bu yerda $K = n + 1/2$, va $n = 0, 1, 2, 3, \ldots$.

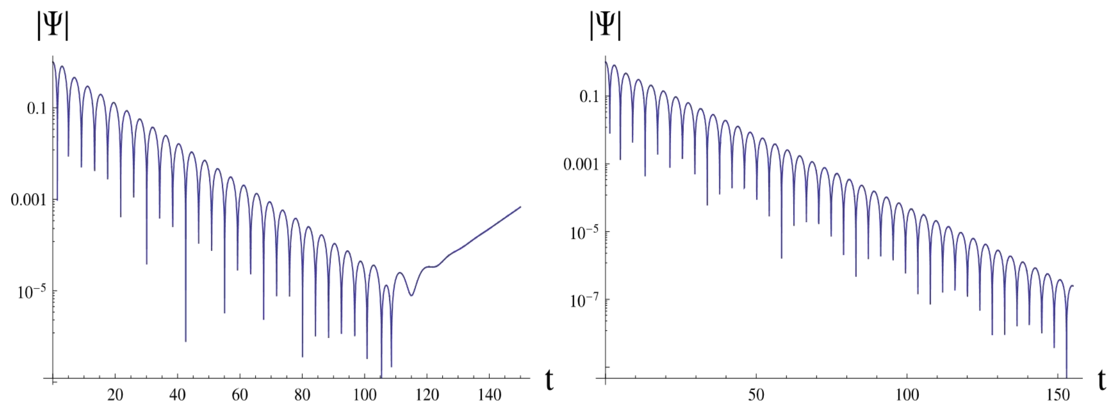
12-rasm tashqi magnit maydonda joylashgan massasiz skalyar maydon uchun WKB usuli yordamida olingan ma'lumotlarga mos keluvchi eng yaxshi polinom funksiyani koʻrsatadi. Rasmda ikki xil egri chiziq tasvirlangan: kulrang va qora – ular fazo-vaqt parametrlarining turli qiymatlariga mos keladi, ya'ni: *a/M*=0.3 (kulrang chiziq) va *a/M*=0.5 (qora chiziq). Bu parametr qiymatlari uchun kvazinormal rejim chastotasi ω ning mavhum qismlari rasmning oʻng tomonida tasvirlangan.

Har bir chiziq zaryad magnit coupling parametri qB ning nol qiymatini quyidagi nuqtalarda kesib oʻtadi: Kulrang chiziq uchun: qB≈0.611711, Qora chiziq uchun: qB≈0.651842.

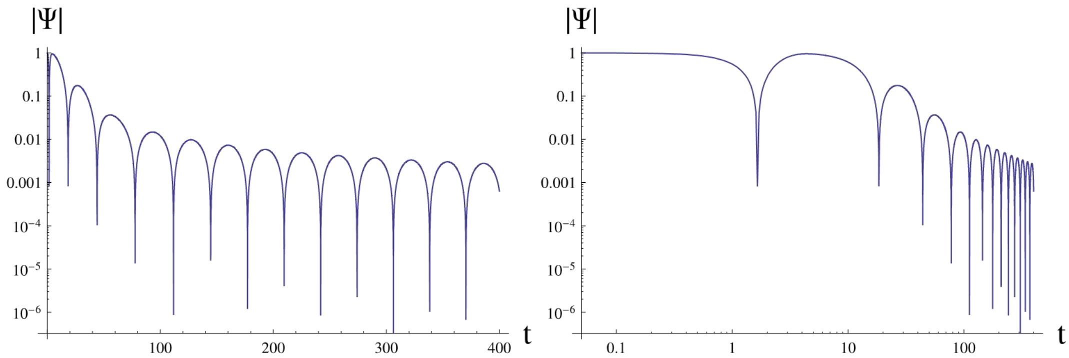


12-Rasm: WKB usuli asosida olingan ma’lumotlar uchun eng yaxshi mostushuvchi polinom funksiyalar turli fazoviy-vaqt parametr qiymatlari uchun $(l = 2, m = -2)$ taqdim etiladi.

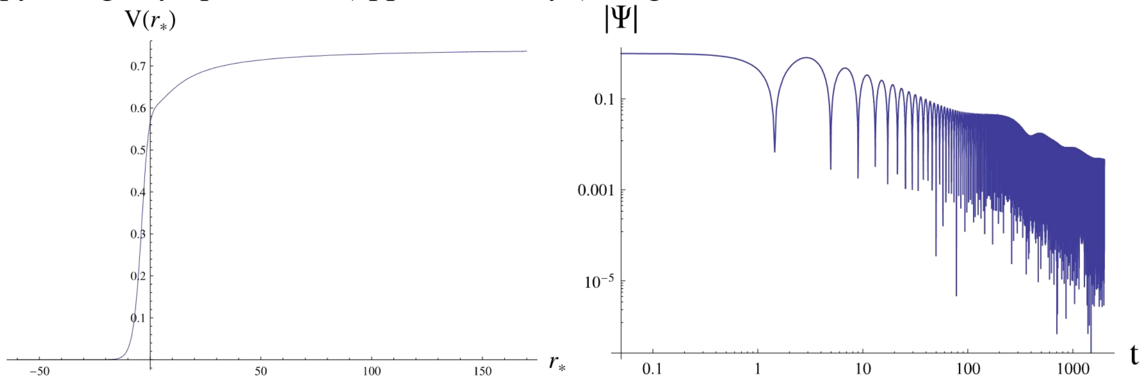
Bu yerda WKB natijalari vaqt sohasidagi integrallash orqali tekshirildi. Biroq, bunday taqqoslashni to‘g‘ri amalga oshirish uchun ikki muhim jihatni hisobga olish zarur: $\ell = 0$ perturbatsiyalar uchun kvazinormal tebranish davri juda qisqa bo‘ladi, chunki u tezda tebranma xarakterdagi kuch qonuni (power-law) quyruqlari bilan almashtiriladi (14-rasmga qarang). Shu sababli, bu holatda vaqt sohasidagi profildan chastotani aniqlik bilan ajratib olish qiyin bo‘ladi. Ikkinchi muhim jihat — bu kechki vaqt oralig‘ida perturbatsiyaning o‘sishi (13-rasmga qarang). Bunday o‘sish, ehtimol, $m > 0$ bo‘lgan holatda katta rr qiymatlarida samarali potensialning manfiy bo‘lishi tufayli yuzaga keladigan barqarorlik buzilishi (noaniqlik) deb talqin qilinishi mumkin.



13-Rasm: Yarim-logarifmik grafiklarda m=3 (chap) va m=−3 (o‘ng) uchun vaqt sohasidagi profil chiziqlari tasvirlangan. Bu yerda parametrlar quyidagicha olingan: ℓ=3, qB=0.1, a=0.3, μ=0.1, M=1.



14-Rasm: Kechikish vaqti oralig‘ida m>0 holatida kuzatilgan barqarorlik buzilishi (instabillik) bu — samarali potensialning faqat qora tuynukdan ma’lum bir masofagacha haqiqiy bo‘lgan yaqinlashma (approksimatsiya)ining artefaktidir.



15-Rasm: Shuningdek, m=−2 uchun (ℓ=2, qB=0.61, a=0.3, μ=0, M=1) samarali potensial va logarifmik ko‘rinishdagi vaqt sohasidagi profil grafigi tasvirlangan.

Shuni ta’kidlash kerakki, regulyar qora tuynuklarning kvazinormal rejimlari oldingi ko‘plab ishlar doirasida keng o‘rganilgan. Biroq, tashqi magnit maydoni mavjud bo‘lgan holatda regulyar qora tuynuklar uchun to‘liq va tizimli tadqiqotlar hali amalga oshirilmagan. Biz ushbu ilmiy bo‘shliqni to‘ldirishga urinib ko‘rdik va quyidagilarni ko‘rsatdik: Magnit maydon zaryadlangan skalyar maydonning spektrini sezilarli darajada o‘zgartiradi hamda cheksiz uzoq yashovchi kvazinormal rejimlarning — ya’ni kvazi-rezonanslarning paydo bo‘lishiga olib keladi.Tashqi magnit maydoni mavjud bo‘lgan holatda perturbatsiyalar evolyutsiyasining yana bir o‘ziga xos xususiyati — bu asimptotik quyruqlarning noodatiy xatti-harakatidir. Ma’lum parametrlar qiymatlarida bu quyruqlar kuch qonuni shaklidagi muqova (envelope)ga emas, balki tebranma (osillatsion) muqovaga ega bo‘ladi.

Xulosa

"Skalyar maydon mavjudligida kompakt obyektlar atrofida relyativistik astrofizik jarayonlar" mavzusida olib borilgan ilmiy tadqiqot asosida quyidagi ilmiy xulosalarga kelindi:

1. Janis– Nyuman–Vinikur (JNW) naked singulyarlik fazoviy-vaqtida skalyar maydonning itaruvchi ta'siri mavjudligi ko'rsatildi. Bu ta'sir eng ichki barqaror aylana orbitaning (ISCO) radiusini skalyar parametr n ga bog'liq holda nomonoton tarzda o'zgartiradi. Schwarzschild qora tuynugidan farqli ravishda, ma'lum kritik qiymatlarda ISCO butunlay yo'qoladi, bu esa singulyarlik atrofida barqaror orbitalarning buzilishini bildiradi.
2. JNW fazoviy-vaqtida zarralar harakati uchun yangi analitik natijalar. JNW fazo vaqtida massiv zarralar uchun effektiv potensial, xos energiya va burchak momentining aniq analitik ifodalari birinchi marta chiqarildi. Tadqiqotlar shuni ko'rsatdiki, skalyar bog'lanish parametri g_s ning qiymati musbat bo'lsa – ISCO kamayadi, manfiy bo'lsa – ortadi. Mazkur fazo vaqtda akretsiya disklarining nurlanish samaradorligi Schwarzschild qora tuynugiga nisbatan 12% gacha yuqori bo'lishi aniqlangan.
3. Ellis wormholida tomog' radiusi r_0 va skalyar bog'lanish parametri g_s barqaror orbitalarning radiusi orasidagi bog'lanish olindi. Radiatsion reaksiyani hisobga olganda, zarralar harakati qochish trayektoriyalarini namoyon etadi, bu esa qora tuynuklardagi ushlab qolish xarakteridan keskin farq qiladi. Tebranishning asosiy chastotalari hisoblab chiqildi va QPO (kvaziperiodik tebranishlar) orqali kuzatiladigan sezilarli siljishlar aniqlangan.
4. Magnitlangan Ellis wormholida zaryadlangan zarra dinamikasi o'rganildi. Magnit maydon ostidagi Ellis wormholida zaryadlangan zarra Lorents kuchi ta'sirida uning fundamental chastotalari bo'linib ketadi. Ma'lum magnit kuchlarida kvazi-rezonans rejimlar vujudga keladi. Nurlanish intensivligi skalyar va elektromagnit o'zaro ta'sirlarga bog'liqligi ko'rsatildi.
5. Markov zanjiri Monte-Karlo (MCMC) usuli birinchi bor JNW fazoviy-vaqti parametrlarini (M,g_s,n) QPO X-nurlari ma'lumotlari (masalan, GRO J1655–40) asosida aniqlashda qo'llanildi. Eng yaxshi mos keluvchi qiymatlar $n \sim 0.75$, $g_s \sim 0.2$ deb topildi, bu zaif skalyar maydon ustunligini ko'rsatadi.
6. Ellis wormholidagi skalyar perturbatsiyalar uchun yechimlar aniq analitik ko'rinishda, ya'ni konflyuent Heun funksiyasi orqali ifodalanishi ko'rsatildi. Bunda gravitatsion perturbatsiyalar esa Regge-Wheeler tipidagi tenglamaga bo'ysunadi va g'ayrioddiy tebranma quyruqlarga ega, bu holat qora tuynuklarda kuzatiladigan Power-Law(quvvat qonuni)ga asoslangan so'nishdan farqlanadi.
7. Tashqi magnit maydonga joylashtirilgan Schwarzschildga o'xshash qora tuynuklar uchun kvazinormal rejimlar WKB yondashuvi va vaqt sohasidagi integratsiya usullari yordamida hisoblab chiqildi. Ular orasida uzoq yashovchi kvazi-rezonanslar va anomaliya ko'rsatuvchi kechki vaqt quyruqlari aniqlandi. Ushbu xususiyatlar kelajakdagi gravitatsiya to'lqinlari kuzatuvlari orqali, skalyar maydon mavjut bo'lgan fazov vaqtlarni farqlash imkonini beradi.

SCIENTIFIC COUNCIL DSc. 03/07.07.2025.FM/T.192.01 ON AWARD OF SCIENTIFIC DEGREE AT INSTITUTE FOR ADVANCED STUDIES AT “NEW UZBEKISTAN” UNIVERSITY

“NEW UZBEKISTAN” UNIVERSITY
ASTRONOMICAL INSTITUTE

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RELATIVISTIC ASTROPHYSICAL PROCESSES AROUND COMPACT OBJECTS COUPLED WITH SCALAR FIELD

01.03.01 – Astronomy
01.04.02 – Theoretical physics

ABSTRACT OF DISSERTATION OF THE DOCTOR OF PHILOSOPHY (PhD) IN PHYSICS AND MATHEMATICS

TASHKENT-2025

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INTRODUCTION (Annotation of PhD dissertation)

Topicality and demand of the dissertation. With the increasing precision of astronomical observations -- such as gravitational wave detections (LIGO/Virgo) and highfrequency quasi-periodic oscillations (QPOs) from X-ray binaries—there is a growing demand for theoretical frameworks that extend beyond classical general relativity. This work provides essential insights into how scalar fields and external magnetic fields modify spacetime dynamics, offering testable predictions for future observations. The study of exotic compact objects (e.g. JNW naked singularities, Ellis wormholes) also aligns with ongoing efforts to probe the limits of Einstein theory and explore alternative gravity models. The dissertation addresses critical questions in modern relativistic astrophysics, particularly the interplay between compact objects (black holes, naked singularities, and wormholes) and scalar fields.

The dissertation's focus on scalar-tensor couplings and their astrophysical implications is highly relevant, given recent developments in modified gravity, dark matter, and dark energy research. Scalar fields are fundamental in theories like \$f(R)\$ gravity and string-inspired models, making this work pertinent to both theoretical and observational cosmology. Additionally, the analysis of quasi-resonant modes and QPOs contributes to the interpretation of data from next-generation telescopes (e.g., LISA, Athena) and X-ray observatories (e.g., NICER). The unusual late-time tails of perturbations in magnetized black holes, as explored in Chapter IV, could even shed light on anomalies in pulsar timing array (PTA) signals, such as those reported by NANOGrav.

By bridging particle dynamics, perturbation theory, and observational constraints, this dissertation advances the toolkit for studying compact objects in extreme environments. Its methodologies, such as the WKB approximation for quasinormal modes and MCMC analysis for QPOs, are transferable to other compact object scenarios, including neutron stars and exotic horizonless objects. The work also underscores the importance of interdisciplinary approaches, combining general relativity, quantum field theory, and data science to tackle open problems in astrophysics. As the community seeks to resolve tensions between general relativity and quantum mechanics, studies like this pave the way for new physics beyond the Standard Model. The topicality, demand, and actuality of the dissertation lie in its direct engagement with cutting-edge questions in relativistic astrophysics, its applicability to current and future observational campaigns, and its potential to influence theoretical paradigms in gravity and cosmology.

Furthermore, the dissertation aligns with the strategic scientific priorities in Uzbekistan and the broader international community. This dissertation work corresponds to the tasks of the following state regulatory documents: Decree of the President of the Republic of Uzbekistan No. UP-4947 "On the Strategy of Actions for the Further Development of the Republic of Uzbekistan" dated February 07, 2017, Decree of the President of the Republic of Uzbekistan No. PP-2789 "On

Measures for Further Improvement of Academy of Sciences, Organization, Management, and Financing of Research Activities from 18.02.2017.

Relevance of the research to the priority areas of science and technology development of the Republic of Uzbekistan. The dissertation research has been carried out in accordance with the priority areas of science and technology in the Republic of Uzbekistan: II. “Power, energy and resource-saving”.

Degree of study the problem. While the motion of test particles in black hole spacetimes has been extensively studied in general relativity (Subrahmanyan Chandrasekhar, Brandon Carter, James Bardeen, Valeri Frolov, Andrei Zelnikov, Pankaj S. Joshi, Remo Ruffini), the inclusion of scalar fields, particularly in the context of naked singularities (JNW spacetime) and wormholes (Ellis spacetime) – introduces novel complexities. This research work builds on foundational studies by Misner, Wheeler, and others but extends them by incorporating scalar couplings, radiation reaction effects, and magnetic fields, which have not been systematically explored in these spacetimes. This represents a significant advance beyond the classical treatments of geodesic motion.

Prior research on scalar fields in astrophysics has largely focused on cosmological applications or black hole perturbations (Claudia de Rham, Thomas P. Sotiriou, Vitor Cardoso, Kostas D. Kokkotas, Toby Crisford, Hideo Kodama), leaving gaps in understanding their role in naked singularities and wormholes. This dissertation fills these gaps by deriving exact analytical solutions (e.g., ISCO radii in JNW spacetime), numerical simulations of particle trajectories, and perturbation analyses (e.g., quasinormal modes in magnetized black holes). The study of quasiperiodic oscillations (QPOs) in scalarmodified spacetimes is particularly original, as it connects theoretical predictions with observational data from X-ray binaries, a link that has rarely been explored in the earlier literature. The inclusion of MCMC methods to constrain spacetime parameters further demonstrates a modern, data-driven approach to testing theoretical models.

Compared with similar studies, this work stands out by integrating scalar fields, electromagnetic interactions, and radiation reaction into a unified framework for multiple types of compact objects. However, certain limitations remain, such as the assumption of weak backreaction (neglecting particle effects on spacetime geometry) and the restriction to static, spherically symmetric solutions. Rotating compact objects (e.g., Kerr-like naked singularities) and nonlinear scalar couplings remain open challenges. Future research could extend this work by incorporating numerical relativity techniques or higher-dimensional theories (e.g., braneworld models), which would deepen the exploration of scalarfield dynamics in more realistic astrophysical scenarios. The dissertation provides a comprehensive but not exhaustive study of the problem, balancing analytical rigor with innovative applications while identifying clear pathways for future research.

Connection of the topic of the dissertation with the scientific research of the higher educational/research institutions, where the dissertation was carried out. The dissertation was done in the framework of the scientific projects funded by the Ministry of Innovative Development: F-FA-2021-510 "Investigations of nuclear matter of neutron stars in modified gravity" (2021-2026).

The aim of the research of the dissertation is to develop a new theoretical model to study scalar field couplings and reveal new findings on particle dynamics near compact objects.

The tasks of the research:

- to develop a theoretical model coupling scalar fields to particle dynamics in JNW and Ellis spacetimes;
- to derive equations of motion for massive particles under scalar field influence;
- to analyze ISCO modifications due to scalar coupling parameters;
- to investigate radiation reaction effects on particle trajectories near naked singularities;
- to compute fundamental frequencies (orbital, radial, vertical) for QPO applications;
- to conduct MCMC analysis to constrain spacetime parameters using X-ray binary data;
- to explore charged particle dynamics in magnetized wormhole and black hole spacetimes.
- to study perturbation spectra (quasinormal modes, late-time tails) in scalar-modified geometries.

The objects of the research are the Janis-Newman-Winicour (JNW) naked singularity spacetime with scalar field coupling; the Ellis wormhole geometry interacting with external scalar fields; Schwarzschild-like regular black holes immersed in external magnetic fields; test particles (neutral and charged) in these modified spacetime geometries.

The subjects of the research are dynamics of massive particles under scalar field influence in exotic spacetimes; modification of orbital mechanics and fundamental frequencies due to scalar couplings;astrophysical implications of scalar-field modified compact objects (QPOs, ISCOs, etc.); perturbation spectra and stability properties of scalarcoupled compact objects.

The methods of the research are analytical modeling was employed to derive exact equations of motion and effective potentials for particles in scalar-coupled spacetimes (JNW, Ellis). Numerical simulations (Runge-Kutta integration) were used to solve geodesic equations and analyze particle trajectories under radiation reaction effects. Semianalytical WKB approximation and time-domain integration techniques were applied to compute quasinormal modes and perturbation spectra. Bayesian MCMC analysis was conducted to fit theoretical QPO frequencies to observational data from X-ray binaries.

The scientific novelty of the research is the follows:

- The research establishes the first complete theoretical model of particle dynamics in JNW naked singularity spacetime with scalar coupling, revealing unique ISCO behavior that distinguishes it from black holes.

- Novel findings demonstrate how scalar fields modify Ellis wormhole physics, including radiation-driven particle escape and characteristic QPO frequencies that differentiate exotic compact objects from black holes.
- The work pioneers MCMC-based constraints on scalar field parameters using X-ray binary data and identifies new spectral signatures for testing modified gravity theories in strong-field regimes.

Practical results of the research are as follows:

- The analytical expressions derived for particle dynamics in scalar-modified spacetimes enable the precise modeling of accretion-disk structures and emission profiles around exotic compact objects.
- The computed QPO frequencies and ISCO radii provide observable signatures to distinguish JNW naked singularities and Ellis wormholes from black holes using Xray binary data.
- The MCMC-fitted scalar field parameters ($n \sim 0.75$, $g_s \sim 0.2$) offer testable constraints for future gravitational wave and multi-messenger astrophysics campaigns.
- The identified perturbation spectra (e.g., confluent Heun solutions, oscillatory tails) serve as benchmarks for detecting scalar-tensor gravity effects in next-generation observatories.

Reliability of the research results is provided by the following:

- The analytical framework was rigorously derived from first principles and cross-verified against established results in general relativity, ensuring mathematical consistency throughout all calculations.
- Numerical simulations employed multiple independent methods (Runge-Kutta integration, WKB approximation) that produced consistent results, confirming the robustness of the particle dynamics and perturbation analyses.
- The MCMC parameter estimation was validated through convergence tests and comparison with existing astrophysical constraints, demonstrating statistical reliability of the scalar field coupling values.
- All theoretical predictions maintain consistency with observational data from X-ray binaries and known black hole phenomenology, while clearly identifying testable deviations for future verification.

Scientific and practical significance of the research.

- The research establishes a comprehensive framework for analyzing scalar field effects in exotic compact objects, providing new insights into the interplay between gravity and fundamental fields beyond classical black hole paradigms.
- The derived QPO frequencies and ISCO modifications serve as crucial observational discriminators between black holes, naked singularities, and wormholes, which are directly applicable to current X-ray telescope data analysis.

- The developed MCMC-QPO analysis technique and perturbation methods offer powerful new tools for constraining modified gravity theories using astrophysical data, significantly enhancing precision gravity tests.
- By identifying unique spectral signatures of scalar-tensor interactions, the results in-
form the design requirements for next-generation gravitational wave detectors and multimessenger astronomy campaigns.

Applications of the research results. The results of the study of the dynamics of particles around black holes in modified gravity theories have been applied as follows:
the theoretical research results and methods, published in the scientific paper "A. Davlataliev, B. Narzilloev, I. Hussain, A. Abdujabbarov, B. Ahmedov, "Probing the Starobinsky-Bel-Robinson gravity by photon motion around the Kerr-type black hole in non-uniform plasma," // Phys.Dark Univ. 42 (2023) 101340, https://doi.org/ 10.1016/j.dark.2023.101340" and presented in the Doctorate (PhD) thesis of Mr. A. Davlataliev have been used in the frame of the programs supported by the Fudan University (Letter from Prof. Cosimo Bambi)

Approbation of the research results. The dissertation results have been discussed in 3 international and 1 local conference.

Publication of the research results. 11 scientific publications have been made on research results, and 11 of them are research papers in refereed journals.

Volume and structure of the dissertation consist of an introduction, four chapters, a conclusion, and a list of references all in 121 pages.

THE MAIN CONTENT OF THE DISSERTATION

In the introduction of the dissertation indicates the relevance and necessity of the topic, the correspondence of the research to the priority directions of development of science and technology of the republic, the degree of knowledge of the problem, its connection with the research plans of the higher educational institution in which the dissertation was carried out, and the purpose, objectives, object of research, brief information about the subject, methods, scientific novelty, practical result, reliability, scientific and practical significance of the results, introduction of the results into practice, approval of the results, publication of the results, as well as the structure and scope of the dissertation..

In the first chapter of the dissertation entitled "Influence of scalar field in massive particle motion in JNW spacetime", we study the particle dynamics around the Janis-Newman-Winicour (JNW) naked singularity. JNW naked singularity represents an exact solution of Einstein's equations with a massless scalar field. We briefly describe the Einstein-scalar-field-massive particle system and derive the equation of motion for the whole system. We also consider the circular motion of massive particles in the presence of the scalar field in the JNW spacetime. We have also studied the circular motion of massive particles including the radiation reaction term and the present particle trajectories.

One of the simplest solutions to these Einstein-scalar field equations is represented by the Janis-Newman-Winicour naked singularity. The corresponding line element is given as

$$
\begin{aligned}
ds^2 &= - f^n dt^2 + f^{-n} dr^2 + f^{1-n} r^2 (d\theta^2 + \sin^2\theta d\phi^2) \ , \\
\varphi(r) &= \frac{\sqrt{1-n^2}}{2} \ln f \ , \qquad f = 1 - \frac{2M}{nr} \ ,
\end{aligned} \tag{1}
$$

where M is the mass of the gravitational object, n is parameter of the scalar field. The singularity of the spacetime is located at $r^*=2M/n$.

The equation of motion for a massive particle can be explicitly written as:

$$
\ddot{t} + \frac{2M\dot{t}\dot{r}}{fr^2}(1-B) = 0 \ , \tag{2}
$$

$$
\begin{aligned}
&\ddot{r} + \frac{Mf^n}{fr^2}\left(f^n \dot{t}^2 - 2B\right) - \frac{M\dot{r}^2}{fr^2}(1+2B) \\
&+ \left(\frac{M(n+1)}{n} - r\right)\left(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2\right) = 0 \ ,
\end{aligned} \tag{3}
$$

$$
\ddot{\theta} - \frac{1}{2}\sin 2\theta\dot{\phi}^2 - \frac{2M\dot{\theta}\dot{r}}{fr^2}\left(1 + \frac{1}{n} - \frac{r}{M} + B\right) = 0 \ , \tag{4}
$$

$$
\ddot{\phi} + 2\cot\theta\dot{\theta}\dot{\phi} - \frac{2M\dot{r}\dot{\phi}}{fr^2}\left(1 + \frac{1}{n} - \frac{r}{M} + B\right) = 0 \ , \tag{5}
$$

where B is defined as

$$B = \frac{2g_s\sqrt{1-n^2}}{n(g_s\sqrt{1-n^2}\ln f + 2)} \, . \tag{6}$$

In Fig. 1, we show dependence of the ISCO position for massive particle from scalar parameter *n* for different value of the interaction parameter g_s.

In Fig. 2, we show the trajectory of a massive particle in the presence of a scalar field, including the radiation reaction term. Using numerical calculations, we have demonstrated that the contribution due to the radiation reaction is not significant. Therefore, to observe the effect of the radiation reaction term in circular motion, we set the damping time to $\tau_0 = 0$ s and $\tau_0 = 0.5$ s. This implies that the damping time is much shorter than 1 s, and the particle orbits many more times than shown in Fig. 2 around the compact object described by JNW spacetime before eventually escaping due to the radiation reaction force. In Fig. 3, we demonstrate the trajectory of a massive particle in two scenarios: (i) considering the effect of the scalar field but without the radiation reaction term and (ii) neglecting effect from the scalar field but including the radiation reaction term. This result shows that indeed the scalar field generates an attractive effect, however, radiation reaction repulsive effect in the motion of massive particle orbiting around the naked singularity.

Our results indicate that the scalar field generates an attractive force, while the radiation reaction exerts a repulsive force. The trajectory of a massive particle around a naked singularity in the JNW spacetime is influenced by both these effect as shown in our numerical simulations.

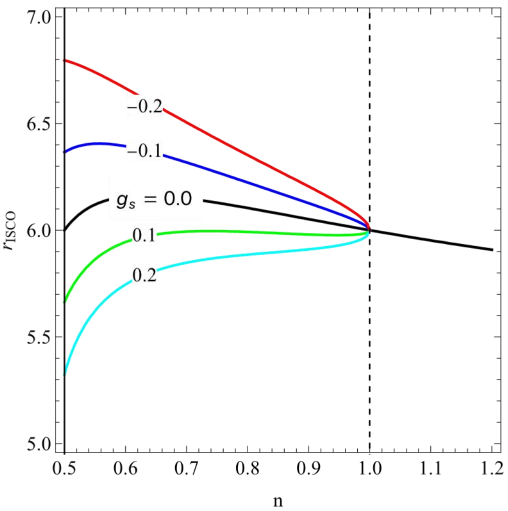

Figure 1: Dependence of the ISCO position of massive particle from the parameter n for the different values of the coupling parameter.

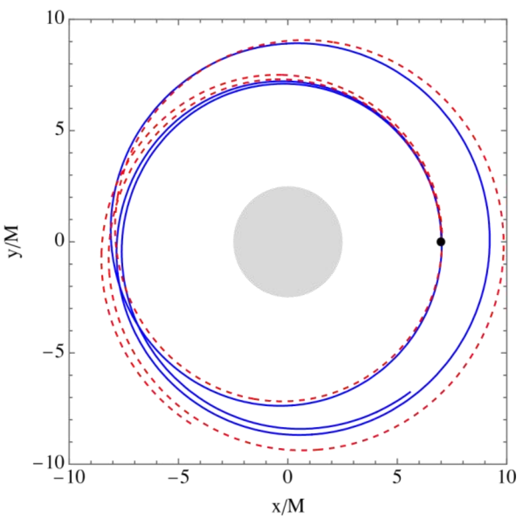

Figure 2: Particle trajectories near the JNW naked singularity without ($\tau0 = 0$, solid blue line) and with ($\tau0 = 1.0$, dashed red line) radiation reaction term for the particular choice of parameters gs = 0.03 and n = 0.8.

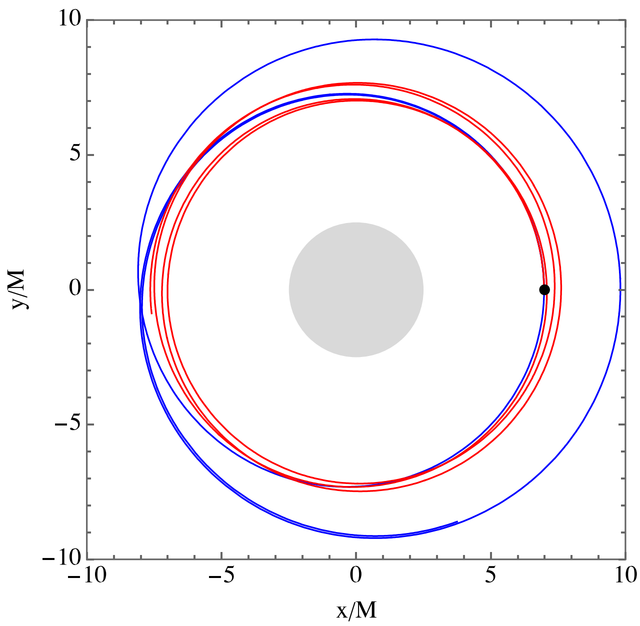

Figure 3: Particle trajectories near the JNW naked singularity the particular choice of parameters. Blue solid line represents for $g_s = 0.03$ and $\tau_0 = 0$ s and red solid line represents for $g_s = 0$ and $\tau_0 = 0.5$ s.

In the second chapter, entitled "Influence of scalar field in massive particle motion in JNW spacetime", we applied the results of the particle dynamics to study the quasiperiodic oscillations (QPO) around JNW compact objects. We considered the oscillatory motion of a massive particle near the stable circular orbit in the JNW spacetime. As an astrophysical consequence we studied application of the oscillatory motion of massive particle twin-peak QPO and using these results obtained the constrains for two main parameters mass of the central object, scalar parameter, and coupling constant.

The oscillation motion of test particles around gravitational compact objects is a key aspect of relativistic astrophysics. This motion can be categorized into radial and vertical oscillations, and provides crucial insights into the dynamics and structure of the spacetime around these objects. (i) Radial oscillations refer to the backand-forth motion of a particle in the radial direction, that is, towards and away from the central object. These oscillations occur around stable circular orbits. The stability and frequency of these oscillations are influenced by the spacetime geometry and the properties of the massive object. For the Schwarzschild black hole, the radial oscillation frequency depends on the mass of the black hole and the radial distance from it. Closer to the black hole, the stronger gravitational pull leads to higher oscillation frequencies. (ii) Vertical oscillations, also known as latitudinal or epicyclic oscillations, refer to the motion of a particle perpendicular to the plane of the orbit. These oscillations provide information about the vertical stability of the orbit and are influenced by the angular momentum and spin of the central object. The vertical epicyclic frequency is the frequency at which a particle oscillates vertically around the equatorial plane of the central object. These two kinds of frequencies are related to the orbital frequency also known as the Keplerian frequency. Together they are called fundamental frequencies and in the Schwarzschild black hole these fundamental frequencies are expressed as $\Omega_r = \Omega_K(1-6M/r)^{1/2}$, and $\Omega_\theta = \Omega_K = \sqrt{M/r^3}$, where Ω_K is Keplerian frequency in the Schwarzschild spacetime.

The radial dependence of fundamental frequencies is illustrated in Fig. 4 with specific parameter settings. The left panel displays plots for different values of the coupling parameters with a fixed value of the parameter n, while the right panel shows similar plots for varying values of the parameter n with a fixed coupling parameter gs. It can be observed that the fundamental frequencies increase due to the interaction of the massive particle with the scalar field, but they decrease with an increase in the n parameter. The combined effect of orbital, radial and vertical oscillations can lead to complex trajectories for test particles. These oscillations are crucial for understanding phenomena such as quasi-periodic oscillations (QPOs) observed in X-ray binaries. QPOs are believed to be related to the oscillatory motion of matter in the accretion disk around a black hole or neutron star. The oscillatory motion affects the stability and structure of accretion disks. Understanding these oscillations helps in modeling the emission spectra and variability of the disks. Observing the frequencies and modes of oscillations can provide insights into the properties of black holes and neutron stars, such as their mass, spin, and the geometry of the surrounding spacetime.

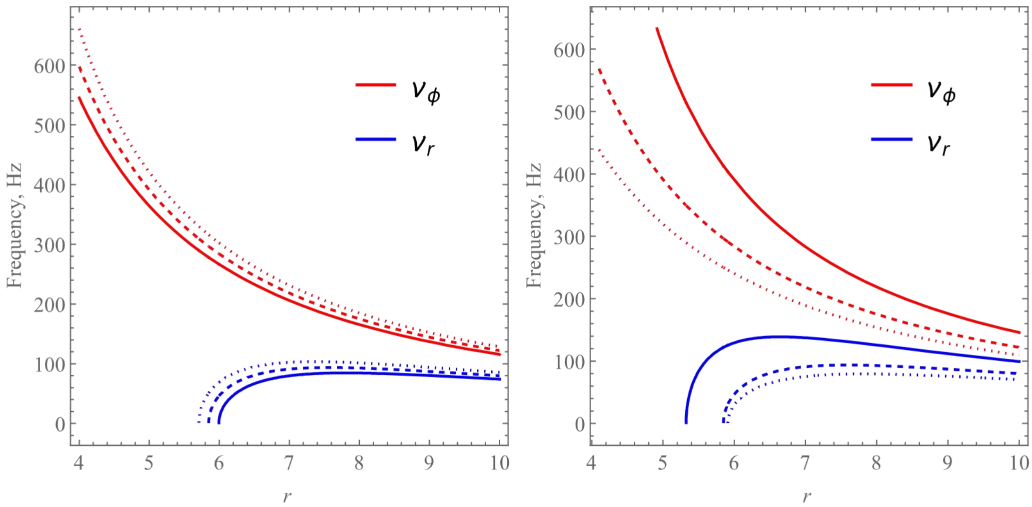

Figure 4: (Left panel) Radial dependence of fundamental frequencies of massive particle for different values of the coupling parameter ($g_s = 0.1$ - solid line, $g_s = 0.2$ - dashed line, and $g_s = 0.3$ -dotted line) for fixed $n = 0.7$. (Right panel) Radial dependence of fundamental frequencies of massive particle for different values of the n parameter ($n = 0.5$ - solid line, $n = 0.7$ - dashed line, and n = 0.9 -dotted line) for fixed $g_s = 0.2$. In both cases, the mass of the central object is taken as $M = 10M_\odot$.

The Relativistic Precession (RP) model offers a theoretical framework to explain the occurrence of quasi-periodic oscillations (QPOs), attributing them to the quasi-harmonic oscillations of charged particles as they move radially and angularly around black holes and wormholes. Within the RP model, the interpretation of twinpeaked QPOs is explained as follows: the higher frequency corresponds to the orbital frequency of the particle, denoted as $\nu_U = \nu_\phi$, while the lower frequency corresponds to the difference between the orbital frequency and the radial oscillation frequency, expressed as $\nu_L = \nu_\phi - \nu_r$. In other words, for twin-peaked QPOs, the upper frequency reflects the particle's orbital frequency, and the lower frequency is obtained by subtracting the radial oscillation frequency from the orbital frequency.

Figure 5 illustrates the radial dependence of the upper and lower frequencies of twin QPOs within the RP model, shown by blue and black lines, respectively. In the left panel, the solid line represents n = 0.5, while the dashed line represents n = 0.7 with a fixed coupling parameter g_s = 0.2. In the right panel, the solid line represents g_s = 0.1, while the dashed line represents g_s = 0.3 with a fixed n = 0.7. It is evident from the figures that the radial position at which the frequency ratio of 3:2 QPOs is observed shifts slightly toward the naked singularity as the two main parameters, n and g_s, are increased.

In Fig. 6, we examine the relationships between the upper and lower frequencies of twin peak QPOs for various *n* and g_s values. The analysis shows that as n increases, both the upper and lower frequencies decrease. Conversely, the effect of gs is opposite, leading to an increase in these frequencies. As can be seen from Fig. 6, the frequencies of the twin peak QPOs are not observable in the shaded area in both plots because these correspond to distances inside the ISCO position.

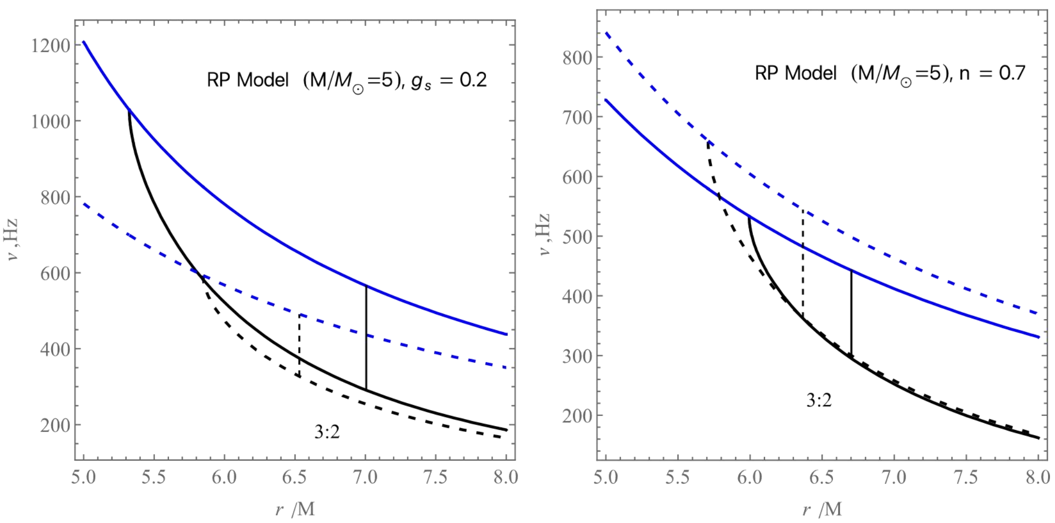

Figure 5: The radial dependence of upper and lower frequencies in RP model that are observed at ratio of 3:2.

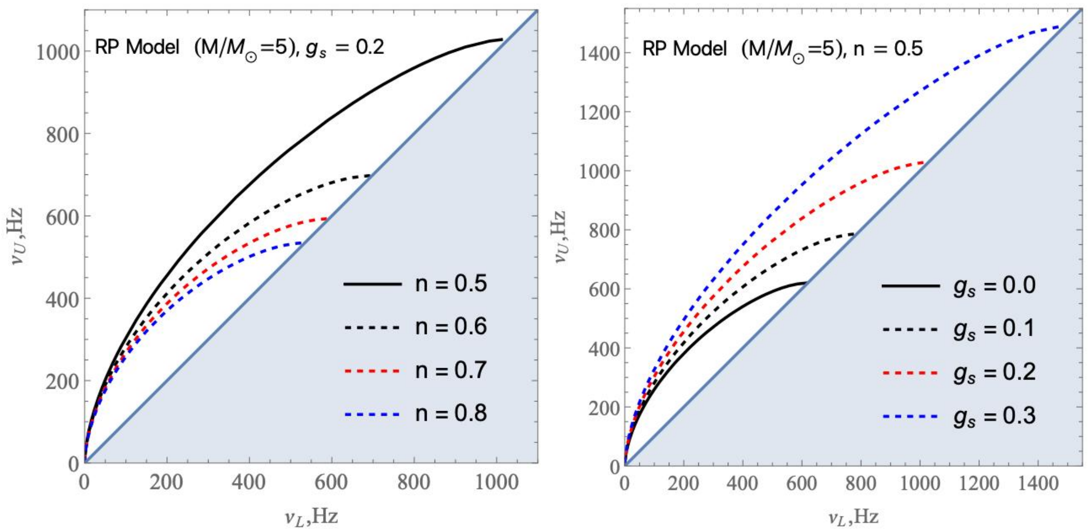

Figure 6: Relationships between the upper and lower peak frequencies of twin-peak QPOs in the RP model with mass M = 5M⊙

Using MCMC analyses at 1σ confidence level in Fig.7, we show the corner plots three different selected sources as mentioned before for the best-fit parameter values (M, g_s, n, r) resembling those of JNW spacetime. Thus the best-fit values are listed in Tab. 1.

The obtained results demonstrate the constraints on the JNW parameters for each microquasar. It is also presented the best-fit parameter values derived from the QPOs data, reflecting the effectiveness of the RP model in describing the QPO phenomena in these X-ray binaries. This study enhances our understanding of the dynamics and properties of compact objects in strong gravitational fields.

Table 1: The best-fit parameter values resembling those of JNW, deduced from the Quasi-Periodic Oscillations (QPOs) for the chosen X-ray sources.

	XTE J1550-564	GRO J1655-40	GRS 1915+105
$M(M_\odot)$	8.96 ± 0.50	5.31 ± 0.27	$12.72^{+0.66}_{-0.82}$
g_s	$0.251^{+0.065}_{-0.096}$	$0.209^{+0.059}_{-0.11}$	$0.138^{+0.077}_{-0.12}$
n	0.734 ± 0.083	0.755 ± 0.092	$0.779^{\pm}0.085$
r/M	6.498 ± 0.032	6.538 ± 0.069	6.700 ± 0.014

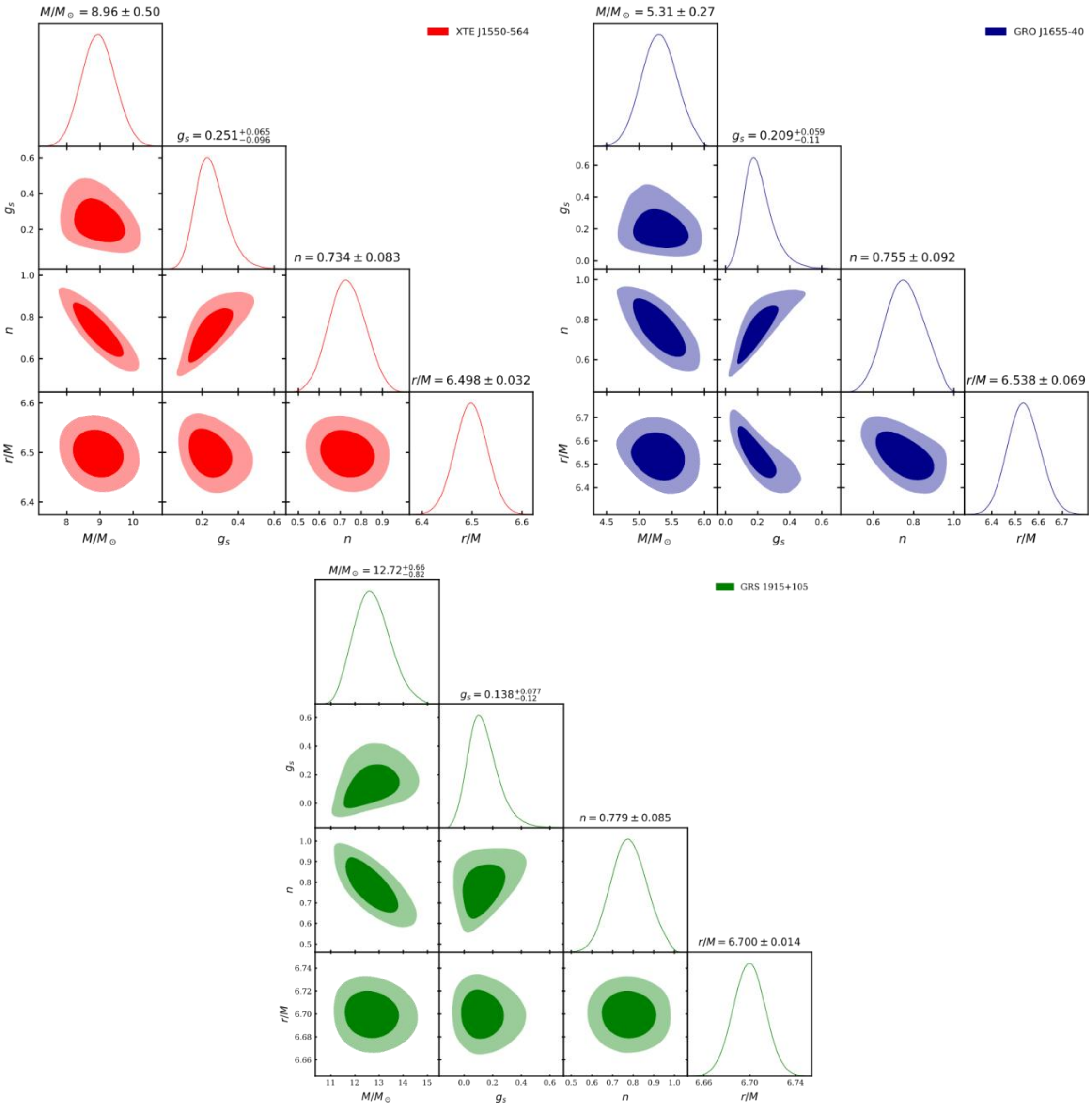

Figure 7: Constraints on JNW mass, the gs and n parameters in the microqusars XTE J1550-564(upper left),GRO J1655-40 (upper right) and GRS 1915+105 (down) using MCMC analysis.

In the third chapter, entitled "Exploring a Novel Feature of Ellis Spacetime: Insights into Scalar Field Dynamics" we tested the Ellis spacetime by considering particle motion around the wormhole in the presence of the external scalar field. We provided the main equations that are related to background spacetime and dynamics motion of test particle in the presence of the external scalar field.We have also studied particle motion, including the radiation reaction.

The Ellis wormhole is governed by the spacetime line element

$$ds^2 = -dt^2 + dr^2 + (r^2 + r_0^2)(d\theta^2 + \sin^2\theta d\phi^2) \ , \tag{7}$$

along with the associated scalar field

$$\Phi = \frac{\pi}{2} - \tan^{-1}\left(\frac{r}{r_0}\right) \ , \tag{8}$$

where r_0 is the throat of the wormhole. Here the radial coordinate r runs between r_0 and infinity, i.e. $r_0 < r < \infty$. The curvature scalar invariants of the spacetime such as Ricci scalar and Kretschmann scalar can be expressed as

$$R = -\frac{2r_0^2}{(r^2+r_0^2)^2}\ , \qquad K = \frac{12r_0^4}{(r+r_0^2)^4}\ , \tag{9}$$

which are regular at any point of spacetime at $r > r_0$. The radial dependence of the scalar field and curvature invariants, Ricci and Kretchmann scalar is shown in Fig.8.

We also discussed fundamental frequencies, namely, orbital and epicyclic frequencies, of massive particle in the Ellis spacetime. In Fig.9 radial dependence of the fundamental frequencies, namely, orbital, radial and vertical frequencies are illustrated.

The Lagrangian for a charged particle in the presence of the external magnetic field is

$$L = \frac{1}{2}m_* g_{\mu\nu}u^\mu u^\nu + qA_\mu u^\mu\ , \tag{10}$$

where q is a charge of the test particle. The Lagrangian equation of motion is derived as

$$\frac{Du^\mu}{d\tau} = \frac{q}{m}F^\mu{}_\nu u^\nu + (g^{\mu\nu} + u^\mu u^\nu)\partial_\nu \ln\frac{m_*}{m}\ , \tag{11}$$

and after simple algebraic manipulations equation of the radial motion yields

$$\dot{r}^2 = \frac{1}{(1+\mathrm{g}_s\Phi)^2}\left[\mathcal{E}^2 - \left(r^2+r_0^2\right)\left(\frac{\mathcal{L}}{r^2+r_0^2} - \omega\right)^2\right] - 1\ , \tag{12}$$

where $\omega = qB/2m$ is a magnetic parameter. The marginally bound circular radius for a charged test particle is determined using conditions $\dot{r} = \ddot{r} = 0$

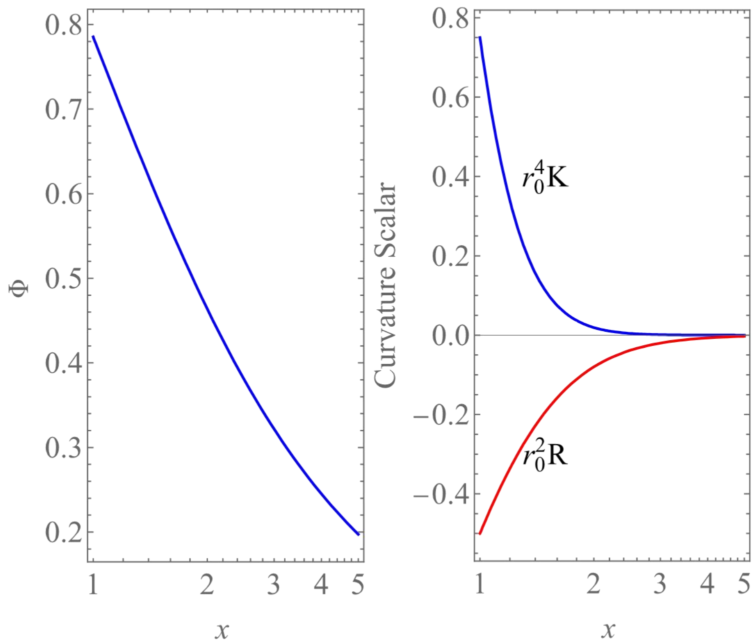

Figure 8: Radial dependence of the scalar field Φ(x) and curvature invariant, dimensionless Ricci scalar $r_0{}^2R$ and dimensionless Kretschmann scalar $r_0{}^4K$.

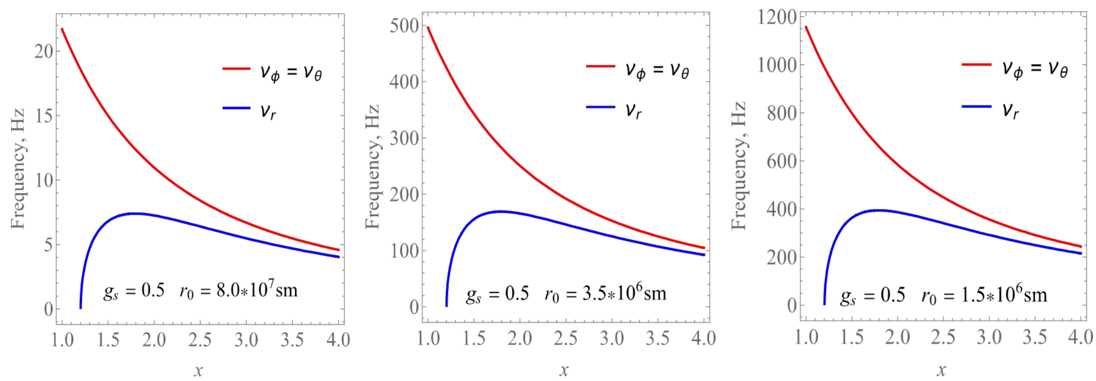

Figure 9: Radial dependence of the fundamental frequencies of a massive particle.

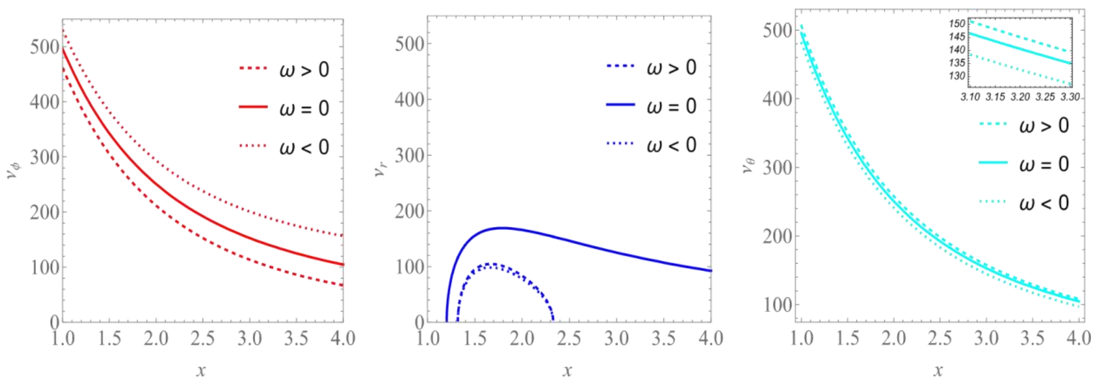

Figure 10: Fundamental frequencies of charged particle around Ellis wormhole on ω=±0.4, parameter $g_s = 0.5$.

$$\mathrm{g}_s x + \left(1 - x^2\right)(1 + \mathrm{g}_s \Phi) + 4\omega^2 x^3 (1 + x^2) = 0\ . \tag{13}$$

The fundamental frequencies of the charged particle in the vicinity of Ellis wormhole in the presence of the external magnetic field can easily be done. The Keplerian frequency of the charged particle is derived from the following expression

$$\Omega^2 + \omega\Omega\sqrt{1 - \Omega^2(x^2+1)} = \frac{\mathrm{g}_s\left[1 - \Omega^2(x^2+1)\right]}{x(x^2+1)(1+\mathrm{g}_s\Phi)}\ . \tag{14}$$

Similarly, epicyclic frequencies for charged particle orbiting around the Ellis wormhole in the presence magnetic field, however we will skip the detailed calculations. The radial dependence of the fundamental frequencies is shown in Fig. 10. As one can see from this figure the fundamental frequencies of charged particle are split due to the external magnetic field. On the other hand, our results show that Keplerian frequencies strongly depend on the magnetic parameter.

The black hole perturbation is one of the hot topic, in particular, after LIGO and Virgo collaboration start detect the gravitational wave from the binary systems. This kind of perturbation cab also be applicable to wormhole spacetime. Here we are interested in considering perturbation of the Ellis spacetime. We have mentioned before that the Ellis spacetime is the solution of Einstein-scalar field equations. So that the scalar field and metric tensor can be expressed as $\Phi = \Phi + \delta\Phi$ and $g_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}$, where Φ and $g_{\mu\nu}$ are the scalar field and background spacetime metric given in Eqs. (7) and (8), while $\delta\Phi$ is perturbed scalar field is

$$\delta\Phi = e^{-i\omega t} F(r) P_\ell(\cos\theta)\ , \tag{15}$$

and perturbed metric tensor components are defined as

$$h_{\mu\nu} = e^{-i\omega t}\begin{pmatrix} H_0 & H_1 & 0 & 0 \\ H_1 & H_2 & 0 & 0 \\ 0 & 0 & K(r^2+r_0^2) & 0 \\ 0 & 0 & 0 & K(r^2+r_0^2)\sin^2\theta \end{pmatrix} P_\ell(\cos\theta)\ , \tag{16}$$

where $H_0(r)$, $H_1(r)$, $H_2(r)$, $K(r)$ and $F(r)$ are unknown radial functions and $P_\ell = P_\ell(\cos\theta)$ is the Legendre polynomial which satisfies the following equation:

$$\frac{1}{\sin\theta}\frac{d}{d\theta}(\sin\theta\frac{d}{d\theta})P_\ell + \ell(\ell+1)P_\ell = 0\ . \tag{17}$$

We discuss the wave solution of the field equations for radial functions. Before going further we introduce a dimensionless frequency which is defined as $\omega_0 = \omega_{r0}$. The equation for the function F can be easily derived as

$$\left[\left(x^2+1\right)F'\right]'+\left[\omega_0^2x^2+\frac{4}{x^2+1}-4\eta\right]F=0\ , \tag{18}$$

where $\eta = \ell^2 + \ell - \omega_0^2$ and the analytical solution to the above equation can be presented in terms of the confluent Heun function (i.e. Heun*C(a, b, c, d, x)*) as follows:

$$F(r)=\left(x^2+1\right)\left[c_1F_{1\ell}(x)+c_2xF_{2\ell}(x)\right]\ , \tag{19}$$

where

$$F_{1\ell}(x)=\mathrm{HeunC}\left[\eta-\frac{3}{2},-\frac{\omega_0^2}{4},\frac{1}{2},3,0,-x^2\right]\ , \tag{20}$$

$$F_{2\ell}(x)=\mathrm{HeunC}\left[\eta-3,-\frac{\omega_0^2}{4},\frac{3}{2},3,0,-x^2\right]\ . \tag{21}$$

As we found that in the Ellis spacetime, the solution to scalar perturbation is described by the analytical expression, namely, confluent Heun function. The radial dependence of the function F (x) is shown in Fig. 11. The radial profile function F (x) oscillates while simultaneously decreasing.

We have explored scalar and gravitational perturbations in the Ellis spacetime. We assume that both scalar gravitational waves propagate at identical frequencies and expressions for these are expanded in terms of the spherical harmonics. It is shown that the equation for the scalar profile function is totally independent from the tensor profile functions, however equations for the tensor profile functions strongly depend on the scalar profile functions in the Ellis spacetime. We have discovered that time-independent solutions for scalar and gravitational disturbances can be expressed using Legendre and associated Legendre functions, where the argument is complex. However, when stationary solutions within the wave zone are considered, the exact analytical solution for scalar disturbances can be achieved, described by the confluent Heun function. It’s noteworthy that the equations governing gravitational disturbances are considerably intricate, but they can be simplified to the familiar Regge–Wheeler–Zerilli equation for the tensor profile function. Finally, we present numerical solutions to the Regge–Wheeler–Zerilli equation for the radial functions.

In the fourth chapter, entitled "Long-lived quasinormal modes and asymptotic tails of regular Schwarzschild-like black holes in the presence of a magnetic field" we first provided an overview of Schwarzschild-like spacetime and discussed the associated magnetic field configuration. We also analyzed the dynamics of a charged scalar field within this framework. We have presented the numerical results obtained using both the WKB approximation and the time-domain analysis.

The gravitational field of a Schwarzschild-like compact object in Boyer–Lindquist coordinates can be expressed through the following line element:

$$ds^2 = -f\,dt^2 + f^{-1}\,dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\phi^2\right)\,, \tag{22}$$

where the metric function f is defined as: (23)

$$f = 1 - \frac{2Me^{-a/r}}{r}\,.$$

In this context, the parameter M represents the mass of the black hole, while a is a deviation parameter introduced by Simpson and Visser. It is important to note that the spacetime metric described above corresponds to the standard Schwarzschild black hole in general relativity when a → 0.

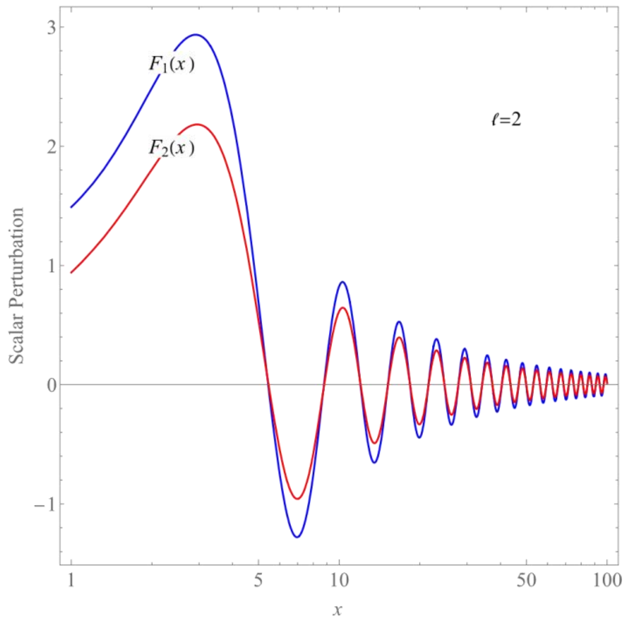

Figure 11: Radial dependence of the scalar perturbation at ℓ = 2.

The relativistic Klein-Gordon equation for a massive, charged scalar field Ψ, in the presence of an electromagnetic field, is expressed as follows:

$$g^{\alpha\beta}(\Delta_\alpha - iqA_\alpha)(\Delta_\beta - iqA_\beta)\Psi - \mu^2\Psi = 0\,, \tag{24}$$

here, μ represents the mass of the scalar field, q is the charge coupling constant between the scalar and electromagnetic fields,∇α denotes the covariant derivative and i is imaginary number. Although separating variables in equation (23) is quite challenging, we can simplify the problem by applying the following physically reasonable assumptions:

- Lorentz gauge condition for the vector potential: ∇αAα = 0;

- In the weak interaction limit, higher-order terms, such as q^2B^2, can be neglected, i.e., $q^2B^2 \rightarrow 0$. Then, eq. (24) becomes

$$\frac{1}{\sqrt{-g}}\partial_\alpha(\sqrt{-g}g^{\alpha\beta}\partial_\beta\Psi) - 2iqA^\alpha\partial_\alpha\Psi - \mu^2\Psi = 0 \ , \tag{25}$$

We can write the solution as:

$$\Psi(t,r,\theta,\phi) = e^{-i\omega t}Y_{lm}(\theta,\phi)\frac{R(r)}{r}. \tag{26}$$

We briefly present the results of numerical calculations of the quasinormal modes for a Schwarzschild-like black hole immersed in an external asymptotically uniform magnetic field. For frequency-domain analysis, we employ the semianalytical WKB method. This method involves expanding the solution at both infinities in a WKB series and matching these asymptotic expansions with a Taylor series near the peak of the effective potential.

The higher order WKB formula is given by

$$\begin{aligned}\omega^2 &= V_0 + A_2(K^2) + A_4(K^2) + A_6(K^2) + \dots \\ &- iK\left(-2V_1 + A_3(K^2) + A_5(K^2) + A_7(K^2) + \dots\right)\end{aligned} \tag{27}$$

where $K = n + 1/2$, with $n = 0, 1, 2, 3, \dots$.

Fig.12 illustrates the best-fit polynomial function for data obtained using the WKB method, applied to a massless scalar field in an external magnetic field. The figure shows two curves, gray and black, corresponding to different spacetime parameter values (a/M = 0.3 and a/M = 0.5, respectively). The imaginary parts of the quasinormal mode frequency ω for these parameter values are shown on the righthand side. Each curve intersects the zero value of qB at approximately 0.611711 (gray line) and 0.651842 (black line), respectively.

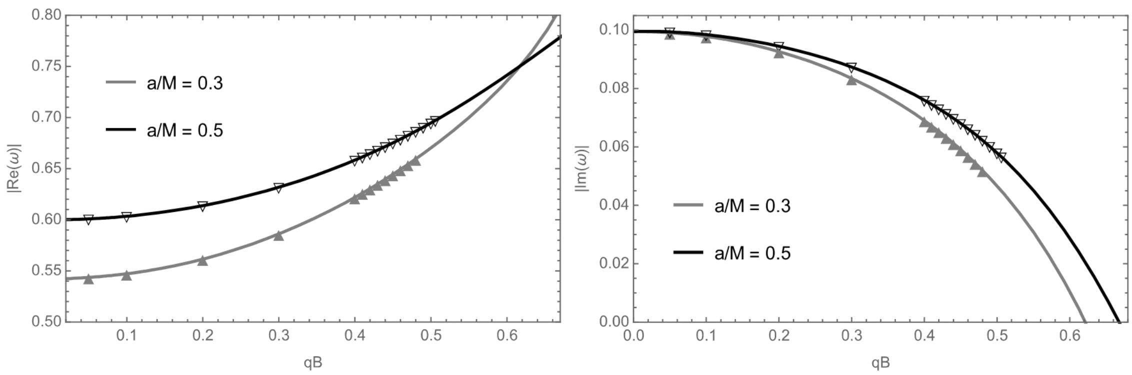

Figure 12: Best fitting polynomial functions of WKB data for different values of spacetime values (l = 2, m = −2).

The WKB data is checked here by time-domain integration. However, two aspects must be taken into account for such a comparison. First of all, the period of quasinormal ringing is very short for $\ell = 0$ perturbations because it is quickly followed by oscillatory power-law tails, as shown in Fig. 14. Therefore, it is difficult to extract the frequency from the time-domain profile with sufficient accuracy in this case. The second aspect is the growth of the perturbation at late times Fig. 13. One could think that this means an instability caused by the negative value of the effective potential at large r when m > 0.

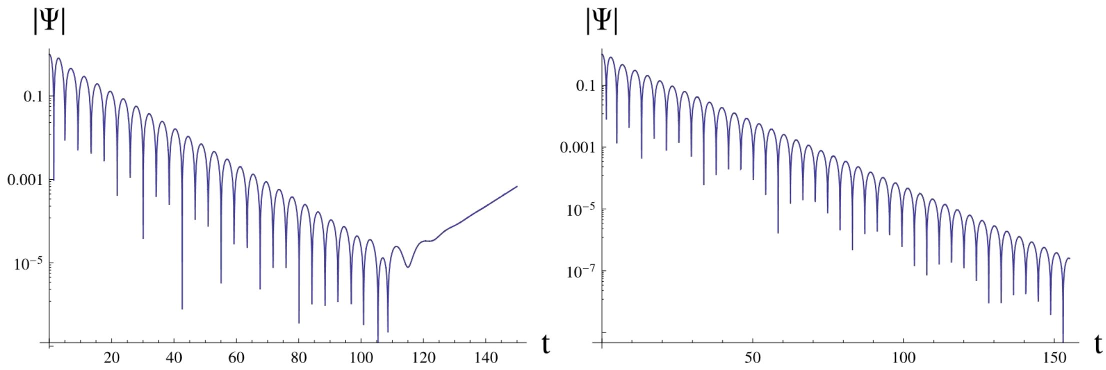

Figure 13: Semi-logarithmic plots of time-domain profiles for m = 3 (left) and m = −3 (right) perturbations; l = 3, qB = 0.1, a = 0.3, μ = 0.1, M = 1. The instability at late times for m > 0 is the artifact of the approximation for the effective potential which is valid only until some distance from the black hole. At the ringdown phase the WKB data is reproduced with high accuracy

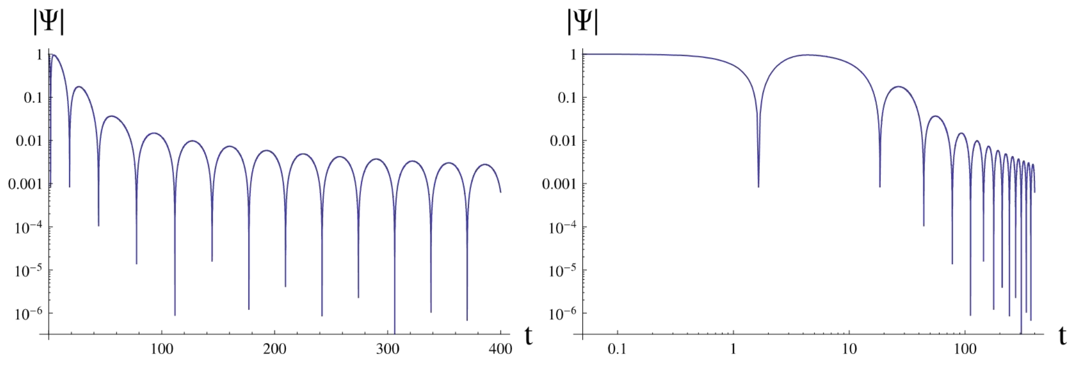

Figure 14: Semi-logarithmic plots of time-domain profiles (left) and logarithmic plot (right); l = m = 0, qB = 0.1, a = 0.3, μ = 0.1, M = 1. The short period of quasinormal ringing is quickly changed by an asymptotic tails, making it unable to extract the frequency from the profile with sufficient accuracy

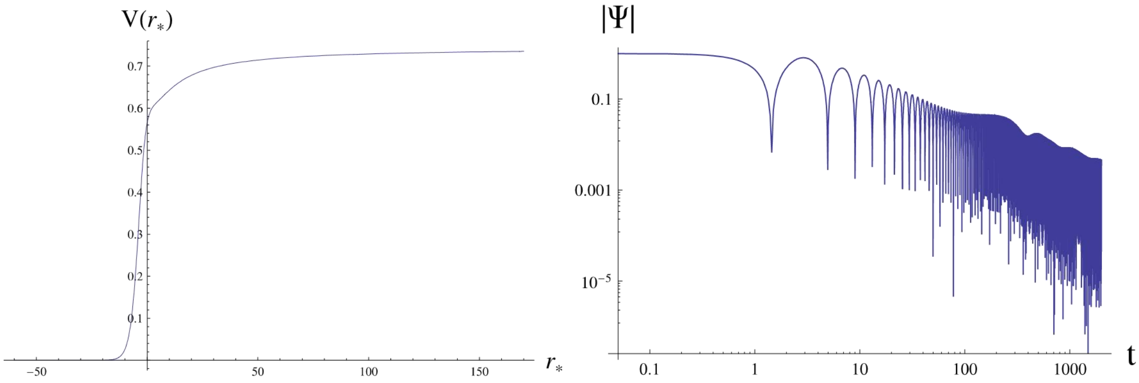

Figure 15: Effective potential and logarithmic plots of time-domain profiles for m = −2 perturbations; l = 2, qB = 0.61, a = 0.3, μ = 0, M = 1

While the quasinormal modes of regular black holes have been extensively studied in numerous works, no comprehensive investigations have been conducted for regular black holes in the presence of an external magnetic field. Here, we address this gap and demonstrate that the magnetic field significantly alters the spectrum of a charged scalar field, leading to the emergence of arbitrarily long-lived quasinormal modes, known as quasi-resonances.

Another distinctive feature of the evolution of perturbations in the presence of a magnetic field is the unusual behavior of asymptotic tails. For certain parameter values, these tails do not exhibit a power-law envelope but instead display an oscillatory envelope.

CONCLUSION

The following conclusions were presented on the basis of research carried out on the topic of "**Relativistic Astrophysics of Compact Objects Coupled with Scalar Field**" for the Doctor of Philosophy (PhD) dissertation:

1. It has been shown that the Janis-Newman-Winicour (JNW) naked singularity spacetime exhibits a repulsive scalar field effect, altering the innermost stable circular orbit (ISCO) radius nonmonotonically with the scalar parameter n. Unlike Schwarzschild black holes, the ISCO vanishes for critical scalar strengths, indicating a breakdown of stable orbits near the singularity.
2. For the first time, analytical expressions for the effective potential, specific energy, and angular momentum of massive particles in the JNW spacetime have been derived, revealing that scalar coupling g_s reduces (increases) the ISCO for positive (negative) values. The radiative efficiency of the accretion disks in this space-time was found to exceed the Schwarzschild predictions by up to 12%.
3. It has been demonstrated that Ellis' throat wormhole r_0 and scalar coupling g_s govern marginally stable orbits, with particle trajectories exhibiting escape paths under radiation reaction, a stark contrast to black hole capture. The fundamental frequencies of oscillatory motion were computed, showing distinct shifts observable in quasiperiodic oscillations (QPOs).
4. It has been found that charged particles in magnetized Ellis wormhole spacetimes follow split fundamental frequencies due to Lorentz forces, with quasi-resonant modes emerging at critical magnetic field strengths. The radiation intensity was shown to depend on both scalar and electromagnetic interactions.
5. For the first time, Markov Chain Monte Carlo (MCMC) analysis was applied to constrain the JNW spacetime parameters (M, g_s, n) using binary QPO Xray data (e.g. GRO J1655-40), yielding best-fit values consistent with weak scalar dominance ($n \sim 0.75$, $g_s \sim 0.2$).
6. It has been shown that scalar perturbations in Ellis wormhole spacetime admit exact solutions in terms of confluent Heun functions, while gravitational perturbations obey a Regge-Wheeler-type equation with unusual oscillatory tails, which varies from power-law decay in black holes.
7. The quasinormal modes of Schwarzschild-like black holes in external magnetic fields were computed via WKB and time-domain methods, revealing long-lived quasiresonances and anomalous late-time tails. These signatures could distinguish scalar-modified spacetimes in future gravitational wave observations.

**НАУЧНЫЙ СОВЕТ DSc.03/07.07.2025. FM/T.192.01
ПО ПРИСУЖДЕНИЮ УЧЕНЫХ СТЕПЕНЕЙ ПРИ ИНСТИТУТЕ ПЕРСПЕКТИВНЫХ ИССЛЕДОВАНИЙ УНИВЕРСИТЕТА "НОВЫЙ УЗБЕКИСТАН"**

**АСТРОНОМИЧЕСКИЙ ИНСТИТУТ
УНИВЕРСИТЕТ "НОВЫЙ УЗБЕКИСТАН"**

ДАВЛАТАЛИЕВ АКБАРЖОН АКМАЛЖОН ЎҒЛИ

РЕЛЯТИВИСТСКИЕ АСТРОФИЗИЧЕСКИЕ ПРОЦЕССЫ В ОКРЕСТНОСТИ КОМПАКТНЫХ ОБЪЕКТОВ ПРИ НАЛИЧИИ СКАЛЯРНОГО ПОЛЯ

**01.03.01-Астрономия
01.04.02-Теоретическая физика**

**АВТОРЕФЕРАТ ДИССЕРТАЦИИ ДОКТОРА ФИЛОСОФИИ (PhD)
ПО ФИЗИКО-МАТЕМАТИЧЕСКИМ НАУКАМ**

Ташкент – 2025

Тема диссертации доктора философии (PhD) по физико-математическим наукам зарегистрирована в Высшей аттестационной комиссии при Министерстве высшего образования науки и инноваций Республики Узбекистан за номером № B2025.2.PhD/FM1325

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Автореферат диссертации на трех языках (узбекский, английский, русский (резюме)) размещен на веб-странице Научного совета (ias.newuu.uz) и Информационно-образовательном портале «Ziyonet» (www.ziyonet.uz).

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Ведущая организация:	Казахский национальный университет имени Аль-Фараби

Защита диссертации состоится «__» ________ 2025 года в ___ часов на заседании Научного совета DSc.03/07.07.2025.FM/T.10.04 при Институте перспективных исследований Университета "Новый Узбекистан" Адрес: 100007, город Ташкент, Мирзо Улугбекский район, улица Мовароуннахр, дом 1, Институте перспективных исследований университета "Новый Узбекистан", Тел.: +998971202411;)

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ВВЕДЕНИЕ (аннотация диссертации доктора философии (PhD))

Цель исследования — разработка новой теоретической модели для изучения скалярных полей и выявления новых данных о динамике частиц вблизи компактных объектов.

Задачи исследования:

- разработать теоретическую модель, связывающую скалярные поля с динамикой частиц в пространствах JNW (Janis-Newman-Winicour) и Эллиса;
- вывести уравнения движения массивных частиц под влиянием скалярного поля;
- проанализировать модификации устойчивых круговых орбит (ISCO) изза параметров скалярной связи;
- исследовать влияние радиационного трения на траектории частиц вблизи голых сингулярностей;
- вычислить фундаментальные частоты (орбитальную, радиальную, вертикальную) для применения в анализе квазипериодических осцилляций (QPO);
- провести MCMC (Markov chain Monte Carlo) анализ для ограничения параметров пространства-времени с использованием данных рентгеновских двойных систем;
- изучить динамику заряженных частиц в намагниченных пространствах кротовых нор и чёрных дыр;
- исследовать спектры возмущений (квазинормальные моды, затухающие «хвосты») в скалярно-модифицированных геометриях.

Объектом исследования являются — Пространство-время голой сингулярности Яниса–Ньюмена–Виникура (JNW) со скалярным полем; геометрия кротовой норы Эллиса, взаимодействующая с внешними скалярными полями; регулярные чёрные дыры, аналогичные шварцшильдовским, в присутствии внешних магнитных полей; пробные частицы (нейтральные и заряженные) в этих модифицированных пространственно-временных геометриях.

Предметом исследования являются Динамика массивных частиц под влиянием скалярного поля в экзотических пространствах-временах; изменение орбитальной механики и фундаментальных частот из-за скалярных связей; астрофизические следствия компактных объектов, модифицированных скалярным полем (квазипериодические осцилляции

(QPO), устойчивые круговые орбиты (ISCO) и др.); спектры возмущений и свойства устойчивости компактных объектов со скалярными связями.

Методы исследования включают методы аналитического моделирования для вывода точных уравнений движения и эффективных потенциалов частиц в пространствах-временах со скалярными полями; численного моделирования (интегрирование методом Рунге–Кутты) для решения уравнений геодезических и анализа траекторий частиц с учётом эффектов радиационного трения; полуаналитические методы (WKB-приближение и интегрирование во временной области) для вычисления квазинормальных мод и спектров возмущений; MCMC-анализ для сопоставления теоретических частот квазипериодических осцилляций (QPO) с наблюдательными данными рентгеновских двойных систем..

Научная новизна исследования заключается в следующем:

Впервые разработана полная теоретическая модель динамики частиц в пространстве-времени голой сингулярности JNW при наличии скалярного поля, выявлено уникальное поведение ISCO, отличающее её от чёрных дыр.

Получены новые результаты, демонстрирующие влияние скалярных полей на физику кротовой норы Эллиса, включая уход частиц под действием излучения и характерные частоты QPO, позволяющие отличить экзотические компактные объекты от чёрных дыр.

Впервые применён MCMC анализ для получения ограничительных значений параметров скалярного поля с использованием данных рентгеновских двойных систем, а также выявлены новые спектральные признаки для проверки модифицированных теорий гравитации в сильных полях.

Практические результаты исследования заключаются в следующем:

Полученные аналитические выражения для динамики частиц в скалярномодифицированных пространствах-временах позволяют точно моделировать структуры аккреционных дисков и профили излучения вокруг экзотических компактных объектов.

Рассчитанные частоты QPO и радиусы ISCO дают наблюдаемые признаки для различия голых сингулярностей JNW и кротовых нор Эллиса от чёрных дыр по данным рентгеновских двойных систем.

Определённые методом MCMC параметры скалярного поля предоставляют проверяемые ограничения для будущих исследований гравитационных волн и мультимессенджерной астрофизики.

Выявленные спектры возмущений (например, решения в виде конфлюэнтных функций Хойна, осциллирующие «хвосты») служат эталонами для обнаружения эффектов скалярно-тензорной гравитации в обсерваториях нового поколения..

Достоверность результатов исследования обеспечивается следующими аналитическая часть строго выведена из первых принципов и перепроверена на соответствие известным результатам общей теории относительности, что гарантирует математическую согласованность всех расчётов;

численные симуляции проводились независимыми методами (интегрирование Рунге–Кутты, WKB-приближение), давшими согласующиеся результаты, что подтверждает надёжность анализа динамики частиц и возмущений; оценка параметров методом MCMC проверена тестами на сходимость и сравнением с существующими астрофизическими ограничениями,

демонстрируя статистическую достоверность значений скалярной связи; все теоретические предсказания согласуются с наблюдательными данными рентгеновских двойных систем и известной феноменологией чёрных дыр, одновременно предлагая проверяемые отклонения для будущих исследований..

Научная и практическая значимость результатов исследования:

Научная значимость результатов исследования заключается в том, что исследование создаёт комплексную основу для анализа влияния скалярных полей на экзотические компактные объекты, расширяя понимание взаимодействия гравитации и фундаментальных полей за пределами классической парадигмы чёрных дыр; полученные частоты QPO и модификации ISCO служат ключевыми наблюдательными критериями для различия чёрных дыр, голых сингулярностей и кротовых нор, что применимо для анализа данных современных рентгеновских телескопов.

Практическая значимость результатов исследования состоит в том, что разработанные методы MCMC-QPO анализа и исследования возмущений предоставляют новые инструменты для проверки модифицированных теорий гравитации с использованием астрофизических данных, значительно повышая точность тестов гравитации; выявленные уникальные спектральные признаки скалярно-тензорных взаимодействий помогают определить требования к проектированию детекторов гравитационных волн нового поколения и мультимессенджерных астрономических кампаний.

Внедрение результатов исследования. Разработанные теоретические модели движения фотонов и частиц в теории ModMax были применены к следующим задачам:

Результаты исследования динамики частиц вокруг чёрных дыр в модифицированных теориях гравитации были использованы в следующих работах:

Теоретические результаты и методы, опубликованные в статье «A. Davlataliev, B. Narzilloev, I. Hussain, A. Abdujabbarov, B. Ahmedov, “Probing the Starobinsky-Bel-Robinson gravity by photon motion around the Kerr-type black hole in non-uniform plasma,” // Phys.Dark Univ. 42 (2023) 101340, https://doi.org/10.1016/j.dark.2023.101340»,

— А также представленные в докторской диссертации (PhD) A. Davlataliev, были применены в рамках программ, поддержанных Фуданьским университетом (письмо от проф. Косимо Бамби).

Апробация результатов исследования.

Результаты исследований были представлены на 3 республиканских и 1 международных научных конференциях.

Опубликованность результатов исследования. По результатам исследования опубликовано 11 научных работ, и все 11 из них являются статьями в рецензируемых журналах.

Структура и объем диссертации.

Диссертация состоит из введения, четырех глав, заключения и списка литературы. Объем диссертации 121 страниц.

ВЫВОДЫ

На основе проведенного исследования по теме «**Релятивистские астрофизические процессы в окрестности компактных объектов при наличии скалярного поля**» для диссертации на соискание ученой степени доктора философии (PhD) были сделаны следующие выводы:

1. Показано, что в пространстве-времени голой сингулярности JNW скалярное поле проявляет эффект отталкивания, приводя к немонотонному изменению радиуса ISCO в зависимости от скалярного параметра. В отличие от чёрных дыр Шварцшильда, ISCO исчезает при критических значениях скалярного поля, что указывает на разрушение устойчивых орбит вблизи сингулярности.

2. Впервые получены аналитические выражения для эффективного потенциала, удельной энергии и углового момента массивных частиц в пространстве-времени JNW. Установлено, что скалярный параметр связи g_s уменьшает (увеличивает) ISCO для положительных (отрицательных) значений. Эффективность излучения аккреционных дисков в такой геометрии превышает предсказания модели Шварцшильда на величину до 12%.

3. Доказано, что в кротовой норе Эллиса параметры горловины и скалярной связи определяют границы маргинально устойчивых орбит, причём траектории частиц могут покидать систему под действием радиационного трения, в отличие от неизбежного захвата чёрной дырой. Рассчитаны фундаментальные частоты колебательного движения, демонстрирующие характерные сдвиги, которые могут наблюдаться в квазипериодических осцилляциях (QPO).

4. Обнаружено, что заряженные частицы в намагниченной кротовой норе Эллиса демонстрируют расщепление фундаментальных частот из-за действия силы Лоренца, с возникновением квазирезонансных мод при критических значениях магнитного поля. Интенсивность излучения зависит как от скалярных, так и от электромагнитных взаимодействий.

5. Впервые применён метод MCMC для определения параметров пространства JNW на основе данных о QPO в рентгеновских двойных системах (например, GRO J1655-40). Наилучшее соответствие дало значения, указывающие на слабое доминирование скалярного поля ($n \approx 0{,}75$, $g_s \approx 0{,}2$).

6. Показано, что скалярные возмущения в кротовой норе Эллиса допускают точные решения в виде конфлюэнтных функций Хойна, в то время как гравитационные возмущения описываются уравнением типа Редже–Уилера с необычными осциллирующими "хвостами", отличающимися от степенного затухания в чёрных дырах.

7. Методами WKB и анализа во временной области вычислены квазинормальные моды чёрных дыр шварцшильдовского типа в присутствии внешних магнитных полей. Обнаружены долгоживущие квазирезонансы и аномальные "хвосты" на поздних временах. Эти особенности могут помочь в идентификации скалярно-модифицированных пространств-времён в будущих наблюдениях гравитационных волн.

E’LON QILINGAN ISHLAR RO‘YXATI
LIST OF PUBLISHED WORKS
СПИСОК ОПУБЛИКОВАННЫХ РАБОТ I бўлим (1 часть; part 1)

1. **A. Davlataliev**, B. Narzilloev, I. Hussain, A. Abdujabbarov, B. Ahmedov, “Probing the Starobinsky-Bel-Robinson gravity by photon motion around the Kerr-type black hole in non-uniform plasma,” // Physics of Dark Universe 42 (2023) 101340, (**Web of Science IF:=5.0**, Q1) https://doi.org/10.1016/j.dark.2023.101340
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3. **A. Davlataliev**, B. Narzilloev, I. Hussain, A. Abdujabbarov, Bobomurat Ahmedov, “Charged and magnetized particle motion around a static black hole in the Starobinsky–Bel–Robinson gravity” // The European Physics Journal C 84 (2024) 7, 694. (**Web of Science IF:=4.9**, Q1) https://doi.org/10.1140/epjc/s10052-024-13039-3
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II бўлим (2 часть; part 2)

1. **A. Davlataliev**, A. Abdujabbarov, Sh. Karshiboev. Radiation Properties of the accretion disk around a BH in SBR gravity “Masoili muosiri fizikai nimnokilho va dielektrikho” Dushanbe 30.04.2025.
2. **A. Davlataliev**, P. Davletova, B. Musinov. Charged particle motion around a static black hole in the Starobinsky-Bel-Robinson gravity “Fizka Fanining Rivojida Iste’dodli Yoshlarning o’rni” RIAK-XVII-2024.
3. **A. Davlataliev**. Influence of scalar field in massive particle motion in JNW spacetime ICTPA 2024 International Conference on Theoretical Physics and Astrophysics.

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